Market Responsive Control: 
A Second Order Approach to Economic Based Controller Design

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Volume of Air 
(the Room) 
\(T_{room}, C_{room}\)

Control Variables: 
\(T_{room} \) and \(C_{room}\)

Manipulated Variables: 
\(F_{ref} \) and \(F_{fresh}\)

Heat Leakage 
\(T_{outside}\) measured

Heat Leakage 
\(T_{outside}\) measured

Contaminant Source: \(S\)

Energy Usage 
\(F_{ref}, T_{room}, C_{room}\)

Air Processing Unit 
\(T_{cool} = 20^\circ C\)

Manipulated Variables: 
\(F_{ref} \) and \(F_{fresh}\)

Disturbances: 
\(T_{outside} \) and \(S\)

BOP with 
more profit

EDOR’s due to 
different controller 
tunings

BOP with 
less profit

OSSOP
Outline

• Economic Based Tuning
• Motivating Examples
• Market Responsive Control
• Application to Examples
Performance in Time Series

![Diagram showing a process flow with variables T and F, and a time series graph with T and F over time.]
Performance in Phase Plane
Elliptical Operating Region

\[ T(t) \]

\[ F(t) \]
Elliptical Operating Region

\[ T(t) \]

\[ F(t) \]
Steady-State Operating Line

$T(t)$

$F(t)$
Different Operating Point

$T(t)$

$F(t)$

Increase $F$  
$\rightarrow$ Increased production rate
Requires Retuning of Controller
Economic Based Controller Tuning

\[ T(t) \]

\[ F(t) \]
Profit Control
(Simultaneous BOP and Controller Selection)

EDOR’s due to different controller tunings

BOP with more profit

BOP with less profit

Max Profit

Peng et al. (2005)
HVAC Control

Demand-Controlled Ventilation (DCV) for Indoor Air Quality (IAQ)

Volume of Air (the Room)

$T_{room}, C_{room}$

Control Variables:
$T_{room}$ and $C_{room}$

Manipulated Variables:
$F_{rec}$ and $F_{fresh}$

Disturbances:
$T_{outside}$ and $S_c$

Heat Leakage
($T_{outside}$ measured)

Solid Material
$T_{solid}$

Contaminant Source: $S_c$

Air Processing Unit
($T_{cool} = 20^\circ C$)

$F_{rec}, T_{room}, C_{room}$

$F_{rec}, T_{cool}, C_{room}$

$F_{fresh}, T_{room}, C_{room}$

$F_{fresh}, T_{cool}, C_{fresh}$

$F_{fresh}, T_{outside}, C_{fresh}$

(Energy Usage)

(Fresh Air)

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HVAC Control

Energy Usage of Traditional Controller: 3.16 kW
Energy Usage of Energy Efficient Controller: 2.55 kW
(a reduction of almost 20%).
Outline

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Thermal Energy Storage (TES)

In HVAC systems TES is used for
Load Leveling and to shift usage to Off-Peak Hours
Cyclical pattern with a phase shift of about 3 hours.
Operation of the TES

- Heat Leakage $T_{\text{outside}}$
- Volume of Air (the Room) $T_{\text{room}}$
- Heat from Room
- Heat to Cooler
- Heat to TES Unit
- Cooling Unit
- TES Unit
- Energy Usage

Graph showing:
- Cents per k hr
- Temperature (°C)
- Time (days)

Legend:
- Electricity Price
- Outside Temperature
Integrated Gasification Combined Cycle (IGCC)
Value of Electric Power Generation

Converted Gas Value (cents / m³)

Time (days)
Synthesis Gas Storage

Coal, Oxygen and Steam → Gasification and Gas Cleaning Units → Gas Storage Unit → Energy Conversion Units (Gas Turbines and Electric Generators) → Electric Power
Synthesis Gas Storage

Gasification and Gas Cleaning Units

Energy Conversion Units (Gas Turbines and Electric Generators)

Gas Storage Unit

Coal, Oxygen and Steam

Electric Power

Converted Gas Value (cents / m³)

Time (days)

0 2 4 6 8 10 12 14 16 18 20

0 2 4 6 8 10

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Response to Market Changes

EDOR’s due to different controller tunings

BOP with less profit

BOP with more profit

OSSOP

Peng et al. (2005)
Outline

• Economic Based Tuning
• Motivating Examples
• Market Responsive Control
• Application to Examples
Electric Price Model
Electric Price Model

- White Noise Input
- Measured Electricity Price
- Shaping Filter
- State Estimator and/or Predictor
- Sequence with Electricity Price Characteristics
- Prediction of Electricity Price
Model Predictive Control

\[
\max_{v_p(t)} \left\{ \int_0^T p_e(t) \ast v_p(t) \, dt \right\}
\]

where \( p_e(t) \) ~ the predicted price (or value)
\( v_p(t) \) ~ the velocity of production

and \( S(t) \) ~ amount in storage

Constraints include:

\[
0 \leq v_p(t) \leq v_p^{\max} \quad \text{and} \quad 0 \leq S(t) \leq S^{\max}
\]
Model Predictive Control

\[ \max_{v_p(t)} \left\{ \int_0^T p_e(t) \cdot v_p(t) \, dt \right\} \approx E[p_e \cdot v_p] = R \]

where \( p_e(t) \sim \text{the predicted price (or value)} \)

\( v_p(t) \sim \text{the velocity of production} \)

and \( S(t) \sim \text{amount in storage} \)

Constraints include:

\[ 0 \leq v_p(t) \leq v_{p_{\text{max}}} \quad \text{and} \quad 0 \leq S(t) \leq S_{\text{max}} \]
System Design

\[
\max_{v_p(t)} \left\{ \int_0^T p_e(t) v_p(t) \, dt \right\} \approx E[p_e v_p] \equiv R
\]

How does \( v_p^{\text{max}} \) and \( S^{\text{max}} \) impact \( R \)?
\[
(0 \leq v_p(t) \leq v_p^{\text{max}} \quad \text{and} \quad 0 \leq S(t) \leq S^{\text{max}})
\]
Expected Revenue

White Noise Input

Shaping Filter $p_e(t)$

Manipulated Variables $v_p(t)$

$E[p_e v_p]$

(Controller is $u = Lx$)
Expected Revenue

\[ z \equiv \begin{bmatrix} p_e \\ v_p \end{bmatrix} \quad E[zz^T] = \begin{bmatrix} E[p_e^2] & E[p_e v_p] \\ E[v_p p_e] & E[v_p^2] \end{bmatrix} = \Sigma_z \]
Maximum Expected Revenue

\[ z \equiv \begin{bmatrix} p_e \\ v_p \end{bmatrix} \quad E[zz^T] = \begin{bmatrix} E[p_e^2] & E[p_e v_p] \\ E[v_p p_e] & E[v_p^2] \end{bmatrix} = \Sigma_z \]

\[
\max_L \left\{ \begin{bmatrix} 1 & 0 \end{bmatrix} \Sigma_z \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\} = R
\]
Maximum Expected Revenue

\[ z \equiv \begin{bmatrix} p_e \\ v_p \end{bmatrix} \]

\[ E[zz^T] = \begin{bmatrix} E[p_e^2] & E[p_e v_p] \\ E[v_p p_e] & E[v_p^2] \end{bmatrix} = \Sigma_z \]

\[ \max_L \left\{ \begin{bmatrix} 1 & 0 \end{bmatrix} \Sigma_z \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\} = R \]

Method Failed
Second Attempt

\[ z \equiv p_e - v_p \quad E\left[z^2\right] = E\left[(p_e - v_p)^2\right] < \varepsilon \equiv 0 \]

Then \[ v_p(t) \cong p_e(t) \]
Second Attempt

\[ z \equiv p_e - \nu_p \quad E[z^2] = E[(p_e - \nu_p)^2] < \varepsilon \equiv 0 \]

Then \( \nu_p(t) \equiv p_e(t) \)

Close but still not right
Re-Scaling of Price

\[ \alpha w(t) \rightarrow \text{Shaping Filter} \rightarrow p'(t) \rightarrow \text{Process Model} \rightarrow \nu_p(t) \]

\[ (p'_e \equiv \alpha \ p_e) \]

\[ E[p'_e * \nu_p] \]

Manipulated Variables

(Controller is \( u=Lx \))
Third Attempt

If $E[(p'_e - \nu_p)^2] < \varepsilon \equiv 0$ and $p'_e \equiv \alpha \, p_e$

then $\nu_p(t) \equiv \alpha \, p_e(t)$
Third Attempt

If $E[(p'_e - v_p)^2] < \varepsilon \equiv 0$ and $p'_e \equiv \alpha p_e$

then $v_p(t) \equiv \alpha p_e(t)$

Also, $E[\alpha^2 p_e^2] - 2E[\alpha p_e v_p] + E[v_p^2] \equiv 0$

$\Rightarrow \alpha^2 E[p_e^2] = \alpha E[p_e v_p] = E[v_p^2]$

$\Rightarrow \alpha E[p_e^2] = E[p_e v_p] \equiv R$
Maximum Expected Revenue

\[ R = \max_{L, \alpha} \{ c_R \alpha \} \quad (c_R = E[p_e^2]) \]

\[ E[(p'_e - \nu_p)^2] < \varepsilon \cong 0 \]

\[ E[\nu_p^2] < (\nu_p^{\text{max}})^2 \]

\[ E[S^2] < (S^{\text{max}})^2 \]
Maximum Expected Revenue

\[ R = \max_{L, \alpha} \{ c_R \alpha \} \]

\[ (c_R = E[p_e^2]) \]

\[ E[(p_e' - v_p)^2] < \varepsilon \equiv 0 \]

\[ E[v_p^2] < (v_p^{\text{max}})^2 \]

\[ E[S^2] < (S^{\text{max}})^2 \]

Method Works
Outline

• Economic Based Tuning
• Motivating Examples
• Market Responsive Control
• Application to Examples
IGCC Example
(Small Storage Unit)

Coal, Oxygen, and Steam → Gasification and Gas Cleaning Units → Energy Conversion Units (Gas Turbines and Electric Generators) → Gas Storage Unit

Graphs showing:
- Converted Gas Value (cents / m³) vs. Time (days)
- Gas Volume in Storage (million m³) vs. Time (days)
- Volumetric Flow (million m³ / day) vs. Time (days)
IGCC Example
(Larger Storage Unit)

Coal, Oxygen and Steam → Gasification and Gas Cleaning Units → Energy Conversion Units (Gas Turbines and Electric Generators) → Electric Power → Gas Storage Unit

Graph showing:
- Converted Gas Value (cents / m³) vs. Time (days)
- Gas Volume in Storage (million m³) vs. Time (days)
- Volumetric Flow (million m³ / day) vs. Time (days)
IGCC Example
(Changes in Revenue)

Average Revenue
- Nominal Case: $1.00 million per day (plot not depicted)
- Case 1: $1.04 million per day.
- Case 2: $1.15 million per day.
Thermal Energy Storage
(Small Storage Unit)

Volume of Air (the Room) $T_{room}$

Heat Leakage $T_{outside}$

Heat from Room

Heat to Cooler

Cooling Unit

Energy Usage

Heat to TES Unit

TES Unit

Heat from Room

Heat to Cooler

Heat to TES Unit

Energy Usage

Graph:

- Red: Heat from Room
- Green: Heat to Cooler
- Blue: Heat to TES Unit

KWh/Day vs. Time (days)

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Thermal Energy Storage
(Medium Storage Unit)

Volume of Air (the Room) $V_{room}$

Heat from Room $T_{room}$

Heat to Cooler $T_{cool}$

Cooling Unit

Heat to TES Unit $T_{tes}$

TES Unit

Heat Leakage $T_{outside}$

Energy Usage

Heat from Room

Heat to Cooler

Heat to TES Unit

Graph showing energy usage over time with 79, 60, 61, and 62 days.

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Thermal Energy Storage
(Large Storage Unit)

- Volume of Air (the Room) $T_{room}$
- Heat from Room
- Heat to Cooler
- Cooling Unit
- Energy Usage
- Heat to TES Unit
- TES Unit
- Heat Leakage $T_{outside}$

Graph:
- Heat from Room
- Heat to Cooler
- Heat to TES Unit

Graph: Time (days) vs. kW hr / day

KWs From Room
KWs to Cooler
KWs to TES Unit

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Thermal Energy Storage
(Comparison of Storage Cases)
Thermal Energy Storage
(Revenue Comparisons)

Average Cooling Costs:
One ton: $8 per day
Five tons: $7 per day (14% savings)
Ten tons: $6 per day (25% savings)
Maximum Levelized Revenue

\[
\max_{L, \alpha, \nu_p^{\text{max}}, S_p^{\text{max}}} \left\{ c_R \alpha - c_{L,1} \nu_p^{\text{max}} - c_{L,2} S_p^{\text{max}} \right\}
\]

\[E\left[(p_e' - \nu_p)^2\right] < \varepsilon \equiv 0\]

\[E\left[\nu_p^2\right] < (\nu_p^{\text{max}})^2\]

\[E\left[S^2\right] < (S^{\text{max}})^2\]
Maximum Levelized Revenue

\[
\max_{L, \alpha, v_p^{\text{max}}, S_p^{\text{max}}} \left\{ c_R \alpha - c_{L,1} v_p^{\text{max}} - c_{L,2} S_p^{\text{max}} \right\}
\]

\[
E\left[ (p_e' - v_p)^2 \right] < \varepsilon \equiv 0
\]

\[
E\left[ v_p^2 \right] < \left( v_p^{\text{max}} \right)^2
\]

\[
E\left[ S^2 \right] < \left( S^{\text{max}} \right)^2
\]

Non-Convex Problem
(but global solution can be found)
Conclusions

1. Response to price variations usually requires controller re-tuning and/or a re-selection of set-points.

2. Direct response to price changes can be implemented with Model Predictive Control.

3. Alternatively, a linear controller can be designed for market responsiveness.

4. Convex optimization used for Market Responsive Controller design.

5. Non-convex, but global methods can be used to size and/or select equipment.
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