Economic Based Control System Design

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BOP with more profit
BOP with less profit

OSSOP

EDOR’s due to different controller tunings

BOP with more profit
OSSOP

BOP with less profit

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\[ x \]

\[ u \]
Process Systems Engineering
(Chmielewski Lab)

Energy Systems

- Power Systems
  - Dry Gasification Oxy-Combustion (DGOC) Process
  - Control of Oxygen Enhanced Boilers
  - Oxygen as Energy Carrier

- Fuel Cell Systems
  - SOFC
  - Fuel Processors
  - PEMFC
  - Hybrid Vehicles

Control Theory

- Profit Control
  - Chemical Processes
  - Inventory Planning
  - Smart Grid Operation
  - Water Resource Management
  - Hybrid Vehicles

- Market Responsive Control
  - Power Plant Dispatch
  - Building HVAC with Thermal Energy Storage
Outline

• Motivating Example
• Pseudo-Constrained Control
• Back-off and Profit Control
• Market Responsive Control
Motivating Example  
(Non-isothermal Reactor)

\[ V \frac{dC_A}{dt} = F(C_{Ain} - C_A) + Vr_A \]

\[ V \frac{dT}{dt} = F(T_{in} - T) + \left( \frac{V\Delta H}{\rho C_p} \right)r_A \]

\[ r_A = -k(T)C_A \]

Increase F  \( \Rightarrow \)  Increased production rate
Motivating Example
(Non-isothermal Reactor)

\[ V \frac{dC_A}{dt} = F(C_{\text{in}} - C_A) + Vr_A \]
\[ V \frac{dT}{dt} = F(T_{\text{in}} - T) + \left( \frac{V \Delta H}{\rho C_p} \right) r_A \]
\[ r_A = -k(T)C_A \]

Increase \( F \) \( \rightarrow \) Increased production rate

Decrease \( F \) \( \rightarrow \) Increase \( T \) \( \rightarrow \) Increase reaction rate
\( \rightarrow \) Increase production
**Limited Operating Region**

**Process Limitations:**

\[
T(t) \leq T^{(\text{max})}
\]
- Catalyst protection or onset of side reactions

\[
F(t) \leq F^{(\text{max})}
\]
- Pump limit or limit on downstream unit
Limited Operating Region

Process Limitations:

\[ T(t) \leq T^{(\text{max})} \]
- Catalyst protection or onset of side reactions

\[ F(t) \leq F^{(\text{max})} \]
- Pump limit or limit on downstream unit

Possible Controller:

\[ F = K_c (T - T^{(sp)}) + F^{(sp)} \]
Performance in Time Series

\[ F(t) \]

\[ T(t) \]

\[ C_A, T \]

\[ F^{(sp)} \]

\[ T^{(sp)} \]

\[ F^{(max)} \]

\[ T^{(max)} \]
Performance in Phase Plane

\[ T(t) \]

\[ F(t) \]
Dynamic Operating Region

\[ T(t) \]

\[ F(t) \]
Dynamic Operating Region

\[ T(t) \]

\[ F(t) \]
Steady-State Relation

Controller:

\[ F = K_c (T - T^{(sp)}) + F^{(sp)} \]

Steady-State Relation:

\[ F^{(sp)} = f (T^{(sp)}) \]
Dynamic Operating Region

\[ T(t) \]

\[ F(t) \]
Steady-State Operating Line

\[ T(t) \]

\[ F(t) \]
Optimal Operating Point

Decrease $F$ → Increase $T$
→ Increase conversion
→ Increase production
Optimal Operating Point:
Another Possibility

$T(t)$

Increase $F$ → Increased production rate

$F(t)$
Optimal Operating Point: Another Possibility

Increase $F$ → Increased production rate
Requires Different Controller Tuning

\[ T(t) \]

\[ F(t) \]
Less Aggressive Tuning
Connects Controller Design to Plant Economics
Outline

• Motivating Example
• Pseudo-Constrained Control
• Back-off and Profit Control
• Market Responsive Control
Covariance Analysis  
(Open-Loop Case)

Process Model:
\[
\begin{align*}
\dot{x} &= A x + G w \\
z &= D x \\
w(t) &\text{ Gaussian white noise with covariance } \Sigma_w
\end{align*}
\]

Steady State Covariance:
\[
\begin{align*}
A \Sigma_x + \Sigma_x A^T + G \Sigma_w G^T &= 0 \\
\Sigma_z &= D \Sigma_x D^T
\end{align*}
\]
Expected Dynamic Operating Region (EDOR)

EDOR defined by:

\[ \Sigma_z = \begin{bmatrix}
\sigma_{11}^2 & \sigma_{12}^2 \\
\sigma_{21}^2 & \sigma_{22}^2 
\end{bmatrix} \]
Closed-Loop Covariance Analysis
(Full State Information Case)

Process Model:
\[
\dot{x} = Ax + Bu + Gw \\
z = Dx x + Du u + Dw w
\]

Controller:
\[
u(t) = Lx(t)
\]

Steady-State Covariance:
\[
(A + BL) \Sigma_x + \Sigma_x (A + BL)^T + G \Sigma_w G^T = 0
\]
\[
\Sigma_z = (D_x + Du L) \Sigma_x (D_x + Du L)^T + Dw \Sigma_w Dw^T
\]
Closed-Loop EDOR

EDOR’s from different controllers

\[ u = L_1 x \]

\[ u = L_2 x \]
Constrained Closed-Loop EDOR

Constraints

\( \sigma_{z_i} < \bar{z}_i \)
Constrained Controller Existence

Does there exist $L$ such that:

$$(A + BL) \Sigma_x + \Sigma_x (A + BL)^T + G \Sigma_w G^T = 0$$

$$\Sigma_z = (D_x + D_u L) \Sigma_x (D_x + D_u L)^T + D_w \Sigma_w D_w^T$$

$$\xi_i = \phi_i \Sigma_z \phi_i^T < \bar{z}_i^2 \quad i = 1 \ldots n_z$$

$$\phi_i = \begin{bmatrix} 0 & 0 & \ldots & 1 & \ldots & 0 & 0 \end{bmatrix}$$

\[\uparrow \text{$i^{th}$ column}\]
Constrained Controller Existence (Convex Condition)

If and only if there exist $X>0$ and $Y$ such that:

$$(AX + BY) + (AX + BY)^T + G \Sigma w G^T < 0$$

$$
\begin{bmatrix}
\xi - \phi_i D_w \Sigma w D_w^T \phi_i^T & \phi_i (D_x X + D_u Y) \\
(D_x X + D_u Y)^T \phi_i^T & X
\end{bmatrix} > 0
$$

$$\xi_i < \bar{z}_i^2 \quad i = 1 \ldots n_z$$

And controller $u = Lx$ is constructed as: $L = YX^{-1}$

$$\phi_i = \begin{bmatrix} 0 & 0 & \ldots & 1 & \ldots & 0 & 0 \end{bmatrix}$$
Unconstrained Controller Existence

Achievable Performance Levels

Unachievable

\(\xi_1\)

\(\xi_2\)
Constrained Controller Existence

\[ \xi_1 < z_1^2 \]

\[ \xi_2 < z_2^2 \]
Pseudo-Constrained Control (PCC)

\[
\min_{X>0,Y,\xi} \sum_i d_i \xi_i
\]

such that:

\[
(AX + BY) + (AX + BY)^T + G \Sigma_w G^T < 0
\]

\[
\begin{bmatrix}
\xi_i - \phi_i D_w \Sigma_w D_w^T \phi_i^T & \phi_i (D_x X + D_u Y) \\
(D_x X + D_u Y)^T \phi_i^T & X
\end{bmatrix} > 0
\]

\[
\xi_i < \bar{z}_i^2 \quad i = 1 \ldots n_z
\]
PCC Controller Equivalence

Theorem 1 (Chmielewski & Manthanwar, 2004):

The controller generated by PCC

is coincident with

the controller generated by some
Unconstrained Model Predictive Controller.
Inverse Optimality

Theorem 2 (Chmielewski & Manthanwar, 2004):

If there exists $P > 0$ and $R > 0$ such that

$$
\begin{bmatrix}
L^T RL - A^T P - PA & -(L^T R + PB) \\
-(L^T R + PB)^T & R
\end{bmatrix} > 0
$$

then $M = -(L^T R + PB)$ and $Q = L^T RL - A^T P + PA$ are such that

$$
\begin{bmatrix}
Q & M \\
M^T & R
\end{bmatrix} > 0
$$

and $P$ and $L$ satisfy

$$
A^T P + PA + Q - (PB + M)R^{-1}(PB + M)^T = 0
$$

$$
L = -R^{-1}(PB + M)^T
$$
Outline

• Model Predictive Controller Tuning
• Pseudo-Constrained Control
• Back-off and Profit Control
• Market Responsive Control
Constrained Operating Region

Constraints

CV’s

MV’s
Real-Time Optimization

Original Nonlinear Process Model:

\[ \dot{s} = f(s,m,p) \quad q = h(s,m,p) \]

\((s,m,p,q) \sim (\text{state, mv, dist, performance}) \sim (x,u,w,z)\)
Real-Time Optimization

Original Nonlinear Process Model:

\[
\dot{s} = f(s, m, p) \quad q = h(s, m, p)
\]

\((s, m, p, q) \sim \text{(state, mv, dist, performance)} \sim (x, u, w, z)\)

Real-Time Optimization (minimize profit loss):

\[
\min_{s, m, q} \left\{ g(q) \right\} \quad \text{s.t.}
\]

\[
0 = f(s, m, p) \quad q = h(s, m, p) \quad q_i^{\text{min}} \leq \phi_i q \leq q_i^{\text{max}}
\]

RTO solution denoted as \((s^{\text{ossop}}, m^{\text{ossop}}, p^{\text{ossop}}, q^{\text{ossop}})\)
Real-Time Optimization

Constraints
Optimal Steady-State Operating Point (OSSOP)

CV’s

MV’s
Backed-off Operating Point (BOP)

CV’s

EDOR

Optimal Steady-State Operating Point (OSSOP)

MV’s
Stochastic BOP Selection
(Loeblein & Perkins, 1999)

Assume controller $L$ is given and calculate $\xi_i$:

$$(A + BL) \Sigma_x + \Sigma_x (A + BL)^T + G \Sigma_w G^T = 0$$

$$\Sigma_z = (D_x + D_u L) \Sigma_x (D_x + D_u L)^T + D_w \Sigma_w D_w^T$$

$$\xi_i = \phi_i \Sigma_z \phi_i^T \quad i = 1 \ldots n_z$$

Solve the following Linear Program:

$$\min \left\{ \begin{array}{c} g_q q' \\ s', m', q' \end{array} \right\} \quad \text{s.t.} \quad 0 = A s' + B m'$$

$$q_i' = \phi_i (D_x s' + D_u m') \quad \quad q_i^{\min} \leq q_i' \leq q_i^{\max}$$

$$\xi_i^{1/2} < q_i'^{\max} - q_i'$$

$$\xi_i^{1/2} < q_i' - q_i'^{\min}$$
Fixed Controller BOP Selection

Loeblein and Perkins (1999):

Controller is fixed $\iff$ EDOR has fixed size and shape
Peng et al. (2005):

Variable Controller BOP Selection

Variable Controller \iff EDOR has variable size and shape
Profit Control
(Simultaneous BOP and Controller Selection)

EDOR’s due to different controller tunings

BOP with more profit

BOP with less profit

Max Profit

Peng et al. (2005)
Profit Control
(Simultaneous BOP and Controller Selection)

\[
\min_{s', m', q', \xi, X, Y} \left\{ g \begin{bmatrix} q \\ q' \end{bmatrix} \right\} \quad \text{s.t.} \quad 0 = As' + Bm' \\
q_i' = \phi_i (D_x s' + D_u m') \quad q_i^{\min} \leq q_i' \leq q_i^{\max} \\
\xi_i^{1/2} < q_i^{\max} - q_i' \quad \xi_i^{1/2} < q_i' - q_i^{\min} \\
(AX + BY) + (AX + BY)^T + G \Sigma_w G^T < 0 \\
\begin{bmatrix}
\xi - \phi_i D_w \Sigma_w D_w^T \phi_i^T & \phi_i (D_x X + D_u Y) \\
(D_x X + D_u Y)^T \phi_i^T & X
\end{bmatrix} > 0
\]

Peng et al. (2005)
\[
\begin{align*}
\min & \left\{ g_q q' \right\} \quad \text{s.t.} \quad 0 = As' + Bm' \\
q_i' &= \phi_i (D_x s' + D_u m') \quad q_i^{\min} \leq q_i' \leq q_i^{\max} \\
\xi_i^{1/2} &< q_i^{\max} - q_i' \quad \xi_i^{1/2} < q_i' - q_i^{\min} \\
(AX + BY) + (AX + BL)^T + G \Sigma_w G^T &< 0 \\
\begin{bmatrix}
\xi_i - \phi_i D_w \Sigma_w D_w^T \phi_i^T & \phi_i (D_x X + D_u Y) \\
(D_x X + D_u Y)^T \phi_i^T & X
\end{bmatrix} &> 0
\end{align*}
\]

Peng et al. (2005)
Profit Control Applications

- Mechanical Systems
- Chemical and Reaction Systems
- Hybrid Vehicle Design
- Inventory Control
- Electric Power System Design
- Building HVAC
- Water Resource Management
Profit Control Applications

- Mechanical Systems
- Chemical and Reaction Systems
- Hybrid Vehicle Design
- Inventory Control
- Electric Power System Design
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- Water Resource Management
Mass-Spring-Damper Example

System Model:

\[
\begin{bmatrix}
\dot{r}
\
\dot{v}
\end{bmatrix}
= \begin{bmatrix}
0 & 1 \\
-2 & -3 \\
\end{bmatrix}
\begin{bmatrix}
r \\
v
\end{bmatrix}
+ \begin{bmatrix}
0 \\
1
\end{bmatrix} f
+ \begin{bmatrix}
0 \\
1
\end{bmatrix} w
\]

where \( r \) is the mass position, \( v \) is the velocity, \( f \) is the input force (MV) and \( w \) is the disturbance force.

System Constraints:

\(-1 \leq r \leq 1 \) and \( 0 \leq f \leq 16 \)
Mass-Spring-Damper Example

System Model:

\[
\begin{bmatrix}
\dot{r} \\
\dot{v}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-2 & -3
\end{bmatrix}
\begin{bmatrix}
r \\
v
\end{bmatrix} +
\begin{bmatrix}
0 \\
1
\end{bmatrix} f +
\begin{bmatrix}
0 \\
1
\end{bmatrix} w
\]

where \( r \) is the mass position, \( v \) is the velocity, \( f \) is the input force (MV) and \( w \) is the disturbance force.

System Constraints:

\(-1 \leq r \leq 1 \quad \text{and} \quad 0 \leq f \leq 16\)
Mass-Spring-Damper Example

System Model:
\[
\begin{bmatrix}
\dot{r} \\
\dot{v}
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
-2 & -3
\end{bmatrix} \begin{bmatrix}
r \\
v
\end{bmatrix} + \begin{bmatrix}
0 \\
1
\end{bmatrix} f + \begin{bmatrix}
0 \\
1
\end{bmatrix} w
\]

where \( r \) is the mass position, \( v \) is the velocity, \( f \) is the input force (MV) and \( w \) is the disturbance force.

System Constraints:
\[-1 \leq r \leq 1 \quad \text{and} \quad 0 \leq f \leq 16\]
Mass-Spring-Damper Example (Phase Plane)
Discrete-time Simulation (Scatter Plot)
MPC and the EDOR

\[
\min_{x,u} \left\{ \int_0^\infty \left( x^T Q x + 2 u^T M x + u^T R u \right) dt \right\}
\]

s.t. \quad \dot{x} = Ax + Bu + Gw

\[z(t) = D_x x + D_u u + D_w w\]

\[z_i^{\text{min}} \leq z_i(t) \leq z_i^{\text{max}} \quad i = 1 \ldots n_z\]
Soft Constraints

\[
\min_{x, u} \left\{ \int_0^\infty \left( x^T Q x + 2u^T M x + u^T R u \right) dt + s^T \Gamma s \right\}
\]

s.t. \quad \dot{x} = Ax + Bu + Gw

\[
z(t) = D_x x + D_u u + D_w w
\]

\[
z_{i, \text{min}} - s_i \leq z_i(t) \leq z_{i, \text{max}} + s_i \quad i = 1 \ldots n_z
\]

\[
s_i \geq 0
\]
Soft Constraints

\[
\min_{x,u} \left\{ \int_{0}^{\infty} \left( x^T Q x + 2 u^T M x + u^T R u \right) dt + s^T \Gamma s \right\}
\]

\[s.t. \quad \dot{x} = A x + B u + G w\]

\[z(t) = D_{x} x + D_{u} u + D_{w} w\]

\[z_{i}^{\text{min}} - s_{i} \leq z_{i}(t) \leq z_{i}^{\text{max}} + s_{i} \quad i = 1 \ldots n_{z}\]

\[s_{i} \geq 0\]
MPC with Soft Constraints

\[ \gamma_m = 10^7 \quad \gamma_f = 10^3 \]

\[ \gamma_m = 10^3 \quad \gamma_f = 10^3 \]
Flexibility in EDOR Definition

\[ \alpha = 1 \rightarrow \text{constraint observance } \sim 84\% \text{ of time} \]

\[ \alpha = 2 \rightarrow \text{constraint observance } \sim 95\% \text{ of time} \]

\[ \alpha = 3 \rightarrow \text{constraint observance } \sim 99.5\% \text{ of time} \]
Impact of EDOR Definition

\[ \alpha = 1 \]

\[ \alpha = 2 \]
MPC with Soft Constraints
EDOR = 2 std dev’s

\[ \gamma_m = 10^7 \quad \gamma_f = 10^3 \]

\[ \gamma_m = 10^3 \quad \gamma_f = 10^3 \]
Impact of EDOR Definition
(Reduced Sensitivity to Soft Weights)

\[ \gamma_m = 10^7 \quad \gamma_f = 10^3 \]

\[ \gamma_m = 10^3 \quad \gamma_f = 10^3 \]
Outline

• Model Predictive Controller Tuning
• Pseudo-Constrained Control
• Back-off and Profit Control
• Market Responsive Control
Thermal Energy Storage (TES)

In HVAC systems TES is used for Load Leveling and to shift usage to Off-Peak Hours.
Energy Prices and Weather

Cyclical pattern with a phase shift of about 3 hours.
Operation of the TES

![Diagram](chart.png)

Heat Loss to Room

Heat to Room

Heat to TES Unit

Energy Usage

Volume of Air (the Room)

Heat from Room

Heat to Cooler

Heat from TES Unit

Cooling Unit

Volume of Air (the Room)

Heat from Room

Heat to Cooler

Heat from TES Unit

Cooling Unit

Energy Usage

- Heat Leakage $T_{outside}$
- $T_{room}$
- $T_{outside}$

![Graph](graph.png)

Cents per kWh

Temperature (°C)

- Electricity Price
- Outside Temperature

59 60 61 62

59 60 61 62
Response to Market Changes

EDOR’s due to different controller tunings

BOP with more profit

BOP with less profit

OSSOP

Peng et al. (2005)
Electric Price Model

White Noise Input → Shaping Filter → Sequence with Electricity Price Characteristics
Electric Price Model

White Noise Input → Shaping Filter → Sequence with Electricity Price Characteristics

Measured Electricity Price → State Estimator and/or Predictor → Prediction of Electricity Price
Model Predictive Control

\[
\min_{v_u(t)} \left\{ \int_0^T p_e(t) \cdot v_u(t) \, dt \right\}
\]

where \( p_e(t) \) \sim \text{the predicted price (or value)}

\( v_u(t) \) \sim \text{the velocity of usage}

and \( S(t) \) \sim \text{amount in storage}

Constraints include:

\[
0 \leq v_u(t) \leq v_u^{\text{max}} \quad \text{and} \quad 0 \leq S(t) \leq S^{\text{max}}
\]
Economic MPC

\[
\min_{v_u(t)} \left\{ \int_0^T p_e(t) \ast v_u(t) \, dt \right\}
\]

where \( p_e(t) \) ~ the predicted price (or value)

\( v_u(t) \) ~ the velocity of usage

and \( S(t) \) ~ amount in storage

Constraints include :

\[
0 \leq v_u(t) \leq v_u^{\max} \quad \text{and} \quad 0 \leq S(t) \leq S^{\max}
\]
Infinite-time Economic MPC

$$\min_{\nu_u(t)} \left\{ \int_{0}^{\infty} p_e(t) * \nu_u(t) \, dt \right\}$$

where \( p_e(t) \sim \text{the predicted price (or value)} \)
\( \nu_u(t) \sim \text{the velocity of usage} \)

and \( S(t) \sim \text{amount in storage} \)

Constraints include:

$$0 \leq \nu_u(t) \leq \nu_u^{\max} \quad \text{and} \quad 0 \leq S(t) \leq S^{\max}$$
Infinite-time Economic MPC

\[
\min_{v_u(t)} \left\{ \int_0^\infty p_e(t) \cdot v_u(t) \, dt \right\} = E[p_e \cdot v_u] = \bar{C}_e
\]

where \( p_e(t) \) ~ the predicted price (or value)

\( v_u(t) \) ~ the velocity of usage

and \( S(t) \) ~ amount in storage

Constraints include:

\[
0 \leq v_u(t) \leq v_u^{\text{max}} \quad \text{and} \quad 0 \leq S(t) \leq S^{\text{max}}
\]
Finite-time Equivalent to Economic MPC

\[
\min_{v_u(t)} \left\{ \int_0^T p_e(t) \cdot v_u(t) \, dt + \Phi(p_e(T)) \right\} = E[p_e \cdot v_u]
\]

where \( p_e(t) \sim \text{the predicted price (or value)} \)

\( v_u(t) \sim \text{the velocity of usage} \)

and \( S(t) \sim \text{amount in storage} \)

Constraints include:

\[
0 \leq v_u(t) \leq v_u^{\text{max}} \quad \text{and} \quad 0 \leq S(t) \leq S^{\text{max}}
\]
Infinite-time MPC
(Chmielewski & Manousiouthakis, 1996)

\[
\min_{x,u} \left\{ \int_0^\infty \left( x^T Q x + 2u^T M x + u^T Ru \right) dt \right\}
\]

s.t. \quad \dot{x} = Ax + Bu + Gw \\
\quad z(t) = D_x x + D_u u + D_w w \\
\quad z_i^\text{min} \leq z_i(t) \leq z_i^\text{max} \quad i = 1 \ldots n_z
Infinite-time MPC  
(Chmielewski & Manousiouthakis, 1996)

\[
\min_{x,u} \left\{ \int_0^\infty \left( x^T Q x + 2u^T M x + u^T R u \right) dt \right\}
\]

s.t.  
\[ \dot{x} = Ax + Bu + Gw \]

\[ z(t) = D_x x + D_u u + D_w w \]

\[ z_i^{\text{min}} \leq z_i(t) \leq z_i^{\text{max}} \quad i = 1 \ldots n_z \]

\[
\min_{x,u} \left\{ \int_0^T \left( x^T Q x + 2u^T M x + u^T R u \right) dt + x^T (T) P x(T) \right\}
\]

s.t.  
\[ \dot{x} = Ax + Bu + Gw \]

\[ z(t) = D_x x + D_u u + D_w w \]

\[ z_i^{\text{min}} \leq z_i(t) \leq z_i^{\text{max}} \quad i = 1 \ldots n_z \]
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