On The Scheduling of Leak Test Personnel

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Outline

• Introduction and Motivation
• Reliability Modeling
• Formulation of The Scheduling Problem
• Some Examples
Fugitive Emissions

• Definition of Fugitive Emissions
  Those emissions which could not reasonably pass through a stack, chimney, vent or other functionally equivalent opening.

• Potential Emission Sources
  Any discontinuity in a solid barrier intended to maintain containment
Emission Sources

Potential Leak Sources

- Valve Packings
- Flange
- Pump Seal

Unit Operation
Typical Process Industry Plant
Leak Detection And Repair (LDAR)

LDAR program consists of four activities:

1. Identify equipment to be included in the program
2. Conduct routine testing to identify leaks
3. Repair leaking equipment in a specified time frame
4. Be prepared to report monitoring results
Can LDAR Be Improved?

• Most Programs employ uniform testing intervals (such as quarterly, semiannually).

• There does not exist a systematic scheduling scheme aimed at balancing:

  Program Cost vs. Effectiveness
LDAR Literature


- Siegell (1997) “Improve your Leak Detection and Repair Program.”


None of these focus on leak test scheduling
Maintenance Scheduling

- Similar to the Leak Test Scheduling Problem Applies reliability theory methods to balance program costs with effectiveness

- Example Papers include:
  - Tan and Kramer (1997)
  - Martorell et al., (1999)
  - Usher et al., (1998)
Reliability Modeling

• Review of Basic Definitions
  – The Probability of Failure Concept
  – The Reliability Function
  – The Hazard Rate Parameter

• A Continuous-Time Model

• Proposed Discrete-Time Model
The Probability of Failure

- Probability part will fail between time $t_0$ and $t$

\[
\text{Pr}[t_{\text{fail}} \leq t]
\]

\[
\text{Pr}[t_{\text{fail}} > t]
\]

*note:* $\text{Pr}[t_{\text{fail}} \leq t] = 1 - \text{Pr}[t_{\text{fail}} > t]$
Influence of Test

- Probability part will fail between $t_{test}$ and $t$, given component is not leaking at time $t_{test}$

\[ \Pr[t_{fail} \leq t \mid t_{fail} > t_{test}] \]

\[ \Pr[t_{fail} > t \mid t_{fail} > t_{test}] \]
The Reliability Function

- Reliability: \( \Pr[t_{\text{fail}} > t] \)

- We assume a zeroth order Weibull function

\[
R(t) = e^{-\beta t}
\]

- Conditional Reliability: \( \Pr[t_{\text{fail}} > t \mid t_{\text{fail}} > t_{\text{test}}] \)

\[
R(t / t_{\text{test}}) = e^{-\beta(t - t_{\text{test}})}
\]
The Hazard Rate Parameter

\[ R(t) = e^{-\beta t} \]

- \( \beta \) has units of 1/time
- \( \frac{1}{\beta} \) is the Expected Life of the Part
- Large \( \beta \) : Low Quality Part
- Small \( \beta \) : High Quality Part
Differential Equation Model

• The Conditional Reliability will satisfy:

\[
\frac{dR}{dt} = -\beta R(t) + [1 - R(t)]U(t)
\]

where

\[
U(t) = \sum_{l=1}^{l} \delta(t - t_{test}^l)
\]

\[t_{test}^l\] is the test time and \[\delta(t)\] is the direct delta function.
Continuous-Time Representation

\[ R(t) \]

\( k \)
Discrete-Time Model

\[ r_i(k + 1) = \begin{cases} 
  a_i(k)r_i(k) & \text{if } u_i(k) = 0 \\
  a_i(k) & \text{if } u_i(k) = 1 
\end{cases} \]

where

\[ a_i(k) = e^{-\beta_i \Delta t_s} \quad ; \beta_i \text{ is the hazard rate for part } i \]

\[ u_i(k) = \begin{cases} 
  1 & \text{if test on part } i \text{ at time } k\Delta t_s \\
  0 & \text{if no test on part } i \text{ at time } k\Delta t_s 
\end{cases} \]
Formulation of the Scheduling Problem

- Total Cost of Detection Program
- Definition of Effectiveness: Leak Fraction
- Optimal Scheduling Problem Statement
Cost of the Detection Program

- Total testing cost is calculated as

\[ \sum_{k=1}^{N} \sum_{i=1}^{n_p} c_i u_i(k) \]

where

- \( c_i \) is the Cost of a single test on part \( i \) at time \( k \)
- \( n_p \) is the number of parts
- \( N \) is the number of test intervals (schedule horizon)
Leak Fraction

- **Expected Leak Fraction:** \( \frac{\text{# of leaking parts}}{\text{Total # parts}} \)

\[
f_L(k) = \frac{1}{n_p} \sum_{i=1}^{n_p} \Pr[t_{\text{fail}}^i < k\Delta t_s] = \frac{1}{n_p} \sum_{i=1}^{n_p} [1 - r_i(k)]
\]

- **Leak Fraction Bound** \( \bar{f}_L \)

\[
f_L(k) < \bar{f}_L \text{ for all } k
\]
Optimal Scheduling

\[
\min \left\{ \sum_{k=0}^{N-1} \sum_{i=1}^{n_p} c_i u_i(k) \right\}
\begin{aligned}
0 \leq r_i(k) \leq 1 \\
u_i(k) \in \{0,1\}
\end{aligned}
\]

Minimum Cost

subject to

\[
r_i(k+1) = \begin{cases} 
    a_i(k)r_i(k) & \text{if } u_i(k) = 0, \\
    a_i(k) & \text{if } u_i(k) = 1,
\end{cases}
\]

Reliability Model

\[
\frac{1}{n_p} \sum_{i=1}^{n_p} (1 - r_i(k)) \leq f_L
\]

Leak Fraction Bound
Solution Methods

- Branch and Bound Algorithm
  - Conversion of MINLP to MILP
    - Nonlinear region expanded to a linear feasible region
    - However, still computationally challenging

- Suboptimal Schedules
  - Heuristic based scheduling Algorithm
    - Expected leak fraction is below the bound but may not be cost optimal
Example I

- Number of Parts: $n_p = 4$
- Time-Invariant Hazard Rates (1/years)
  \[ \beta_1 = 1/5, \quad \beta_2 = 1/10, \quad \beta_3 = \beta_4 = 1/20 \]
- Leak Fraction Bound: $\bar{f}_L = 0.005$ (0.5%)  
- Sampled Period: $\Delta t_s = 1/52$ years
- Schedule Horizon: $N = 5$ ($N\Delta t_s = 35$ days)
Reliability vs. Time
Heuristic Schedule (5 tests total)

![Graph showing reliability over time for different parts with varying beta values.]
Reliability vs. Time
Optimal Schedule (4 tests total)
Expected Leak Fraction
Optimal vs. Heuristic

![Graph showing expected leak fraction over time for heuristic and optimal schedules.](image-url)
Example II

- Number of Parts: \( n_p = 50 \)
- Hazard Rates: Randomly Selected
  \[ \beta_i \in [1/10 - 1/20] \]
- Sampled Period: \( \Delta t_s = 1 \text{ day} \)
- Schedule Horizon: \( N = 300 \)
Expected Leak Fraction

\[ \bar{f}_L = 0.5, 1.0, 1.5\% \]
Leak Fraction Bound vs. Number of Tests

Graph showing the relationship between reliability and time for different leak fraction bounds:
- $f_L = 0.5\%$
- $f_L = 1.0\%$
- $f_L = 1.5\%$
Non-Ideal Testing
Testing Quality

• Most tests are not perfect

• New Reliability Model has

\[ r_i(k+1) = a_i(k) \left[ 1 - (1 - r_i(k))(1 - q) \right] u_i(k) \]

• Where 0 ≤ q ≤ 1 is the test quality
Testing Quality vs. Number of Tests
Additional Generalizations

• Personnel Availability
  – Limits on the number of tests per interval

• Time-Varying Hazard Rates

\[ a_i(k) = \exp\left(-\int_{k\Delta t_s}^{(k+1)\Delta t_s} \beta_i(\tau) d\tau\right) \]

A function of time but still a constant with respect to the optimization
Availability

• Example: one person can do 30 tests/day and we have 200 parts to check/week

• Solution – hire more personnel or put a constraint on the scheduling problem

Total number of parts tested/week $\leq$ available personnel
Conclusion

• We have developed a cost vs. risk model

• Also using sub-optimal methods we can generate schedules of industrially relevant size.
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