Economic Based Control System Design

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EDOR’s due to different controller tunings

BOP with less profit

BOP with more profit

OSSOP

\[ x \]

\[ u \]
Outline

• Motivating Example
• Pseudo-Constrained Control
• Back-off and Profit Control
• Market Responsive Control
Motivating Example
(Non-isothermal Reactor)

\[
V \frac{dC_A}{dt} = F(C_{Ain} - C_A) + Vr_A
\]

\[
V \frac{dT}{dt} = F(T_{in} - T) + \left( \frac{V \Delta H}{\rho C_p} \right) r_A
\]

\[
r_A = -k(T)C_A
\]

Increase F  \rightarrow  Increased production rate
Motivating Example
(Non-isothermal Reactor)

Increase $F$ $\rightarrow$ Increased production rate

Decrease $F$ $\rightarrow$ Increase $T$ $\rightarrow$ Increase reaction rate $\rightarrow$ Increase production
Limited Operating Region

Process Limitations:

\[ T(t) \leq T^{(\text{max})} \]  
- Catalyst protection or onset of side reactions

\[ F(t) \leq F^{(\text{max})} \]  
- Pump limit or limit on downstream unit
Limited Operating Region

Process Limitations:

\[ T(t) \leq T^{(\text{max})} \]  - Catalyst protection or onset of side reactions

\[ F(t) \leq F^{(\text{max})} \]  - Pump limit or limit on downstream unit

Possible Controller:

\[ F = K_c (T - T^{(sp)}) + F^{(sp)} \]
Performance in Time Series

$F(t)$

$F^{(sp)}$

$T(t)$

$T^{(sp)}$

$T^{(max)}$

$F^{(max)}$

$F^{(sp)}$

$C_A, T$

$t$

image
Performance in Phase Plane

\[ T(t) \]

\[ F(t) \]
Dynamic Operating Region

\[ T(t) \]

\[ F(t) \]
Dynamic Operating Region

\[ T(t) \]

\[ F(t) \]
Steady-State Relation

Controller:

\[ F = K_c (T - T^{(sp)}) + F^{(sp)} \]

Steady-State Relation:

\[ F^{(sp)} = f (T^{(sp)}) \]
Dynamic Operating Region

\[ T(t) \]

\[ F(t) \]
Steady-State Operating Line

\[ T(t) \]

\[ F(t) \]
Optimal Operating Point

Decrease $F$ → Increase $T$
→ Increase conversion
→ Increase production

$F(t)$

$T(t)$
Optimal Operating Point:
Another Possibility

Increase $F$ → Increased production rate
Optimal Operating Point: Another Possibility

Increase $F$ → Increased production rate
Requires Different Controller Tuning
Less Aggressive Tuning

\[ T(t) \]

\[ F(t) \]

\[ T^{(max)} \]

\[ T^{(sp)} \]

\[ F^{(max)} \]

\[ F^{(sp)} \]

\[ \text{time} \]

\[ \text{time} \]
Connects Controller Design to Plant Economics
Outline

• Motivating Example
• Pseudo-Constrained Control
• Back-off and Profit Control
• Market Responsive Control
Mass-Spring-Damper Example

$r$ is the mass position
$v$ is the velocity
$f$ is the input force (MV) and
$w$ is the disturbance force
Mass-Spring-Damper Example

System Model:

\[
\begin{bmatrix}
\dot{r} \\
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\end{bmatrix} =
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\end{bmatrix}
\begin{bmatrix}
r \\
v
\end{bmatrix} +
\begin{bmatrix}
0 \\
1
\end{bmatrix} f +
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\(r\) is the mass position
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\end{bmatrix} w
\]

\( r \) is the mass position
\( v \) is the velocity
\( f \) is the input force (MV) and
\( w \) is the disturbance force

System Constraints:

\(-1 \leq r \leq 1\)

and

\(0 \leq f \leq 16\)
Mass-Spring-Damper Example

System Model:

\[
\begin{align*}
\dot{r} &= \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} r \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w \\
\dot{v} &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} r \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f + \begin{bmatrix} 0 \\ 0 \end{bmatrix} w
\end{align*}
\]

System Constraints:

\[
\begin{align*}
-1 &\leq r \leq 1 \\
0 &\leq f \leq 16
\end{align*}
\]

\[
\begin{align*}
\dot{x} &= Ax + Bu + Gw \\
z &= Dx \dot{x} + Du u + Dw w
\end{align*}
\]

\[
-\bar{z}_i \leq z_i(t) \leq \bar{z}_i \\
i = 1 \ldots n_z
\]
Covariance Analysis
(Open-Loop Case)

Process Model:

\[
\dot{x} = Ax + Gw
\]
\[
z = Dx
\]

\(w(t)\) Gaussian white noise with covariance \(\Sigma_w\)

Steady State Covariance:

\[
A \Sigma_x + \Sigma_x A^T + G \Sigma_w G^T = 0
\]
\[
\Sigma_z = D \Sigma_x D^T
\]
Expected Dynamic Operating Region (EDOR)

EDOR defined by:

$$\Sigma_z = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 \end{bmatrix}$$
Closed-Loop Covariance Analysis
(Full State Information Case)

Process Model:
\[ \dot{x} = Ax + Bu + Gw \]
\[ z = Dx x + Du u + Dw w \]

Controller:
\[ u(t) = Lx(t) \]

Steady-State Covariance:
\[ (A + BL) \Sigma_x + \Sigma_x (A + BL)^T + G \Sigma_w G^T = 0 \]
\[ \Sigma_z = (D_x + Du L) \Sigma_x (D_x + Du L)^T + Dw \Sigma_w Dw^T \]
Closed-Loop EDOR

EDOR’s from different controllers

\[ u = L_1 x \]
\[ u = L_2 x \]
**Constrained Closed-Loop EDOR**

\[ z_1 \]

\[ z_2 \]

Constraints

\[ (\sigma_{zi} < \bar{z}_i) \]
Pseudo-Constrained Control

\[ \min_{X>0,Y,\xi} \sum_i d_i \xi_i \]

such that:

\[ (AX + BY) + (AX + BY)^T + G\Sigma_w G^T < 0 \]

\[ \begin{bmatrix} \xi_i - \phi_i D_w \Sigma_w D_w^T \phi_i^T & \phi_i (D_x X + D_u Y) \\ (D_x X + D_u Y)^T \phi_i^T & X \end{bmatrix} > 0 \]

\[ \xi_i < \bar{z}_i^2 \quad i = 1 \ldots n_z \]
Controller Equivalence

Theorem 1 (Chmielewski & Manthanwar, 2004):

The controller generated by CMV is coincident with the controller generated by some Unconstrained Model Predictive Controller.
Outline

• Model Predictive Controller Tuning
• Pseudo-Constrained Control
• Back-off and Profit Control
• Market Responsive Control
Constrained Operating Region

CV’s

Constraints

MV’s
Real-Time Optimization

Original Nonlinear Process Model:

\[
\dot{s} = f(s,m,p) \quad q = h(s,m,p)
\]

\[(s,m,p,q) \sim \text{(state, mv, dist, performance)} \sim (x,u,w,z)\]
Real-Time Optimization

Original Nonlinear Process Model:

\[ \dot{s} = f(s, m, p) \quad q = h(s, m, p) \]

\[(s, m, p, q) \sim (\text{state, mv, dist, performance}) \sim (x, u, w, z)\]

Real-Time Optimization (minimize profit loss):

\[ \min_{s, m, q} \{ g(q) \} \quad \text{s.t.} \]

\[ 0 = f(s, m, p) \quad q = h(s, m, p) \quad q_i^{\min} \leq \phi_i q \leq q_i^{\max} \]

RTO solution denoted as \((s^{ossop}, m^{ossop}, p^{ossop}, q^{ossop})\)
Real-Time Optimization

Optimal Steady-State Operating Point (OSSOP)

Constraints

CV’s

MV’s
Backed-off Operating Point (BOP)

Backed-off Operating Point (BOP)

Optimal Steady-State Operating Point (OSSOP)

EDOR

CV’s

MV’s
Stochastic BOP Selection
(Loeblein & Perkins, 1999)

Assume controller $L$ is given and calculate $\xi_i$:

$$(A + BL)\Sigma_x + \sum_x (A + BL)^T + G\Sigma_w G^T = 0$$

$$\Sigma_z = (D_x + D_u L)\Sigma_x (D_x + D_u L)^T + D_w \Sigma_w D_w^T$$

$$\xi_i = \phi_i \Sigma_z \phi_i^T \quad i = 1 \ldots n_z$$

Solve the following Linear Program:

$$\min_{s', m', q'} \left\{ g_q q' \right\} \quad \text{s.t.} \quad 0 = As' + Bm'$$

$$q_i' = \phi_i (D_x s' + D_u m') \quad q_i^\min \leq q'_i \leq q_i^\max$$

$$\xi_i^{1/2} < q_i^{\max} - q_i'$$

$$\xi_i^{1/2} < q_i' - q_i^{\min}$$
Fixed Controller BOP Selection

Loeblein and Perkins (1999):

Controller is fixed $\iff$ EDOR has fixed size and shape
Peng et al. (2005):

Variable Controller BOP Selection

Variable Controller $\iff$ EDOR has variable size and shape
Profit Control
(Simultaneous BOP and Controller Selection)

- EDOR's due to different controller tunings
- BOP with more profit
- BOP with less profit
- Max Profit

Peng et al. (2005)
Profit Control
(Simultaneous BOP and Controller Selection)

\[
\min_{s',m',q', \xi_i, X, Y} \left\{ g_q q' \right\} \quad \text{s.t.} \quad 0 = As' + Bm'
\]

\[
q_i' = \phi_i (D_x s' + D_u m') \quad q_i^{\min} \leq q_i' \leq q_i^{\max}
\]

\[
\xi_i^{1/2} < q_i^{\max} - q_i' \quad \xi_i^{1/2} < q_i' - q_i^{\min}
\]

\[
(AX + BY) + (AX + BY)^T + G \Sigma_w G^T < 0
\]

\[
\begin{bmatrix}
\xi_i - \phi_i D_w \Sigma_w D_w^T \phi_i^T & \phi_i (D_x X + D_u Y) \\
(D_x X + D_u Y)^T \phi_i^T & X
\end{bmatrix} > 0
\]

Peng et al. (2005)
Computational Aspects of Profit Control

\[
\min_{s',m',q'} \left\{ g_{q} q' \right\} \quad \text{s.t.} \quad 0 = As' + Bm' \\
q_i' = \phi_i(D_x s' + D_u m') \\
q_i^{\text{min}} \leq q_i' \leq q_i^{\text{max}} \\
\xi_i^{1/2} < q_i' - q_i'_{\text{max}} \\
\xi_i^{1/2} < q_i' - q_i'_{\text{min}} \\
(AX + BY) + (AX + BL)^T + G \Sigma_w G^T < 0 \\
\begin{bmatrix}
\xi_i - \phi_i D_w \Sigma_w D_w^T \phi_i^T \\
(D_x X + D_u Y)^T \phi_i^T \\
X
\end{bmatrix} > 0
\]

Peng et al. (2005)
Profit Control Applications

- Mechanical Systems
- Chemical and Reaction Systems
- Hybrid Vehicle Design
- Inventory Control
- Electric Power System Design
- Building HVAC
- Water Resource Management
Profit Control Applications

- Mechanical Systems
- Chemical and Reaction Systems
- Hybrid Vehicle Design
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- Electric Power System Design
- Building HVAC
- Water Resource Management
Fluidized Catalytic Cracker

Regenerator and Separator (dynamic):

\[ W \frac{dC_{rgc}}{dt} = F_{cat}(C_{st} - C_{rgc}) - R_{cb} \]
\[ W_{a} \frac{dO_{a}}{dt} = \frac{F_{air}}{M_{air}}(O_{in} - O_{a}) - \frac{1 + 1.5\sigma R_{cb}}{1 + \sigma} \]
\[ W_{c_{pc}} \frac{dT_{reg}}{dt} = F_{cat}c_{pc}T_{st} + F_{air}c_{pair}T_{air} \]
\[ - (F_{cat}c_{pc} + F_{air}c_{pair})T_{reg} - \left(\Delta H_{CO} + \frac{\sigma}{1 + \sigma} \Delta H_{CO_{2}}\right) \frac{R_{cb}}{M_{c}} \]
\[ W_{st} \frac{dc_{at}}{dt} = F_{cat}(C_{sc} - C_{st}) \]
\[ W_{st}c_{pc} \frac{dt_{st}}{dt} = F_{cat}c_{pc}(T_{ro} - T_{st}) \]
\[ T_{cy} = T_{reg} + 5.555O_{a} \]

Riser (pseudo steady state):

\[ \frac{dy_{f}}{dz} = -K_{1}y_{f}[COR]\phi t_{c}, \quad y_{f}(z = 0) = 1 \]
\[ \frac{dy_{g}}{dz} = (K_{2}y_{f}^{2} - K_{3}y_{g})[COR]\phi t_{c}, \quad y_{g}(z = 0) = 0 \]
\[ \frac{d\theta}{dz} = \frac{\Delta H_{f}F_{feed}}{T_{ri}(F_{cat}c_{cp} + F_{feed}c_{pf} + \lambda F_{feed}c_{pc})} \frac{dy_{f}}{dz} \]
\[ \theta(z) = \frac{T(z) - T_{ri}}{T_{ri}}, \quad \theta(z = 0) = 0, \quad T_{ro} = T(z = 1) \]

(adapted from Loeblein & Perkins, 1999)
FCC Constraints and Economics

Process Constraints:

\begin{align*}
400 \, K &\leq T_{st} \leq 1000 \, K \\
600 \, K &\leq T_{reg} \leq 1000 \, K \\
T_{reg} &\leq T_{cy} \leq 1000 \, K \\
100 \frac{kg}{s} &\leq F_{cat} \leq 400 \frac{kg}{s} \\
0 &\leq F_{air} \leq 60 \, kg/s
\end{align*}

Profit Function:

\[
\Phi = 86400 \left( P_{gs}F_{gs} + P_{gl}F_{gl} + P_{ugo}F_{ugo} - P_{uog}F_{Feed} \right)
\]

$F_{gs}$, $F_{gl}$ and $F_{ugo}$ are product flows (gasoline, light gas and unconverted oil).

(adapted from Loeblein & Perkins, 1999)
Profit Control vs. Fixed Controller Back-off

- Regenerator Temp (K)
- Coke Fraction in Separator
- Catalyst Flow (kg/s)
- Fraction of Coke in Regenerator
- Cyclone Temperature (K)
- Separator Temperature (K)
- Inlet Air (kg/s)
- Oxygen Mass Fraction

Graphs showing the comparison of fixed and free controller back-off in terms of various process variables.
### FCC Profit

<table>
<thead>
<tr>
<th></th>
<th>Gross Profit ($/day)</th>
<th>Diff from OSSOP ($/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OSSOP</strong></td>
<td>$36,905</td>
<td>$0.0</td>
</tr>
<tr>
<td><strong>Fixed Control</strong></td>
<td>$34,631</td>
<td>- $2,274</td>
</tr>
<tr>
<td><strong>Profit Control</strong></td>
<td>$35,416</td>
<td>- $1,489</td>
</tr>
</tbody>
</table>

**Improves profit by 2%**
Mass-Spring-Damper Example

System Model:

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\end{bmatrix} f +
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\end{bmatrix} w
\]

where \( r \) is the mass position, \( v \) is the velocity, \( f \) is the input force (MV) and \( w \) is the disturbance force.

System Constraints:

\(-1 \leq r \leq 1 \) and \( 0 \leq f \leq 16 \)
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System Constraints:
\(-1 \leq r \leq 1 \) and \( 0 \leq f \leq 16 \)
Mass-Spring-Damper Example (Phase Plane)

OSSOP

Steady-State Line

Constraints

BOP
Mass-Spring-Damper Example
(Phase Plane Solution)

FSI Case:
Full State Information:
Controller is $u(t) = Lx(t)$

PSI Case:
Partial State Information:
One Velocity Sensor $u(t) = \hat{L}\hat{x}(t)$
Controller is
Mass-Spring-Damper Example
(Impact of Constraints)

Case A
Case B
Case C

Mass Position (m)
Input Force (N)
Discrete-time Simulation
(Scatter Plot)
MPC and the EDOR

\[
\min_{x,u} \left\{ \int_0^\infty \left( x^T Q x + 2u^T M x + u^T R u \right) dt \right\}
\]

s.t. \quad \dot{x} = Ax + Bu + Gw

\[
z(t) = D_x x + D_u u + D_w w
\]

\[
z_{i \text{ min}} \leq z_i(t) \leq z_{i \text{ max}} \quad i = 1 \ldots n_z
\]
Soft Constraints

\[
\min_{x,u} \left\{ \int_{0}^{\infty} \left( x^T Q x + 2 u^T M x + u^T R u \right) dt + s^T \Gamma s \right\}
\]

s.t. \[ \dot{x} = A x + B u + G w \]

\[ z(t) = D_x x + D_u u + D_w w \]

\[ z_i^{\text{min}} - s_i \leq z_i(t) \leq z_i^{\text{max}} + s_i \quad i = 1 \ldots n_z \]

\[ s_i \geq 0 \]
Soft Constraints

\[
\min_{x,u} \left\{ \int_{0}^{\infty} \left( x^T Q x + 2u^T M x + u^T R u \right) dt + s^T \Gamma s \right\}
\]

\[
\text{s.t.} \quad \dot{x} = Ax + Bu + Gw
\]

\[
z(t) = D_x x + D_u u + D_w w
\]

\[
z_{i}^{\text{min}} - s_i \leq z_i(t) \leq z_{i}^{\text{max}} + s_i \quad i = 1 \ldots n_z
\]

\[
s_i \geq 0
\]
MPC with Soft Constraints

\[ \gamma_m = 10^7 \quad \gamma_f = 10^3 \]

\[ \gamma_m = 10^3 \quad \gamma_f = 10^3 \]
Flexibility in EDOR Definition

\( \alpha = 1 \rightarrow \text{constraint observance} \sim 84\% \text{ of time} \)

\( \alpha = 2 \rightarrow \text{constraint observance} \sim 95\% \text{ of time} \)

\( \alpha = 3 \rightarrow \text{constraint observance} \sim 99.5\% \text{ of time} \)
Impact of EDOR Definition

\[ \alpha = 1 \]

\[ \alpha = 2 \]
MPC with Soft Constraints

EDOR = 2 std dev’s

\[ \gamma_m = 10^7 \quad \gamma_f = 10^3 \]

\[ \gamma_m = 10^3 \quad \gamma_f = 10^3 \]
Impact of EDOR Definition
(Reduced Sensitivity to Soft Weights)

\( \gamma_m = 10^7 \quad \gamma_f = 10^3 \)

\( \gamma_m = 10^3 \quad \gamma_f = 10^3 \)
Building HVAC

Volume of Air (the Room)  
\( T_{room}, C_{room} \)

Contaminant Source: \( S_c \)

Solid Material \( T_{solid} \)

Control Variables:  
\( T_{room} \) and \( C_{room} \)

Manipulated Variables:  
\( F_{rcy} \) and \( F_{fresh} \)

Disturbances:  
\( T_{outside} \) and \( S_c \)

Air Processing Unit  
\( (T_{cool} = 20^\circ C) \)

Energy Usage

Heat Leakage  
\( (T_{outside} \text{ measured}) \)

\( F_{rcy}, T_{room}, C_{room} \)

\( F_{rcy}, T_{cool}, C_{room} \)

\( F_{fresh}, T_{room}, C_{room} \)

\( F_{fresh}, T_{cool}, C_{fresh} \)

\( F_{fresh}, T_{outside}, C_{fresh} \)

\( (C_{fresh} = 0) \)
HVAC Control

Energy Usage of Traditional Controller: 3.16 kW
Energy Usage of Energy Efficient Controller: 2.55 kW
(a reduction of almost 20%).
Outline

• Model Predictive Controller Tuning
• Pseudo-Constrained Control
• Back-off and Profit Control
• Market Responsive Control
In HVAC systems TES is used for Load Leveling and to shift usage to Off-Peak Hours.
Energy Prices and Weather

Cyclical pattern with a phase shift of about 3 hours.
Operation of the TES

Heat Leakage $T_{outside}$

Volume of Air (the Room) $T_{room}$

Heat from Room

Heat to Cooler

Cooling Unit

Heat to TES Unit

TES Unit

Energy Usage

Volume of Air

Cooling Unit

Heat to Cooler

Heat from Room

Heat Leakage $T_{outside}$

TES Unit

Energy Usage

Electricity Price

Outside Temperature

Time (days)

Cents per k hr

Temperature (°C)

59 60 61 62

0

10

20

30

40

62
Response to Market Changes

EDOR’s due to different controller tunings

BOP with more profit

BOP with less profit

OSSOP

Peng et al. (2005)
Electric Price Model

White Noise Input ⟷ Shaping Filter ⟷ Sequence with Electricity Price Characteristics
Electric Price Model

White Noise Input → Shaping Filter → Sequence with Electricity Price Characteristics

Measured Electricity Price → State Estimator and/or Predictor → Prediction of Electricity Price
Model Predictive Control

\[
\min_{\nu_u(t)} \left\{ \int_0^T p_e(t) \nu_u(t) \, dt \right\}
\]

where \( p_e(t) \) \( \sim \) the predicted price (or value)

\( \nu_u(t) \) \( \sim \) the velocity of usage

and \( S(t) \) \( \sim \) amount in storage

Constraints include:

\[
0 \leq \nu_u(t) \leq \nu_u^{\text{max}} \quad \text{and} \quad 0 \leq S(t) \leq S^{\text{max}}
\]
Model Predictive Control

\[
\min_{v_u(t)} \left\{ \int_0^T p_e(t) * v_u(t) \, dt \right\} \approx E[ p_e * v_u ] = \bar{C}_e
\]

where \( p_e(t) \) ~ the predicted price (or value)

\( v_u(t) \) ~ the velocity of usage

and \( S(t) \) ~ amount in storage

Constraints include:

\[
0 \leq v_u(t) \leq v_u^{\text{max}} \quad \text{and} \quad 0 \leq S(t) \leq S^{\text{max}}
\]
System Design

\[
\min_{v_u(t)} \left\{ \int_0^T p_e(t) * v_u(t) \, dt \right\} \approx E[p_e * v_u] \equiv \bar{C}_e
\]

How does \( v_u^{\text{max}} \) and \( S^{\text{max}} \) impact \( \bar{C}_e \)?

\( (0 \leq v_u(t) \leq v_u^{\text{max}} \text{ and } 0 \leq S(t) \leq S^{\text{max}}) \)
System Design

\[
\min_{v_u(t)} \left\{ \int_0^T p_e(t) * v_u(t) \, dt \right\} \equiv E[p_e * v_u] \equiv \bar{C}_e
\]

How does \( v_u^{\text{max}} \) and \( S^{\text{max}} \) impact \( \bar{C}_e \)?

\( 0 \leq v_u(t) \leq v_u^{\text{max}} \) and \( 0 \leq S(t) \leq S^{\text{max}} \)

MPC cannot answer this question!
Expected Cost of Electricity

White Noise Input → Shaping Filter → $p_e(t)$ → $E[p_e v_u]$ → $E[p_e* v_u]$

Manipulated Variables (Controller is $u=Lx$) → Process Model → $v_u(t)$
Re-Scaling of Price

\[ p'_e \equiv \alpha \ p_e \]

\[ E[p'_e \nu_u] \]

\[ \alpha w(t) \rightarrow \text{Shaping Filter} \rightarrow p'_e(t) \]

\[ \text{Manipulated Variables} \rightarrow \text{Process Model} \rightarrow \nu_u(t) \]

(Controller is \( u = Lx \))
Correlating Price and Usage

If \( E[ (p'_e - v_u)^2 ] < \varepsilon \cong 0 \) and \( p'_e \equiv \alpha \ p_e \)

then \( v_u (t) \cong \alpha \ p_e (t) \)
Correlating Price and Usage

If \( E[(p'_e - v_u)^2] < \varepsilon \approx 0 \) and \( p'_e \equiv \alpha \ p_e \)

then \( v_u(t) \approx \alpha \ p_e(t) \)

Also, \( E[\alpha^2 p_e^2] - 2E[\alpha p_e v_u] + E[v_u^2] \approx 0 \)

\[ \Rightarrow \alpha^2 E[p_e^2] = \alpha E[p_e v_u] = E[v_u^2] \]

\[ \Rightarrow \alpha E[p_e^2] = E[p_e v_u] = \overline{C_e} \]
Minimum Cost of Electricity

\[ \bar{C}_e = \min_{L, \alpha} \{ c_R \alpha \} \quad (c_R = E[p_e^2]) \]

\[ E[(p'_e - v_u)^2] < \varepsilon \approx 0 \]

\[ E[v_u^2] < (v_u^{\text{max}})^2 \]

\[ E[S^2] < (S^{\text{max}})^2 \]
Thermal Energy Storage (Small Storage Unit)

Volume of Air (the Room)

Heat from Room

Heat to Cooler

Cooling Unit

Energy Usage

Heat to TES Unit

TES Unit

Heat Leakage $T_{outside}$

$T_{room}$

Heat from Room

Heat to Cooler

Heat to TES Unit

Energy Usage

Graph showing energy usage over time with different heat sources and sinks.
Thermal Energy Storage
(Medium Storage Unit)

Volume of Air (the Room) $T_{room}$
Heat Leakage $T_{outside}$
Heat from Room
Heat to Cooler
Energy Usage
Cooling Unit
Heat to TES Unit
TES Unit

Heat from Room
Heat to Cooler
Heat to TES Unit

kW hr / day

Time (days)

Energy Usage

Illinois Institute of Technology
Department of Chemical and Biological Engineering
Thermal Energy Storage
(Large Storage Unit)

Volume of Air (the Room)
Heat from Room
Heat to Cooler
Energy Usage

Heat Leakage $T_{outside}$

Heat to TES Unit

TES Unit

Heat from Room
Heat to Cooler
Heat to TES Unit

Time (days)

kW hr / day

Heat from Room
Heat to Cooler
Heat to TES Unit
Thermal Energy Storage
(Comparison of Storage Cases)
Thermal Energy Storage
(Cost Comparisons)

Average Cooling Costs:
One ton: $8 per day
Five tons: $7 per day (14% savings)
Ten tons: $6 per day (25% savings)
Minimum Levelized Cost

\[
\min_{L, \alpha, \nu_u^{\text{max}}, S^{\text{max}}} \left\{ c_R \alpha - c_{L,1} \nu_u^{\text{max}} - c_{L,2} S^{\text{max}} \right\}
\]

\[
E \left[ (p'_e - \nu_u)^2 \right] < \varepsilon \equiv 0
\]

\[
E \left[ \nu_u^2 \right] < (\nu_u^{\text{max}})^2
\]

\[
E \left[ S^2 \right] < (S^{\text{max}})^2
\]
Minimum Levelized Cost

\[
\min_{L, \alpha, \nu_u^\text{max}, S^\text{max}} \left\{ c_R \alpha - c_{L,1} \nu_u^\text{max} - c_{L,2} S^\text{max} \right\}
\]

\[
E\left[(p'_e - \nu_u)^2\right] < \varepsilon \equiv 0
\]

\[
E\left[\nu_u^2\right] < (\nu_u^\text{max})^2
\]

\[
E\left[S^2\right] < (S^\text{max})^2
\]

Non-Convex Problem
(but global solution from branch and bound)
Integrated Gasification Combined Cycle (IGCC)
Conclusions

• Relationship between control system performance and plant profit quantified.
• Enables profit guided control system design.
• Broad set of applications from a variety of disciplines.
• Linear controller can be designed for market responsiveness.
• Non-convex, but global methods can be used to size and/or select equipment.
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  Chemical & Biological Engineering Department, IIT
Value of Electric Power Generation

![Graph showing the value of electric power generation over time with converted gas value plotted against time (days).]
Synthesis Gas Storage

Coal, Oxygen and Steam → Gasification and Gas Cleaning Units → Gas Storage Unit → Energy Conversion Units (Gas Turbines and Electric Generators) → Electric Power
IGCC Example
(Small Storage Unit)

Coal, Oxygen and Steam

Gasification and Gas Cleaning Units

Energy Conversion Units (Gas Turbines and Electric Generators)

Electric Power

Gas Storage Unit

![Graphs showing gasification, conversion, and storage units over time]

Converted Gas Value (cents / m³)

Gas Volume in Storage (million m³)

Volumetric Flow (million m³ / day)
IGCC Example
(Larger Storage Unit)

Coal, Oxygen and Steam → Gasification and Gas Cleaning Units → Gas Storage Unit → Energy Conversion Units (Gas Turbines and Electric Generators) → Electric Power

Graphs:
- Converted Gas Value (cents / m\(^3\))
- Gas Volume in Storage (million m\(^3\))
- Volumetric Flow (million m\(^3\) / day)
IGCC Example
(Changes in Revenue)

Average Revenue
- Nominal Case: $1.00 million per day (plot not depicted)
- Case 1: $1.04 million per day.
- Case 2: $1.15 million per day.
Electric Power System Design

Gas Turbine

PC Boiler

Renewable

Transmission Grid

Consumer Demand

Energy Storage
System Disturbances

Consumer Demand

Renewable Power Generated
Electric Power System Model

**Power Limits**

**PC Boiler**

\[ \dot{P}_C = r_C \]

\[ P_C^{\text{min}} \leq P_C \leq P_C^{\text{max}} \]

\[ P_C^{\text{min}} = 0.8 \cdot P_C^{\text{max}} \]

\[ P_C^{\text{max}} = 1200 \]

**Rate Limits**

\[ r_C^{\text{min}} \leq r_C \leq r_C^{\text{max}} \]

\[ r_C^{\text{max}} = 0.05 \cdot P_C^{\text{max}} \]

**Gas Turbine**

\[ \dot{P}_T = r_T \]

\[ P_T^{\text{min}} \leq P_T \leq P_T^{\text{max}} \]

\[ P_T^{\text{min}} = 0.2 \cdot P_T^{\text{max}} \]

\[ P_T^{\text{max}} = 1000 \]

**Rate Limits**

\[ r_T^{\text{min}} \leq r_T \leq r_T^{\text{max}} \]

\[ r_T^{\text{max}} = 6 \cdot P_T^{\text{max}} \]

**Pumped Hydro**

\[ \dot{E}_S = P_S \]

**Energy Limits**

\[ 0 \leq E_S \leq E_S^{\text{max}} \]

**Power Limits**

\[ P_S^{\text{min}} \leq P_S \leq P_S^{\text{max}} \]
Manipulated Variables

PC Boiler
\[ \dot{P}_C = r_C \]

Power Limits
\[ P_C^{\text{min}} \leq P_C \leq P_C^{\text{max}} \]
\[ P_C^{\text{min}} = 0.8 \cdot P_C^{\text{max}} \]
\[ P_C^{\text{max}} = 1200 \]

Rate Limits
\[ r_C^{\text{min}} \leq r_C \leq r_C^{\text{max}} \]
\[ r_C^{\text{max}} = 0.05 \cdot P_C^{\text{max}} \]

Gas Turbine
\[ \dot{P}_T = r_T \]

Power Limits
\[ P_T^{\text{min}} \leq P_T \leq P_T^{\text{max}} \]
\[ P_T^{\text{min}} = 0.2 \cdot P_T^{\text{max}} \]
\[ P_T^{\text{max}} = 1000 \]

Rate Limits
\[ r_T^{\text{min}} \leq r_T \leq r_T^{\text{max}} \]
\[ r_T^{\text{max}} = 6 \cdot P_T^{\text{max}} \]

Pumped Hydro
\[ \dot{E}_S = P_S \]

Energy Limits
\[ 0 \leq E_S \leq E_S^{\text{max}} \]
Case Study

Average of Power Generators

- 32% Gas Turbine
- 48% PC Boiler
- 20% Renewable

Pumped Hydro Equipment Costs

- Energy Storage: $55 /kWh
- Power Rating: $1300/kW
Results Case 1

Gas Turbine

Coal

Storage
## Results Summary

<table>
<thead>
<tr>
<th>Case</th>
<th>Coal Power</th>
<th>Gas Turbine</th>
<th>Renewable</th>
<th>Storage Size</th>
<th>Storage Power</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>48%</td>
<td>32%</td>
<td>20%</td>
<td>12.9 GWh</td>
<td>948 MW</td>
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<tr>
<td>2</td>
<td>18%</td>
<td>32%</td>
<td>50%</td>
<td>26.8 GWh</td>
<td>1398 MW</td>
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<tr>
<td>3</td>
<td>75%</td>
<td>5%</td>
<td>20%</td>
<td>61.1 GWh</td>
<td>1188 MW</td>
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</tbody>
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