Economic MPC with Infinite Horizon

Donald J Chmielewski and Benjamin P Omell

Department of Chemical & Biological Engineering
Illinois Institute of Technology
Outline

- Economic MPC and Issues with Finite-Horizon
- Infinite-Horizon EMPC
- Tuning of Infinite-Horizon EMPC
- Implementation with Historic Price Data
Nonlinear Model Predictive Control

\[ \min_{x,u,w} \int_{t}^{t+T} g(x,u,w) \, d\tau \]

s.t. \[ \dot{x} = f(x,u,w) \]
\[ z = h(x,u,w) \]
\[ z_{\min} \leq z(\tau) \leq z_{\max} \]
Traditional MPC Objective

Quadratic Objective

\[ g(x, u, w) = x^T Q x + u^T R u \]

\[ \min_{x, u, w} \int_{t}^{t+T} g(x, u, w) \, d\tau \]

s.t. \( \dot{x} = f(x, u, w) \)

\( z = h(x, u, w) \)

\( z_{\text{min}} \leq z(\tau) \leq z_{\text{max}} \)
Economic MPC

Economic Objective

\[ g(x, u, w) = - \text{(Instantaneous Profit)} \]

\[
\min_{x,u,w} \int_{t}^{t+T} g(x, u, w) \, d\tau
\]

s.t.

\[ \dot{x} = f(x, u, w) \]

\[ z = h(x, u, w) \]

\[ z_{\text{min}} \leq z(\tau) \leq z_{\text{max}} \]
Literature on EMPC

• **Conceptual Development and Stability Issues:** Rawlings and Amrit (2009); Diel, Amrit and Rawlings (2010); Yang, Omell and Chmielewski (2010); Heidarinejad, Liu and Christofides (2011); Rao (2011)

• **Scheduling Problems:** Baumrucker and Biegler (2010) Lima, Grossman and Jiao (2011)

• **Power Systems:** Zavala, Constantinescu, Krause and Anitescu (2009); Xie and Ilić (2009), Hovgaard, Edlund and Jørgensen (2011)
Smart Grid and Real-time Pricing

Electricity Price
July 19 - 21, 2009

In summer conditions, electricity prices increase when temperature increases.
Smart Grid Applications

• Power Plant Dispatch
Smart Grid Applications

- Power Plant Dispatch
- Smart Buildings (HVAC with TES)
Smart Grid Applications

- Power Plant Dispatch
- Smart Buildings (HVAC with TES)
- Flexible Manufacturing
Smart Grid Applications

• Power Plant Dispatch

• Smart Buildings (HVAC with TES)

• Flexible Manufacturing

• Smart Homes with Plug-in Electric Vehicles
Economic MPC Example

Power Plant \( P_G \)

Value of Electricity \( C_e \)

Revenue = \( P_G C_e \)
Economic MPC for Power Generation

Economic Objective

\[ g(x, u, w) = -P_G C_e \]

\[
\min_{x, u, w} \int_{t}^{t+T} g(x, u, w) \, d\tau
\]

s.t. \quad \dot{x} = f(x, u, w)

\[ z = h(x, u, w) \]

\[ z_{\min} \leq z(\tau) \leq z_{\max} \]
Integrated Gasification Combined Cycle
Integrated Gasification Combined Cycle
IGCC Motivating Example

\[ \dot{M}_{H_2} = \nu_{H_2} - P_G / \beta \]

\[ 0 \leq M_{H_2} \leq M_{H_2}^{\text{max}} \]

\[ 0 \leq P_G \leq P_G^{\text{max}} \]

\[ P_G = \beta \nu_{H_2,G} \]
IGCC Motivating Example

\[ \dot{M}_{H_2} = \nu_{H_2} - \frac{P_G}{\beta} \]

\[ 0 \leq M_{H_2} \leq M_{H_2}^{\text{max}} \]

\[ 0 \leq P_G \leq P_G^{\text{max}} \]

\[ P_G = \beta \nu_{H_2,G} \]
Economic MPC For IGCC

\[
\min_{P_G} \int_{t}^{t+T} -C_e P_G d\tau \\
\text{s.t.} \quad \dot{M}_{H_2} = \nu_{H_2} - P_G / \beta \\
0 \leq M_{H_2} \leq M_{H_2}^{\text{max}} \\
0 \leq P_G \leq P_G^{\text{max}}
\]
In Deviation Variables

\[
\min_{u} \int_{t}^{t+T} -C_e u \, dt \tau
\]

s.t. \( \dot{x} = \frac{1}{6.79} u \)

\(-352 \leq x \leq 352 \)

\(-611 \leq u \leq 611 \)
In summer conditions, electricity prices increase when temperature increases.
Electric Price Model

White Noise Input → Shaping Filter → Sequence with Electricity Price Characteristics
Electricity Price Shaping Filter

Third-order shaping filter

\[ C_e = f_1 + \bar{C}_e \]

\[ \dot{f}_1 = f_2 \]

\[ \dot{f}_2 = \omega_c^2 (w - f_3) - \omega_c^2 f_1 - 2\chi \omega_c f_2 \]

\[ \dot{f}_3 = (w - f_3) / \tau_h \]

\[ \omega_c = 2\pi / \tau_c \]

\[ \chi = 0.1 \]

\[ S_w = \left( \frac{4\chi}{\omega_c} \right) \left( \frac{\omega_c^2 \tau_h^2 + 2\chi \omega_c \tau_h + 1}{\omega_c^2 \tau_h^2} \right) \sum C_e \]
Realization of Value of Electricity
Realization of Value of Electricity

![Graph showing the relationship between frequency (rad/hr) and spectral density ($/MW^2/hr) vs. time (days) and electricity value ($/MW hr). The graphs depict fluctuations and trends in these values over time.](image-url)
Prediction of Electricity Price
Prediction of Electricity Price

White Noise Input

Shaping Filter

Sequence with Electricity Price Characteristics

State Estimator and/or Predictor

Prediction of Electricity Price

Measured Electricity Price
Estimated Price

Predicted Price Model

Real
Predicted

Price ($)

0 0.5 1 1.5 2 2.5 3

20 40 60 80 100 120

t (Days)

Illinois Institute of Technology
Department of Chemical and Biological Engineering
Economic MPC For IGCC

\[
\min_{u} \int_{t}^{t+T} -\hat{C}_e ud\tau
\]

s.t. \quad \dot{x} = \frac{1}{6.79} u

\[-352 \leq x \leq 352\]

\[-611 \leq u \leq 611\]
Economic MPC Simulation of IGCC

![Graph showing Power, Inventory, and Price over time. The graph displays three lines: Power (MW) in blue, Inventory (tonnes) in red, and Price ($/hr) in cyan. The x-axis represents time in days, ranging from 4 to 7 days, and the y-axis represents various units such as Power and Inventory. The graph highlights fluctuations in these variables over the specified time period.]
Economic MPC Simulation of IGCC

![Graph showing economic MPC simulation of IGCC](image)

- **Power (MW)**
- **Inventory (tonnes)**
- **Price ($ / hr)**

The graph illustrates the changes in power and inventory over time, along with the price fluctuations. The x-axis represents time in days (t), and the y-axis represents power, inventory, and price.
Economic MPC Simulation of IGCC
Effect of Changing Horizon

\[ \min_u \int_{t}^{t+T} -\hat{C}_e u d\tau \]

s.t. \[ \dot{x} = \frac{1}{6.79} u \]

\[-352 \leq x \leq 352 \]

\[-611 \leq u \leq 611 \]
Effects of Horizon Size on Power

- **12 hr Horizon**
  - Power (MW) vs. time (t) (Days)
  - Plot shows fluctuations over days.

- **5 hr Horizon**
  - Similar to 12 hr horizon but with shorter intervals.

- **2 hr Horizon**
  - Shortest interval, showing more frequent fluctuations.

---

Illinois Institute of Technology
Department of Chemical and Biological Engineering
Effects of Horizon Size on Inventory

Horizon Affect on Inventory

Inventory (tonnes) vs. t (Days) for different horizons:
- $h=12$ hrs (blue)
- $h=5$ hrs (red)
- $h=2$ hrs (black)
Inventory Depletion

Outline

- Economic MPC and Issues with Finite-Horizon
- Infinite-Horizon EMPC
- Tuning of Infinite-Horizon EMPC
- Implementation with Historic Price Data
Infinite Horizon Economic MPC For IGCC

\[
\min_u \int_0^\infty -\hat{C}_e u \, d\tau
\]

\[
s.t. \quad \dot{x} = \frac{1}{6.79} u
\]

\[-352 \leq x \leq 352\]

\[-611 \leq u \leq 611\]
### Standard Infinite Horizon MPC

\[
\begin{align*}
\min_{x,u} & \left\{ \int_{t}^{\infty} g(x,u,w) d\tau \right\} \\
\text{s.t.} & \quad \dot{x} = f(x,u,w) \\
& \quad z = h(x,u,w) \\
& \quad z_{\text{min}} \leq z(\tau) \leq z_{\text{max}} \quad t < \tau < \infty
\end{align*}
\]
Infinite Horizon Finitely Constrained MPC

\[
\min_{x,u} \left\{ \int_{t}^{\infty} g(x,u,w) d\tau \right\}
\]

s.t. \quad \dot{x} = f(x,u,w)

\[
z = h(x,u,w)
\]

\[
z_{\text{min}} \leq z(\tau) \leq z_{\text{max}} \quad t < \tau < t + T
\]
Infinite Horizon Finitely Constrained MPC

\[
\min_{x,u} \left\{ \int_{t}^{t+T} g(x,u,w) d\tau + \int_{t}^{\infty} g(x,u,w) d\tau \right\}
\]

s.t. \quad \dot{x} = f(x,u,w) \\
\quad z = h(x,u,w) \\
\quad z_{\text{min}} \leq z(\tau) \leq z_{\text{max}} \quad t < \tau < t + T
Infinite Horizon Finitely Constrained MPC

$$\min_{x,u} \left\{ \int_{t}^{t+T} g(x,u,w) d\tau + \Phi(x(t+T)) \right\}$$

s.t. \quad \dot{x} = f(x,u,w)

$$z = h(x,u,w)$$

$$z_{\min} \leq z(\tau) \leq z_{\max} \quad t < \tau < t+T$$
Infinite Horizon Unconstrained Value Function

\[
\Phi(x_0) = \min_{x,u} \left\{ \int_0^\infty g(x,u,w) d\tau \right\}
\]

s.t. \( \dot{x} = f(x,u,w) \)

\( z = h(x,u,w) \)

\( x(0) = x_o \)
Infinite Horizon Unconstrained Value Function

$$\Phi(x_0) = \min_{x,u} \left\{ \int_0^\infty (x^T Qx + u^T Ru) \, d\tau \right\}$$

s.t. \quad \dot{x} = Ax + Bu + Gw

$$z = D_x x + D_u u + D_w w$$

$$x(0) = x_0$$
Infinite Horizon Unconstrained Value Function

\[ \Phi(x_o) = \min_{x,u} \left\{ \int_{0}^{\infty} (x^T Qx + u^T Ru) \, d\tau \right\} = x^T Px \]

s.t. \hspace{1cm} \dot{x} = Ax + Bu + Gw

\[ z = D_x x + D_u u + D_w w \]

\[ x(0) = x_o \]
Infinite Horizon Value Function For IGCC

\[
\Phi(x(0)) = \min \left\{ \int_{0}^{\infty} -\hat{C}_e u \, d\tau \right\}
\]

s.t. \[ \dot{x}_1 = \frac{1}{6.79} u \]
The Economic Quadratic Finite MPC Problem

\[
\min_{x,u} \left\{ \int_{0}^{\infty} -x_2 u d\tau \right\}
\]

\[s.t. \quad \dot{x}_1 = \frac{1}{6.79} u\]
\[\dot{x}_2 = x_3\]
\[\dot{x}_3 = \omega_c^2 (w - x_4) - \omega_c^2 x_2 - 2 \chi \omega_c x_3\]
\[\dot{x}_4 = (w - x_4) / \tau_h\]
The Economic Quadratic Finite MPC Problem

\[
\min_{x,u} \left\{ \int_{0}^{\infty} \begin{bmatrix} x^T & u^T \end{bmatrix} \begin{bmatrix} Q & M^T \\ M & R \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} d\tau \right\}
\]

s.t. \[ \dot{x}_1 = \frac{1}{6.79} u \]
\[ \dot{x}_2 = x_3 \]
\[ \dot{x}_3 = \omega_c^2 (w - x_4) - \omega_c^2 x_2 - 2\chi \omega_c x_3 \]
\[ \dot{x}_4 = (w - x_4) / \tau_h \]

\[
Q = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]
\[
R = 0 , \quad M = \begin{bmatrix} 0 & -1 & 0 & 0 \end{bmatrix}^T
\]
Outline

• Economic MPC and Issues with Finite-Horizon

• Infinite-Horizon EMPC

• Tuning of Infinite-Horizon EMPC

• Implementation with Historic Price Data
Attempt at Infinite Horizon Formulation Solution

\[ \min_{x,u} \left\{ \int_0^t \begin{bmatrix} x^T & u^T \end{bmatrix} \begin{bmatrix} Q & M \\ M^T & R \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} d\tau \right\} + x^T P x \]

s.t. \[ \dot{x}_1 = \frac{1}{6.79} u \]
\[ \dot{x}_2 = x_3 \]
\[ \dot{x}_3 = \omega_c^2 (w - x_4) - \omega_c^2 x_2 - 2 \chi \omega_c x_3 \]
\[ \dot{x}_4 = (w - x_4) / \tau_h \]

\[ Q = \begin{bmatrix} \varepsilon_p & 0 & 0 & 0 \\ 0 & \varepsilon_c & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad R = \varepsilon_p, \quad M = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}^T \]
Attempt at Infinite Horizon Formulation Solution

Infinite Control, $\varepsilon_M=10^{-6}$ and $\varepsilon_M=0.1692$

- Power (MW)
- Inventory (tonnes)

Solves problems of inventory, but still not very desirable
Comparison to Linear Solution

![Comparison to Linear Solution](image)
\[ Q = \begin{pmatrix}
0.0079 & -131.37 & 1.039 & -0.071 \\
-131.37 & 2,172,700 & -17,198 & 1175.7 \\
1.039 & -17,198 & 136.21 & -9.252 \\
-0.071 & 1175.7 & -9.252 & 3.231
\end{pmatrix} \]

\[ R = 26,874 \]

\[ M = \begin{bmatrix}
-14.611 & 241,640 & -1,912.7 & 130.75
\end{bmatrix}^T \]
Infinite Horizon Formulation Comparison

![Graph showing comparison of linear (h=12hrs), unconstrained QP, and constrained QP over time (days) with power (MW) on the y-axis and time (days) on the x-axis. The graph displays the power output over ten days for the different formulations.]
Infinite Horizon Formulation Comparison
Infinite Horizon Formulation Comparison

![Graph showing comparison of different formulations over time.](image-url)
Inventory

![Graph showing inventory over time with different lines representing LP, Unconstrained QP, and Constrained QP]
Inventory

![Graph showing inventory over time with different methods: LP, Unconstrained QP, and Constrained QP.](image)
Horizon Size Sensitivity

\[ \min_{x,u} \left\{ \int_t^{t+T} \begin{bmatrix} x^T \\ u^T \end{bmatrix} \begin{bmatrix} Q & M \\ M^T & R \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} d\tau \right\} + x^T P x \]

s.t. \[ \dot{x}_1 = \frac{1}{6.79} u \]
\[ \dot{x}_2 = x_3 \]
\[ \dot{x}_3 = \omega_c^2 (w - x_4) - \omega_c^2 x_2 - 2 \chi \omega_c x_3 \]
\[ \dot{x}_4 = (w - x_4) / \tau_h \]
\[ z_{\text{min}} \leq z(\tau) \leq z_{\text{max}} \quad t < \tau < t + T \]
15 Minute Horizon

Affects of Horizon Size

Power (MW)

Inventory (tonnes)

0 1 2 3 4 5 6 7 8 9 10
0 200 400 600
0 1500 1000 500

0 1 2 3 4 5 6 7 8 9 10
0 1 2 3 4 5 6 7 8 9 10

12 hour
2 hour
15 min
Outline

- Economic MPC and Issues with Finite-Horizon
- Infinite-Horizon EMPC
- Tuning of Infinite-Horizon EMPC
- Implementation with Historic Price Data
Historic Data from Illinois Hub 09

![Graph showing power (MW) over time (Days) with two lines representing Constrained QP and Linear models.](image-url)
Historic Data from Illinois Hub 09

![Graph showing inventory over days for Constrained QP and Linear models.]
Tuning???

There are 15 weights to adjust. How did we find the right weights for our infinite controller?

\[
Q = \begin{pmatrix}
0.0079 & -131.37 & 1.039 & -0.071 \\
-131.37 & 2,172,700 & -17,198 & 1175.7 \\
1.039 & -17,198 & 136.21 & -9.252 \\
-0.071 & 1175.7 & -9.252 & 3.231
\end{pmatrix}
\]

\[R = 26,874\]

\[M = \begin{bmatrix}
-14.611 & 241,640 & -1,912.7 & 130.75
\end{bmatrix}^T\]
Tuning???

There are 15 weights to adjust. How did we find the right weights for our infinite controller?

Answer: We used Market Responsive Control (MRC)

Chmielewski, Omell (2010)
Conclusions

• Illustrated issues with Finite Horizon EMPC
  • Inventory creep
  • Manipulated variable chattering
• Proposed a Infinite Horizon Formulation
  • Provided IH-EMPC tuning methods
  • Sensitivity to computational horizon nearly eliminated
• Implementation with historic data yielded identical performance
Acknowledgements

• Current and Former Students:
  David Mendoza-Serrano
  Ming Yang (Taiwan Electric)

• Personal Communications:
  Ignacio E. Grossman and Ricardo M Lima

• Funding:
  National Science Foundation (CBET – 0967906)
  Chemical Engineering, CMU
  (Sabbatical Support for D. Chmielewski)