IGCC Power Plant Dispatch Using Infinite Horizon Economic MPC

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Outline

- Economic MPC
- IGCC Case Study
- Issues with Finite-Horizon EMPC
- Infinite-Horizon EMPC
  - Economic Linear Optimal Control
- Implementation with Imperfect Forecasting
Model Predictive Control

\[ \min_{x,u} \int_{t}^{t+T} g(x,u,w) \, d\tau \]

s.t. \[ \dot{x} = f(x,u,w) \]
\[ z = h(x,u,w) \]
\[ z^{\text{min}} \leq z(\tau) \leq z^{\text{max}} \]
Traditional MPC

Quadratic Objective

\[ g(x,u,w) = x^T Q x + u^T R u \]

\[
\min_{x,u} \int_{t}^{t+T} g(x,u,w) \, d\tau
\]

s.t.  \[ \dot{x} = f(x,u,w) \]

\[ z = h(x,u,w) \]

\[ z_{\min} \leq z(\tau) \leq z_{\max} \]
Economic MPC

**Economic Objective**

\[
g(x,u,w) = - \text{(Instantaneous Profit)}
\]

\[
\min_{x,u} \int_t^{t+T} g(x,u,w) \, d\tau
\]

s.t. \quad \dot{x} = f(x,u,w)

\[
z = h(x,u,w)
\]

\[
z^{\text{min}} \leq z(\tau) \leq z^{\text{max}}
\]
Literature on EMPC

- **Conceptual Development and Stability Issues:** Rawlings and Amrit (2009); Diehl, et al. (2011); Huang and Biegler (2011); Heidarinejad, et al. (2012)

- **Process Scheduling:** Karwana and Keblisb (2007); Baumrucker and Biegler (2010); Lima et al. (2011); Kostina et al. (2011)

- **Power Systems:** Zavala et al. (2009); Xie and Ilić (2009), Hovgaard, et al. (2011), Omell and Chmielewski (2011)

- **HVAC Systems:** Braun (1992); Morris et al. (1994); Kintner-Meyer and Emery (1995); Henze et al. (2003); Braun (2007); Oldewurtel et al. (2010), Ma et al. (2012); Mendoza and Chmielewski (2012)
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Overview of Smart Grid

Generators

Transmission

Consumer Demand

Generator Dispatch

Renewable

Smart Homes

Smart Manufacturing

with Storage

Commercial Buildings
Integrated Gasification Combined Cycle
Hydrogen Gas Storage
Conventional IGCC

\[ P_G = \beta v_{H_2,G} \]
IGCC with Dispatch Capability

\[ \dot{M}_{H_2} = \nu_{H_2} - \frac{P_G}{\beta} \]

\[ 0 \leq \dot{M}_{H_2} \leq \dot{M}_{H_2}^{\text{max}} \]

\[ 0 \leq P_G \leq P_G^{\text{max}} \]
Economic MPC For IGCC

\[
\min_{P_G} \int_{t}^{t+T} -C_e P_G \, d\tau
\]

s.t. \( \dot{M}_{H2} = \nu_{H2} - P_G / \beta \)

\[
0 \leq M_{H2} \leq M_{H2}^{\text{max}}
\]

\[
0 \leq P_G \leq P_G^{\text{max}}
\]

\( C_e(t) \) is the cost (or value) of electricity
Economic MPC Simulation with Perfect Forecasting of $C_e(t)$
Impact of Horizon Size on EMPC

- Energy Value ($/MWhr)
- Generated Power (MW)
- H₂ in Storage (tonnes)

Time (days)
Inventory Depletion


...An additional time horizon of six months ... on the future demand [is required] to avoid a “myopic” inventory policy at the end of the scheduling period of 18 months.
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Infinite Horizon MPC

\[
\min_{x,u} \left\{ \int_{t}^{\infty} g(x,u,w) d\tau \right\}
\]

s.t. \[
\dot{x} = Ax + Bu + Gw \\
z = D_x x + D_u u \\
z_{\min} \leq z(\tau) \leq z_{\max} \quad t < \tau < \infty
\]
Infinite Horizon Finite Constrained MPC

\[
\min_{x,u} \left\{ \int_{t}^{\infty} g(x,u,w) d\tau \right\}
\]

s.t. \[ \dot{x} = Ax + Bu + Gw \]

\[ z = D_x x + D_u u \]

\[ z_{\text{min}} \leq z(\tau) \leq z_{\text{max}} \quad t < \tau < t + T \]
Infinite Horizon Finite Constrained MPC

\[
\min_{x,u} \left\{ \int_{t}^{t+T} g(x,u,w) d\tau + \int_{t}^{\infty} g(x,u,w) d\tau \right\}
\]

s.t. \quad \dot{x} = Ax + Bu + Gw

\[z = Dx x + Du u\]

\[z_{\min} \leq z(\tau) \leq z_{\max} \quad t < \tau < t + T\]
Finite Horizon MPC with Terminal Cost

\[
\min_{x,u} \left\{ \int_{t}^{t+T} g(x,u,w)d\tau + \Phi(x(t+T)) \right\}
\]

s.t. \[ \dot{x} = Ax + Bu + Gw \]

\[
z = D_x x + D_u u
\]

\[
z_{\min} \leq z(\tau) \leq z_{\max} \quad t < \tau < t + T
\]
Unconstrained Value Function

\[ \Phi(x(t + T)) = \min_{x,u} \left\{ \int_{t+T}^{\infty} g(x,u,w) d\tau \right\} \]

s.t. \hspace{10pt} \dot{x} = Ax + Bu + Gw
Unconstrained Value Function

\[ \Phi(x(t+T)) = \min_{x,u} \left\{ \int_{t+T}^{\infty} g(x,u,w) d\tau \right\} \]

s.t. \( \dot{x} = Ax + Bu + Gw \)

If \( g(x,u,w) = - \) (Instantaneous Profit)
Unconstrained Value Function

\[ \Phi(x(t + T)) = \min_{x,u} \left\{ \left. \int_{t+T}^{\infty} g(x,u,w) \, d\tau \right| s.t. \quad \dot{x} = Ax + Bu + Gw \right\} \]

If \( g(x,u,w) = - \) (Instantaneous Profit)

Then \( \Phi(\bullet) \) does not exist
Statistically Constrained Function

\[ \Phi_{ELOC}(x(t+T)) = \min_{x,u} \left\{ \int_{t+T}^{\infty} g(x,u,w)d\tau \right\} \]

s.t. \quad \dot{x} = Ax + Bu + Gw

\[ z = D_x x + D_u u \]

\[ E[z^2] \leq \left( \min \left\{ -z^{\min}, z^{\max} \right\} \right)^2 \]
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ELOC Development

1. Stochastic Electricity Price Model
2. Constrained Stochastic Control
3. Analytic Expression for Revenue
4. Controller Synthesis
Electricity Price Model

White Noise Input → Shaping Filter → Sequence with Electricity Price Characteristics
Realization of Electricity

![Graph showing spectral density and electricity value over time.](image)
Scaled Electricity Prices

Overall Process Model:
\[ \dot{x} = Ax + Bu + \alpha Gw \]
ELOC Development

1. Stochastic Electricity Price Model
2. Constrained Stochastic Control
3. Analytic Expression for Revenue
4. Controller Synthesis
Stochastic Constrained Control

Assume $u = Lx$ and find $L$ such that

$$0 = (A + BL)\Sigma_x + \Sigma_x (A + BL)^T + \alpha^2 G S_w G^T$$

$$\zeta_j = \rho_j (D_x + D_u L)\Sigma_x (D_x + D_u L)^T \rho_j^T$$

$$\sigma_j = \sqrt{\zeta_j}$$

$$2\sigma_j < z_j^{\max} \quad \text{and} \quad 2\sigma_j < -z_j^{\min}, \, j = 1...n_z$$

Based on standard deviations ($\sigma_j$'s)

Constraints $z_j^{\min}$ and $z_j^{\max}$
ELOC Development

1. Stochastic Electricity Price Model
2. Constrained Stochastic Control
3. Analytic Expression for Revenue
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ELOC Objective Function

\[
\lim_{T \to \infty} \left\{ \frac{1}{T} \int_0^T C_e P_G dt \right\} = E[C_e P_G]
\]

\[
= E[\widetilde{C}_e \widetilde{P}_G] + \overline{C}_e \overline{P}_G
\]

where

\[
\widetilde{C}_e = C_e - \overline{C}_e , \quad \overline{C}_e = [C_e]
\]

\[
\widetilde{P}_G = P_G - \overline{P}_G , \quad \overline{P}_G = [P_G]
\]
ELOC Objective Function

\[
\lim_{T \to \infty} \left\{ \frac{1}{T} \int_0^T C_e P_G \, dt \right\} = E[\tilde{C}_e \tilde{P}_G] + \bar{C}_e \bar{P}_G
\]

Then, enforce the condition \( \tilde{P}_G = \alpha \tilde{C}_e \)

\[
E[\tilde{C}_e \tilde{P}_G] = E[\tilde{C}_e \alpha(\tilde{C}_e)]
\]

\[
= \alpha E[\tilde{C}_e^2]
\]

\[
= \alpha \Sigma_{C_e}
\]
ELOC Development

1. Stochastic Electricity Price Model
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4. Controller Synthesis
ELOC Synthesis

\[
\begin{align*}
\min_{L, \Sigma_x \geq 0, \zeta_j, \sigma_j, \alpha} & \left\{ \alpha \Sigma_{Ce} + \overline{C_e} \overline{P_G} \right\} \\
\text{s.t.} & \\
0 &= (A + BL) \Sigma_x + \Sigma_x (A + BL)^T + \alpha^2 G S_w G^T \\
\zeta_j &= \rho_j (D_x + D_u L) \Sigma_x (D_x + D_u L)^T \rho_j^T \\
\sigma_j &= \sqrt{\zeta_j} \\
2\sigma_j &< z_j^{\max} \quad \text{and} \quad 2\sigma_j < -z_j^{\min}, \; j = 1 \ldots n_z
\end{align*}
\]

\[
\Rightarrow u = L_{ELOC} x
\]

Can be solved with a Convex Optimization Problem
Comparison of ELOC and EMPC

- Power Generated (MW)
- \( \text{H}_2 \) in Storage (tonnes)

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Linear Quadratic Optimal Control

\[
\Phi(x(t+T)) = \min_{x,u} \left\{ \int_{t+T}^{\infty} g(x,u,w) d\tau \right\}
\]

s.t. \quad \dot{x} = Ax + Bu
Linear Quadratic Optimal Control

\[ \Phi(x(t+T)) = \min_{x,u} \left\{ \int_{t+T}^{\infty} g(x,u,w) d\tau \right\} \]

s.t. \[ \dot{x} = Ax + Bu \]

If \[ g(x,u,w) = \begin{bmatrix} x^T \\ u^T \end{bmatrix} \begin{bmatrix} Q & M \\ M^T & R \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} \]

\[ \Phi(x(t+T)) = x(t+T)^T P_{LQR} x(t+T) \]

\[ u = L_{LQR} x \]
Infinite Horizon EMPC

\[ \Phi(x(t+T)) = \min_{x,u} \left\{ \int_{t+T}^{\infty} g(x,u,w) d\tau \right\} \]

s.t. \[ \dot{x} = Ax + Bu + Gw \]

\[ g(x,u,w) = \begin{bmatrix} x^T \\ u^T \end{bmatrix} \begin{bmatrix} Q_{ELOC} & M_{ELOC} \\ M_{ELOC}^T & R_{ELOC} \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} \]

\[ \Phi(x(t+T)) = x(t+T)^T P_{ELOC} x(t+T) \]

\[ u = L_{ELOC} x \]
Inverse Optimality Theorem

Chmielewski & Manthanwar (2004):

If there exists $P > 0$ and $R > 0$ such that

$$
\begin{bmatrix}
L^T R L - A^T P - PA & -(L^T R + PB) \\
-(L^T R + PB)^T & R
\end{bmatrix} > 0
$$

Then $M = -(L^T R + PB)$ and $Q = L^T R L - A^T P + PA$ are such that

$$
\begin{bmatrix}
Q & M \\
M^T & R
\end{bmatrix} > 0 \quad \text{and } P \text{ and } L \text{ satisfy}
$$

$$
A^T P + PA + Q - (PB + M)R^{-1}(PB + M)^T = 0
$$

$$
L = -R^{-1}(PB + M)^T
$$
IGCC Example

\[ L_{ELOC} = \begin{bmatrix} 0.067 & -14.15 & 12.94 & -0.6943 \end{bmatrix} \]

\[ Q_{ELOC} = \begin{bmatrix} 15.66 & -3303.5 & 3020.6 & -162.13 \\ -3303.3 & 6.96e5 & -6.37e5 & 34198 \\ 3020.6 & -6.37e5 & 5.826e5 & -31270 \\ -162.13 & 34198 & -31270 & 1684.2 \end{bmatrix} \]

\[ M_{ELOC} = \begin{bmatrix} -233.52 \\ 49255 \\ -45038 \\ 2417.5 \end{bmatrix} \]

\[ R_{ELOC} = \begin{bmatrix} 3481.9 \end{bmatrix} \]

\[ P_{ELOC} = \begin{bmatrix} 7.26e-5 & 0.0022 & 0.0125 & 0.0002 \\ 0.0022 & 0.498 & 0.540 & -0.019 \\ 0.0125 & 0.540 & 8.76 & -0.224 \\ 0.0002 & -0.019 & -0.224 & 6.66 \end{bmatrix} \]
Infinite Horizon EMPC

\[
\min_{x,u} \left\{ \int_t^{t+T} \begin{bmatrix} x^T & u^T \end{bmatrix} \begin{bmatrix} Q_{ELOC} & M_{ELOC}^T \\ M_{ELOC} & R_{ELOC} \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} d\tau + x(t+T)^T P_{ELOC} x(t+T) \right\}
\]

s.t. \[ \dot{x} = Ax + Bu + Gw \]
\[ z = D_x x + D_u u \]
\[ z_{\text{min}} \leq z(\tau) \leq z_{\text{max}} \quad t < \tau < t+T \]
ELOC and Unconstrained IH-EMPC

![Graph showing Power Generated and H₂ in Storage over time]

- **Power Generated (MW)**
  - Y-axis: -4000 to 4000
  - X-axis: Time (days)

- **H₂ in Storage (tonnes)**
  - Y-axis: -2000 to 4000
  - X-axis: Time (days)
Constrained IH-EMPC
24-hr horizon
Horizon Size Sensitivity

Difference in computation times for 30 day simulation:
- EMPC 24 hr horizon ~ 22.8 sec
- IH-EMPC 1 hr horizon ~ 0.02 sec
- Reduction of 99.9%
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Prediction of Electricity Price

White Noise Input \rightarrow Shaping Filter \rightarrow Sequence with Electricity Price Characteristics

Measured Electricity Price \rightarrow State Estimator and/or Predictor \rightarrow Prediction of Electricity Price
Imperfect Forecast Simulation

- Energy Value ($/MWhr)
- Generated Power (MW)
- $H_2$ in Storage (tonnes)

IH-EMPC, EMPC
## Revenue

<table>
<thead>
<tr>
<th>Case</th>
<th>Revenue (10^3 $/day)</th>
<th>Revenue Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Dispatch</td>
<td>557.6</td>
<td>-</td>
</tr>
<tr>
<td>EMPC: Perfect Forecast</td>
<td>765.7</td>
<td>27.2%</td>
</tr>
<tr>
<td>EMPC: Imperfect Forecast</td>
<td>683.9</td>
<td>18.5%</td>
</tr>
<tr>
<td>IH-EMPC: Imperfect Forecast</td>
<td>736.0</td>
<td>24.0%</td>
</tr>
</tbody>
</table>
Conclusions

- Illustrated issues with Finite Horizon EMPC
  - Inventory creep
  - Manipulated variable chattering
- Proposed a Infinite Horizon Formulation
  - Provided IH-EMPC tuning methods
  - Sensitivity to horizon size nearly eliminated
- IH-EMPC less sensitive to forecast errors.
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