Identifying Sensor Performance Targets from a System Perspective

2002 AIChE Annual Meeting
Indianapolis, Indiana

Jui-Kun Peng
Donald J. Chmielewski

Department of Chemical and Environmental Engineering
Illinois Institute of Technology
Outline

• Goals of sensor-technology developers

• Closed-loop perspective on sensor selection

• Sensor development performance targets from a closed-loop perspective

• PFR example
Goals of Sensor-Technology Developers

- What are the performance needs?
- What is the most appropriate price?
Closed-Loop Perspective

- Sensor Types
- Sensor Placement
- The Number of Sensors
Sensor Selection in Dynamic Systems

Min \{ \text{Total Instrument Cost} \}

Subject to

\{ \text{System Performance Requirements} \}
PFR Example

Steady State Profiles:

Temperature (K):
- 0 0.5 1 1.5 2 2.5 3 3.5 4
- 700 750 800 850 900

Reactor Length (m):
- 0 0.5 1 1.5 2 2.5 3 3.5 4

Concentration (kmol/m$^3$):
- 0 0.2 0.4 0.6 0.8
- 1
Covariance Analysis

(Open-Loop)

- Process Model:
  \[ \dot{x}(t) = Ax(t) + Gw(t) \]
  \[ z(t) = Cx(t) \]

- Disturbance Size:
  \[ \Sigma_w \]

- Steady State Covariance of State Vector:
  \[ A\Sigma_x + \Sigma_x A^T + G\Sigma_w G^T = 0 \]

- Steady State Covariance of the Output Vector:
  \[ \Sigma_z = C\Sigma_x C^T \]
Open Loop Covariance

![Graph showing reactor length vs. temperature with two lines: one for steady state profile and another for open loop variances. The temperature range is from 650 K to 1000 K, and the reactor length is from 0 m to 4 m.]
Open Loop Covariance

![Graph showing concentration vs. reactor length for steady state profile and open loop variances.]

- Steady State Profile
- Open Loop Variances
Closed-Loop System
(Perfect Measurements)
Closed-Loop Design

*Perfect Measurements*

- Closed-loop Process
  \[
  \dot{x} = (A - BL)x(t) + Gw(t)
  \]

- Does there exist a matrix \( L \) s. t.
  \[
  \phi_i \Sigma_x \phi_i^T < \bar{x}_i^2
  \]
  \[
  \phi_i L \Sigma_x L \phi_i^T < \bar{u}_i^2
  \]

  Where, \((A - BL)\Sigma_x + \Sigma_x (A - BL)^T + G \Sigma_w G^T = 0\)

  And \( \phi_i \) is the \( i^{th} \) row of the identity matrix
• Actuators at places 2 and 4
Closed-Loop Covariance
(Perfect Measurements)

- Actuators at places 2 and 4
Closed-Loop System (Imperfect Measurement)

Actuator System → Actuator Constraints → Base Process → Measurement System

Disturbance Inputs
Sensor Noise

Compensator

$u(t) \rightarrow -L \rightarrow \hat{x}(t) \rightarrow \text{State Estimator} \rightarrow y(t)$

Output Performance Specifications
Optimal State Estimation

• Process Model:
  \[ \dot{x} = Ax + Bu + Gw \quad w \sim \Sigma_w \]
  \[ y = Cx + v \quad v \sim \Sigma_v \]

• Optimal Estimator:
  \[ \dot{\hat{x}} = A\hat{x} + Bu + \Sigma_e C^T \Sigma_v^{-1} (y - C\hat{x}) \]
  where \( \Sigma_\hat{x} \) satisfies the equation:
  \[ (A - BL)\Sigma_\hat{x} + \Sigma_\hat{x} (A - BL)^T + \Sigma_e C^T \Sigma_v^{-1} C \Sigma_e = 0 \]

• Error System:
  \[ \dot{e} = (A - \Sigma_e C^T \Sigma_v^{-1} C)e + Gw + \Sigma_e C^T \Sigma_v^{-1} v \]
  where \( \Sigma_e \) satisfies the Riccati equation:
  \[ A \Sigma_e + \Sigma_e A^T + G \Sigma_w G^T - \Sigma_e C^T \Sigma_v^{-1} C \Sigma_e = 0 \]

• Separation Principle:
  \[ \Sigma_x = \Sigma_\hat{x} + \Sigma_e \]
How Sensor Placement Effects the Closed-loop Performance

- Actuators at places 2 and 4
Systematic Sensor Selection Scheme

\[
\min_{\gamma_i \in \{0,1\}} \sum \{ \beta_i \gamma_i \}
\]

- \( \exists L, \Sigma \hat{x} \) s.t.

\[
\phi_i \Sigma \hat{x} \phi_i^T < \bar{x}^2
\]

\[
\phi_i L \Sigma \hat{x} L^T \phi_i^T < \bar{u}^2
\]

\[
A \Sigma_e + \Sigma_e A^T + G \Sigma_w G^T - \Sigma_e C^T \Sigma_v^{-1} C \Sigma_e = 0
\]

\[
(A - BL) \Sigma \hat{x} + \Sigma \hat{x} (A - BL)^T + \Sigma_e C^T \Sigma_v^{-1} C \Sigma_e = 0
\]
Problem Setup

- Disturbance Size: \( \Sigma_w = 0.05 \)

- Bounds on State Deviation \( (\bar{x}_i) \)

<table>
<thead>
<tr>
<th>Sensor Position</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reactor Temp. (K)</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Concentration (kmol/m³)</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Jacket Temp. (K)</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
</tbody>
</table>

- Sensor Noise Deviation \( (\bar{\sigma}_{v_i}) \)

<table>
<thead>
<tr>
<th>Temp. Sensor</th>
<th>Deviation</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type1</td>
<td>3</td>
<td>$3</td>
</tr>
</tbody>
</table>
Bounds on Closed Loop Performance

Steady State Profile
Open Loop Variance
Variance Bounds
Sensor Selection Results
(Only for Temperature Sensors)

- Temp. Sensor Type1: \( \overline{\sigma_v} = 3 \)

<table>
<thead>
<tr>
<th>Sensor Position</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>Total No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sol. #1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Sol. #2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Sol. #3</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Sol. #4</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Sol. #5</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Sol. #6</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

Minimum Total Cost = $6
Sensor Selection Results

- Actuators at places 2 and 4
The Strategies for Sensor Development

Original Sensors

Develop the same type of sensors

Better Performance & Higher Price
- Temp. Sensor ($\sigma=2$)

Less Performance & Less Price
- Temp. Sensor ($\sigma=5$)

Develop the different type of sensors

Temp. Sensor ($\sigma=3$)

Composition Analyzer
Postulated New Sensor Qualities

- Sensor Noise Deviation $\left( \overline{\sigma}_{v_i} \right)$

<table>
<thead>
<tr>
<th>Temperature Sensors</th>
<th>Type1</th>
<th>Type2</th>
<th>Type3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation</td>
<td>3</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>
Sensor Selection Results
(for higher quality sensors)

- for Temp. Sensor Type 2: \( (\bar{v}_i = 2) \)

<table>
<thead>
<tr>
<th>Sensor Position</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>So1. #1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>So1. #2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
## Maximum Cost to Compete

<table>
<thead>
<tr>
<th></th>
<th>Total Number</th>
<th>Total Min. Cost</th>
<th>One Sensor Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temp. Sensor Type 1 ( \bar{v}_i = 3 )</td>
<td>2</td>
<td>$6</td>
<td>$3</td>
</tr>
<tr>
<td>Temp. Sensor Type 2 ( \bar{v}_i = 2 )</td>
<td>1</td>
<td>( \leq $6 )</td>
<td>( \leq $6 )</td>
</tr>
</tbody>
</table>
Sensor Selection Results
(for lower quality sensors)

- for Temp. Sensor Type 3: \( \overline{\sigma}_{i} = 5 \)

<table>
<thead>
<tr>
<th>Sensor Position</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sol. #1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total No.</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
# Maximum Cost to Compete

<table>
<thead>
<tr>
<th></th>
<th>Total Number</th>
<th>Total Min. Cost</th>
<th>One Sensor Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Temp. Sensor Type 1</strong></td>
<td>2</td>
<td>$6</td>
<td>$3</td>
</tr>
<tr>
<td>((\bar{\sigma}_v = 3))</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Total Number</th>
<th>Total Sensor Cost</th>
<th>One Sensor Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Temp. Sensor Type 3</strong></td>
<td>3</td>
<td>(\leq 6)</td>
<td>(\leq 2)</td>
</tr>
<tr>
<td>((\bar{\sigma}_v = 5))</td>
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<td></td>
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</tr>
</tbody>
</table>
Performance Targets for Composition Analyzer

- Sensor Noise Deviation \( \overline{\sigma_{v_i}} \)

<table>
<thead>
<tr>
<th>Composition Analyzer</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.01</td>
</tr>
</tbody>
</table>
Sensor Selection Results
(for Composition Analyzer)

- for Composition Analyzer: \( \bar{\sigma}_{v_i} = 0.01 \)

<table>
<thead>
<tr>
<th></th>
<th>Sensor Position</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Sol. #1</td>
<td>1</td>
</tr>
</tbody>
</table>
## Maximum Cost to Compete

<table>
<thead>
<tr>
<th></th>
<th>Total Number</th>
<th>Total Min. Cost</th>
<th>One Sensor Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Temp. Sensor Type1</strong></td>
<td>2</td>
<td>$6</td>
<td>$3</td>
</tr>
<tr>
<td>($\bar{\sigma}_{v_i} = 3$)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Total Number</th>
<th>Total Sensor Cost</th>
<th>One Sensor Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Comp. Analyzer</strong></td>
<td>3</td>
<td>$\leq$ $6$</td>
<td>$\leq$ $2$</td>
</tr>
<tr>
<td>($\bar{\sigma}_{v_i} = 0.01$)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Conclusions

• Sensor value is system dependent.
• Closed-loop analysis is required.
• A method was presented to assess the value of candidate new sensors.
Acknowledgements

• Department of Chemical and Environmental Engineering, IIT

• Armour College of Engineering and Science, IIT