Infinite Horizon Economic MPC: from Predictive Controller Tuning to Smart Grid Applications

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Presentation Outline

• Economic MPC
  ➢ Instability in EMPC
  ➢ Infinite Horizon EMPC (part 1)
  ➢ Economic Linear Optimal Control (ELOC)
  ➢ Infinite Horizon EMPC (part 2)
  ➢ Controller Embedded Process Design

• Smart Grid and the Chemical Industry
  ➢ EMPC and the Smart Grid
  ➢ Chemical Process Example
  ➢ Smart Grid Workshop for the CPI
Review of MPC

Process to be Controlled:

\[ s_{i+1} = f'(s_i, m_i, p_i) \quad q_i = h'(s_i, m_i, p_i) \quad q_{\text{min}} \leq q_i \leq q_{\text{max}} \]

Process in Deviation Variables:

\[ x_{i+1} = f(x_i, u_i, w_i) \quad z_i = h(x_i, u_i, w_i) \quad z_{\text{min}} \leq z_i \leq z_{\text{max}} \]
\[ x_i = s_i - s^{ss} \quad u_i = m_i - m^{ss} \quad w_i = p_i - p^{ss} \quad z_i = q_i - q^{ss} \]

Predictive Form:

\[ x_{k+1|i} = f(x_{k|i}, u_{k|i}, w_{k|i}) \quad i \text{ is actual time} \]
\[ z_{k|i} = h(x_{k|i}, u_{k|i}, w_{k|i}) \quad k \text{ is predictive time} \]
\[ z_{\text{min}} \leq z_{k|i} \leq z_{\text{max}} \quad k = i \ldots i + N - 1 \]
Review of MPC

At time the current time, \( i \), solve:

\[
\min_{x_{k|i}, u_{k|i}} \left\{ \sum_{k=i}^{i+N-1} l(x_{k|i}, u_{k|i}, w_{k|i}) + l_f(x_{i+N|i}) \right\} \quad \text{s.t.}
\]

\[
x_{k+1|i} = f(x_{k|i}, u_{k|i}, w_{k|i})
\]

\[
z_{k|i} = h(x_{k|i}, u_{k|i}, w_{k|i})
\]

\[
z_{\text{min}} \leq z_{k|i} \leq z_{\text{max}} \quad k = i \ldots i + N - 1
\]

\[
x_{i|i} = x_i
\]

Then, the manipulated variable is set as: \( u_i = u_{i|i}^* \).
Traditional vs. Economic MPC

In traditional MPC the objective is regulation:

\[
\min \left\{ \sum_{k=i}^{i+N-1} \left( x_{k|i}^T Q x_{k|i} + u_{k|i}^T R u_{k|i} \right) + x_{i+N|i}^T P x_{i+N|i} \right\}
\]

In EMPC the objective is maximize average profit (integral of instantaneous profit):

\[
\max \left\{ \frac{1}{N} \sum_{k=i}^{i+N-1} g^{(IP)}(s_{k|i}, m_{k|i}, p_{k|i}) \right\}
\]
Economic MPC Literature

- **Conceptual Development and Stability Issues**: Rawlings and Amrit (2009); Diehl, et al. (2011); Huang and Biegler (2011); Heidarinejad, et al. (2012)

- **Process Scheduling**: Karwana and Keblisb (2007); Baumrucker and Biegler (2010); Lima et al. (2011); Kostina et al. (2011)

- **Building HVAC Systems**: Braun (1992); Morris et al. (1994); Kintner-Meyer and Emery (1995); Henze et al. (2003); Braun (2007); Oldewurtel et al. (2010), Ma et al. (2012); Mendoza and Chmielewski (2012)

- **Power Scheduling**: Zavala et al. (2009); Xie and Ilić (2009), Hovgaard, et al. (2011), Omell and Chmielewski (2013)
Economic MPC Example

A non-isothermal CSTR:

\[
\begin{align*}
\frac{dC_A}{dt} &= \frac{\nu}{V} (C_{A0} - C_A) - k_0 e^{-E/RT} C_A \\
\frac{dC_B}{dt} &= -\frac{\nu}{V} C_B + k_0 e^{-E/RT} C_A \\
\frac{dT}{dt} &= \frac{\nu}{V} (T_0 - T) + \left( \frac{\Delta H_r}{\rho C_p} \right) k_0 e^{-E/RT} C_A
\end{align*}
\]

\[T \leq 390K\]

Disturbance: \(C_{A0} = C_{A0}^{ss} \pm 0.15 C_{A0}^{ss}\)

Manipulation: \(T_0 \leq 385K\)

Instantaneous Profit:

\[g^{(IP)}(C_B) = 10 \nu C_B\]
Economic MPC Example

\[
\max_{s_{k|i}, T_{o,k|i}} \left\{ \frac{1}{N} \sum_{k=i}^{i+N-1} 10 \nu C_{B,k|i} \right\} \quad \text{s.t.}
\]

\[
s_{k+1|i} = f(s_{k|i}, T_{o,k|i}, C_{A0,k|i})
\]

\[
s_{k|i} = \begin{bmatrix} C_{A,k|i} & C_{B,k|i} & T_{k|i} \end{bmatrix}^T
\]

\[
T_{k|i} \leq 390K
\]

\[
T_{o,k|i} \leq 385K \quad k = i \ldots i + N - 1
\]

\[
s_{i|i} = s_i
\]

\[
N = 8
\]

*Model converted to discrete-time using Euler’s explicit method.*
Economic MPC Simulation

![Diagram showing temperature (T) and concentration (C) over time (minutes)]
Economic MPC Simulation

![Graphs showing concentration changes over time for different species](image)

- **C_A** (k mole/m$^3$) over time (minutes)
- **C_B** (k mole/m$^3$) over time (minutes)
- Scatter plot of **C_A** vs **C_B**

---

**Economic MPC Simulation**

- **C_A** (k mole/m$^3$) over time (minutes)
- **C_B** (k mole/m$^3$) over time (minutes)
- Scatter plot of **C_A** vs **C_B**

---

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Economic MPC Example

A non-isothermal CSTR:

\[ C_{A_0}, T_0, \nu \]

\[ C_A, C_B, T \]
Economic MPC Example

A non-isothermal CSTR with preheater:

\[ T_f = 303K \]

Manipulation: \( 303K \leq T_0 \leq 385K \)

Instantaneous Profit:

\[ g^{(IP)}(C_B, T_0) = \nu [10C_B - 0.005(T_0 - 303)] \approx \nu [5 - 0.4] \]
Economic MPC Simulation

- $C_{A0}$ (kmole/m$^3$)
- $T_0$ (K)
- $T$ (K)

Graphs showing the changes in concentration and temperature over time.
Economic MPC Simulation

\[ g^{(IP)}(C_B, T_0) = \nu [10C_B - 0.005(T_0 - 303)] \]
Economic MPC Simulation

Change horizon from $N = 8$ to $N = 10$
Stability of EMPC

1) If $N=8$ and objective $\nu[10C_B]$, then STABLE

2) If $N=8$ and objective $\nu[10C_B - 0.005(T_0 - 303)]$, then UNSTABLE

3) If $N=10$ and objective $\nu[10C_B - 0.005(T_0 - 303)]$, then STABLE

4) If $N=10$ and objective $\nu[10C_B - 0.025(T_0 - 303)]$, then UNSTABLE

5) If $N=25$ and objective $\nu[10C_B - 0.025(T_0 - 303)]$, then STABLE
Why the Instability?

\[ T_f = 303K \]

\[ \nu \left[ 10C_B - 0.025(T_0 - 303) \right] \]

**Answer:** Myopic Behavior

- EMPC tries to lower costs by lowering \( T_0 \)
- In the short run \( T, C_B \) and Profit will remain high
- In the long run \( T, C_B \) and Profit will drop-off
- If \( N \) is small, then EMPC will not know about the drop-off
- Short term gains lead to long term losses
EMPC Example Summary

Observations:
• Objective function influences stability
• Larger horizon improves stability
• Larger horizon increases computational burden

Conclusions:
• Consider infinite horizon formulation
• Exploit analytic simplicity of infinite horizon formulation
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Infinite Horizon EMPC

\[
\max_{s_k, m_k} \left\{ \frac{1}{N} \sum_{k=0}^{N-1} g^{(IP)}(q_k) \right\} \quad \text{s.t.}
\]

\[
\lim_{N \to \infty} \begin{cases} 
  s_{k+1} = f(s_k, m_k) & k = 0 \ldots N - 1 \\
  q_k = h(s_k, m_k) & k = 0 \ldots N - 1 \\
  q^{\min} \leq q_k \leq q^{\max} & k = 0 \ldots N - 1 
\end{cases}
\]

\approx \max_{m(s)} \left\{ \lim_{N \to \infty} \frac{1}{N} \sum_{k=0}^{N-1} g^{(IP)}(q_k) \right\}

\text{s.t.} \quad \begin{cases} 
  s_{k+1} = f(s_k, m_k, p_k) \\
  q_k = h(s_k, m_k) \\
  q^{\min} \leq q_k \leq q^{\max} 
\end{cases}

\approx \max_{m(s)} \left\{ \lim_{N \to \infty} \frac{1}{N} \sum_{k=0}^{N-1} g^{(IP)}(q_k) \right\}

\text{s.t.} \quad \begin{cases} 
  s_{k+1} = f(s_k, m_k, p_k) \\
  q_k = h(s_k, m_k) \\
  q^{\min} \leq E[q_k] \leq q^{\max} \\
  E[z_k z_k^T] \leq M(q^{\min}, q^{\max}) \\
  z_k = q_k - E[q_k]
\end{cases}
Statistically Constrained Stochastic Infinite Horizon EMPC Problem

\[
\max_{m(s)} \left\{ \lim_{N \to \infty} \frac{1}{N} \sum_{k=0}^{N-1} g^{(IP)}(q_k) \right\}
\]

s.t. \[ s_{k+1} = f(s_k, m_k, p_k) \]
\[ q_k = h(s_k, m_k) \]
\[ q_{\text{min}} \leq E[q_k] \leq q_{\text{max}} \]
\[ E[z_k z_k^T] \leq M(q_{\text{min}}, q_{\text{max}}) \]
\[ z_k = q_k - E[q_k] \]
Analytic Expression for Objective Function

\[
\lim_{N \to \infty} \frac{1}{N} \sum_{k=0}^{N-1} g^{(IP)}(q_k)
\]

\[
= \lim_{k \to \infty} E\left[g^{(IP)}(q_k)\right]
\]

\[
= \lim_{k \to \infty} G^{(IP)}(\bar{q}_k, \Sigma_{z,k})
\]

\[
= G^{(IP)}(\bar{q}, \Sigma_z)
\]

\[
? \equiv g^{(IP)}(\bar{q})
\]

\[
G^{(IP)}(\bar{q}_k, \Sigma_{z,k}) = E[g^{(IP)}(q_k)]
\]

\[
\bar{q}_k = E[q_k]
\]

\[
\Sigma_{z,k} = E[z_k z_k^T]; \quad z_k = q_k - \bar{q}_k
\]

\[
\bar{q} = \lim_{k \to \infty} \bar{q}_k \quad \Sigma_z = \lim_{k \to \infty} \Sigma_{z,k}
\]
Statistically Constrained Stochastic Infinite Horizon EMPC Problem

\[
\max_{m(s), \bar{q}, \Sigma_z} \left\{ G^{(IP)}(\bar{q}, \Sigma_z) \right\} \\
\text{s.t.} \quad s_{k+1} = f(s_k, m_k, p_k) \\
q_k = h(s_k, m_k) \\
q_{\min} \leq E[q_k] \leq q_{\max} \\
E[z_k z_k^T] \leq M(q_{\min}, q_{\max}) \\
z_k = q_k - E[q_k]
\]
Steady-State Operating Point (SSOP)

In general:

\[ s_{k+1} = f(s_k, m_k, p_k) \quad \Rightarrow \quad \bar{s} = F(s, m, \bar{p}, \Sigma_z) \]
\[ q_k = h(s_k, m_k) \quad \Rightarrow \quad \bar{q} = H(s, m, \Sigma_z) \]

In many cases:

\[ s_{k+1} = f(s_k, m_k, p_k) \quad \Rightarrow \quad \bar{s} \approx f(s, m, \bar{p}) \]
\[ q_k = h(s_k, m_k) \quad \Rightarrow \quad \bar{q} \approx h(s, m) \]
Given a nonlinear process model:

\[ s_{k+1} = f(s_k, m_k, p_k) \]
\[ q_k = h(s_k, m_k) \]

And a known feedback element:

\[ m_k = m(s_k) \]

Then, determination of \( \Sigma_z \) requires solution of the:

Fokker-Planck Equation
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Analytic Expression for Variance

Given a linear process model:

\[ x_{k+1} = Ax_k + Bu_k + Gw_k \]

\[ z_k = D_x x_k + D_u u_k \]

And a linear feedback form:

\[ u_k = Lx_k \]

Then, \( \Sigma_z \) is easily determined from:

\[ \Sigma_z = (D_x + D_u L) \Sigma_x (D_x + D_u L)^T \]

\[ \Sigma_x = (A + BL) \Sigma_x (A + BL)^T + G \Sigma_w G^T \]
Economic Linear Optimal Control (ELOC)

\[
\max_{L,q,\Sigma_z} \left\{ G^{(IP)}(\bar{q}, \Sigma_z) \right\}
\]

s.t. \( \bar{q} = H(\bar{s}, \bar{m}, \Sigma_z) \quad \bar{s} = F(\bar{s}, \bar{m}, \bar{p}, \Sigma_z) \)

\[
\Sigma_z = (D_x + D_u L) \Sigma_x (D_x + D_u L)^T
\]

\[
\Sigma_x = (A + BL) \Sigma_x (A + BL)^T + G \Sigma_w G^T
\]

\[ q^{\text{min}} \leq \bar{q} \leq q^{\text{max}} \]

\[
\Sigma_z \leq M(q^{\text{min}}, q^{\text{max}})
\]
Review of Notation

\[ q_k = h(s_k, m_k) \quad \quad q^\text{min} \leq q_k \leq q^\text{max} \]

\[ \bar{q} = \lim_{k \to \infty} E[q_k] \]

\[ \sum_z = \lim_{k \to \infty} E[z_k z_k^T] \quad \quad z_k = q_k - \bar{q} \]

\[
\Sigma_z = \begin{bmatrix}
\sigma_1^2 & \sigma_{12}^2 & \sigma_{13}^2 \\
\sigma_{12}^2 & \sigma_2^2 & \sigma_{23}^2 \\
\sigma_{13}^2 & \sigma_{23}^2 & \sigma_3^2
\end{bmatrix}
\]

\[
\sigma_1^2 = \lim_{k \to \infty} E[(z_{k,1})^2] \\
\sigma_2^2 = \lim_{k \to \infty} E[(z_{k,2})^2] \\
\sigma_3^2 = \lim_{k \to \infty} E[(z_{k,3})^2]
\]
Expected Dynamic Operating Region (EDOR)

\[ (\bar{q}_1, \bar{q}_2) \]

\[ \sigma_1 \]

\[ \sigma_2 \]
A feasible EDOR is such that:

\[ \sigma_j < q_j^{\text{max}} - \bar{q}_j \]

\[ \sigma_j < \bar{q}_j - q_j^{\text{min}} \]
Economic Linear Optimal Control (ELOC)

\[
\max_{L, \bar{q}, \Sigma_z} \left\{ G^{(IP)}(\bar{q}, \Sigma_z) \right\}
\]

s.t. \quad \bar{q} = H(\bar{s}, \bar{m}, \Sigma_z) \quad \bar{s} = F(\bar{s}, \bar{m}, \bar{p}, \Sigma_z)

\[
\sigma_j \leq q_j^{\max} - \bar{q}_j \quad \sigma_j \leq \bar{q}_j - q_j^{\min}
\]

\[
\Sigma_z = (D_x + D_u L) \Sigma_x (D_x + D_u L)^T
\]

\[
\Sigma_x = (A + BL) \Sigma_x (A + BL)^T + G \Sigma_w G^T
\]

\[
\Rightarrow \bar{q}, \bar{s}, \bar{m} \text{ and } u_k = Lx_k
\]
Economic MPC Example

Instantaneous Profit:
\[ g^{(IP)}(q_k) = \nu \left[ 10 C_B - 0.025 (T_0 - 303) \right] \]
\[ G^{(IP)}(\overline{q}) = \nu \left[ 10 \overline{C}_B - 0.025 (\overline{T}_0 - 303) \right] \]

First Order Statistics:
\[ s_{k+1} = f(s_k, m_k, p_k) \Rightarrow \overline{s} \approx f(\overline{s}, \overline{m}, \overline{p}) \]
\[ q_k = h(s_k, m_k) \Rightarrow \overline{q} \approx h(\overline{s}, \overline{m}) \]

Solve Steady-State Optimization:
\[ \min \{ G^{(IP)}(\overline{q}) \} \quad s.t. \quad \overline{s} = f(\overline{s}, \overline{m}, \overline{p}) \quad \overline{q} = h(\overline{s}, \overline{m}) \]
Optimal Steady-State Operating Point (OSSOP)
Economic MPC Example

Linearize around OSSOP:

\[ s_{k+1} = f(s_k, m_k, p_k) \]

\[ q_k = h(s_k, m_k) \]

\[ \Rightarrow x_{k+1} = Ax_k + Bu_k + Gw_k \]

\[ z_k = D_x x_k + D_u u_k \]

Solve ELOC Optimization:

\[ \Rightarrow \bar{q}^*, L^* \text{ and } \Sigma_z^* \]
Optimal EDOR and Operating Point

from $\overline{q}^*$ and $\Sigma_z^*$
with \( u_k = L^* x_k \)
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Predictive form of ELOC

Optimal Control

\[
\min_{x_{k|i}, u_{k|i}} \left\{ \sum_{k=i}^{i+N-1} (x_{k|i}^T Q x_{k|i} + u_{k|i}^T R u_{k|i}) + x_{i+N|i}^T P x_{i+N|i} \right\} \quad \text{s.t.}
\]

\[
x_{k+1|i} = A x_{k|i} + B u_{k|i}
\]

\[
u_i = L_{LQR} x_i
\]

Inverse Optimality

\[
\min_{x_{k|i}, u_{k|i}} \left\{ \sum_{k=i}^{i+N-1} (x_{k|i}^T Q_{ELOC} x_{k|i} + u_{k|i}^T R_{ELOC} u_{k|i}) + x_{i+N|i}^T P_{ELOC} x_{i+N|i} \right\} \quad \text{s.t.}
\]

\[
x_{k+1|i} = A x_{k|i} + B u_{k|i}
\]

\[
u_i = L_{ELOC} x_i
\]

* see Chmielewski & Manthanwar (2004) for details
Infinite Horizon EMPC

\[
\min_{x_{k|i}, u_{k|i}} \left\{ \sum_{k=i}^{i+N-1} \left( x_{k|i}^T Q_{ELOC} x_{k|i} + u_{k|i}^T R_{ELOC} u_{k|i} \right) + x_{i+N|i}^T P_{ELOC} x_{i+N|i} \right\}
\]

s.t.

\[
x_{k+1|i} = A x_{k|i} + B u_{k|i}
\]

\[
z_{k|i} = D_x x_{k|i} + D_u u_{k|i}
\]

\[
z_{\text{min}} \leq z_{k|i} \leq z_{\text{max}}
\]
Infinite Horizon EMPC

![Graph showing temperature vs. temperature at different time points](image)

- T(K): Temperature
- T_0(K): Reference Temperature

**Diagrams**

1. **Main Graph:**
   - X-axis: T_0(K)
   - Y-axis: T(K)
   - Data points shown for different scenarios

2. **Inset Graph:**
   - X-axis: T_0(K)
   - Y-axis: T(K)
   - Expanded view of a specific scenario

**Legend:**

- Red line: Current scenario
- Blue line: Previous scenario
- Green star: Reference point

Note: The diagrams illustrate the behavior of temperature over time, with multiple scenarios represented for analysis.
Infinite vs. Finite Horizon EMPC

![Graph showing temperature over time for ELOC and IH-ELOC](image)

- **ELOC (N=20)**
- **IH-ELOC (N=10)**
Impact of Horizon Size

![Graph showing temperature changes over time for different horizon sizes.](image-url)

- **ELOC (N=20)**
- **IH-ELOC (N=10)**
- **IH-ELOC (N=1)**

![Graph showing temperature changes over time for different horizon sizes.](image-url)

- **EMPC (N=20)**
- **IH-EMPC (N=10)**
- **IH-EMPC (N=1)**
Influence of Final Cost in IH-EMPC

\[
\min_{x_{k|i}, u_{k|i}} \left\{ \sum_{k=i}^{i+N-1} \left( x_{k|i}^T Q_{ELOC} x_{k|i} + u_{k|i}^T R_{ELOC} u_{k|i} \right) + x_{i+N|i}^T P_{ELOC} x_{i+N|i} \right\} \\
\text{s.t.}
\]

\[
s_{k+1|i} = f(s_{k|i}, m_{k|i}) \quad s_{i|i} = s_i
\]

\[
q_{k|i} = h(s_{k|i}, m_{k|i})
\]

\[
q_{\min} \leq q_{k|i} \leq q_{\max}
\]
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Economic MPC Example

A non-isothermal CSTR with preheater:

\[ T_f = 303K \]

\[ Q \]

MV Constraint:

\[ T_0 \leq 385K \]

What if:

\[ T_0 \leq 384K \]

What if:

\[ T_0 \leq 386K \]
Estimated Profit from EMPC Simulation

Estimated Profit:

EMPC: $2.41/sec $2.56/sec $2.62/sec
Economic Linear Optimal Control (ELOC)

\[
\max_{L, \bar{q}, \Sigma_z} \left\{ G^{(IP)}(\bar{q}, \Sigma_z) \right\}
\]

s.t. \( \bar{q} = H(\bar{s}, \bar{m}, \Sigma_z) \quad \bar{s} = F(\bar{s}, \bar{m}, \bar{p}, \Sigma_z) \)

\[
\sigma_j \leq \left[ q_j^{\max} - \bar{q}_j \right] \quad \sigma_j \leq \bar{q}_j - \left[ q_j^{\min} \right]
\]

\[
\Sigma_z = (D_x + D_u L) \Sigma_x (D_x + D_u L)^T
\]

\[
\Sigma_x = (A + BL) \Sigma_x (A + BL)^T + G \Sigma_w G^T
\]
Estimated Profit from ELOC Solution

Estimated Profit:

**EMPC:**
- $2.41/sec
- $2.56/sec
- $2.62/sec

**ELOC:**
- $2.41/sec
- $2.49/sec
- $2.58/sec
Controller Embedded Process Design

\[
\max_{L, \bar{q}, \Sigma_z} \left\{ G^{(IP)}(\bar{q}, \Sigma_z) + \text{CapCost}(q_{\text{max}}, q_{\text{min}}) \right\}
\]

s.t.  
\[
\bar{q} = H(\bar{s}, \bar{m}, \Sigma_z) \quad \bar{s} = F(\bar{s}, \bar{m}, \bar{p}, \Sigma_z)
\]

\[
\sigma_j \leq q^\text{max}_j - \bar{q}_j \quad \sigma_j \leq \bar{q}_j - q^\text{min}_j
\]

\[
\Sigma_z = (D_x + D_u L)\Sigma_x (D_x + D_u L)^T
\]

\[
\Sigma_x = (A + BL)\Sigma_x (A + BL)^T + G\Sigma_w G^T
\]
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Real-time Pricing for Electricity

Real-time Pricing for Electricity

Texas Hub: July 2012

Electricity Price ($/MWhr)

Outside Temperature (°C)

Time (days)
Integrated Gasification Combined Cycle with Dispatch Capability

Gasification Unit \( \nu_{\text{coal}} \) → \( \nu_{\text{H}_2} \) → Gasification Unit

Storage Tank \( \nu_{\text{H}_2,S} \) → \( \nu_{\text{H}_2,G} \) → Power Block

\( P_G \)
EMPC Applied to IGCC

Energy Value ($/MWhr)

Generated Power (MW)

H₂ in Storage (tonnes)

Time (days)
EMPC and Inventory Creep
IH-EMPC

![Graph showing energy value, generated power, and hydrogen storage over time.](image)
IH-EMPC and Horizon Insensitivity

The figure illustrates the power generated over time, with two horizons: 6 hr horizon and 24 hr horizon. The graphs show that the power generated varies significantly over time, with fluctuations in the range of 0 to 1500 MW. The 6 hr horizon data is represented by a solid line, while the 24 hr horizon data is represented by a dashed line. The 1 hr horizon data is also shown for comparison. The time is measured in days, ranging from 0 to 10 days. The power generated is measured in MW (Megawatts).
Commercial Buildings with Thermal Energy Storage

Heat from Environment → Building

Heat from Building → Chiller

Heat to Chiller → TES

Heat to TES

Chiller → Power Consumption

Thermal Energy Storage
Building EMPC and Inventory Creep

![Graph showing heat to chiller and energy in storage over time.](image)

- Heat to Chiller (kW)
- Energy in Storage (MWhr)
- Time (hours)

- 1 hr horizon
- 12 hrs horizon
IH-EMPC and Horizon Insensitivity

EMPC: $N = 12$ hours  
IH-EMPC: $N = 1$ hour
### IH-EMPC and Horizon Insensitivity

#### Simulation Time:
- **EMPC 12 hr horizon:** 21,504 sec
- **IH-EMPC 1 hr horizon:** 2.8 sec

*99.987% reduction in computational effort*

#### Operating Cost:
- **EMPC 12 hr horizon:** $746
- **IH-EMPC 1 hr horizon:** $774

*3.75% increase in operating costs*

Without TES the operating cost is $845 for the 6 days (~9% reduction)
Capital Cost of Thermal Energy Storage

?? ⇐ $$$
Presentation Outline

- Economic MPC
  - Instability in EMPC
  - Infinite Horizon EMPC (part 1)
  - Economic Linear Optimal Control (ELOC)
  - Infinite Horizon EMPC (part 2)
  - Controller Embedded Process Design

- Smart Grid and the Chemical Industry
  - EMPC and the Smart Grid
  - Chemical Process Example
  - Smart Grid Workshop for the CPI
A Chemical Plant
Steam Utilities

Diagram showing a network of steam utilities with nodes labeled $P_1$, $P_2$, $P_3$, and $P_4$. The diagram illustrates the flow of steam through these nodes, ultimately leading to a connection with the Utility Plant.
Possible Electric Utilities

Diagram showing the possible electric utilities with components labeled as $P_1$, $P_2$, $P_3$, and $P_4$, connected to the Utility Plant and Electric Grid.
Flexible Utilities

Steam Plant

Fuel

Steam Utilities

Electric Utilities

Electric Node

Steam Bank

Electric Node

Electric Node

Electric Node
Option of Co-generation

```
\begin{aligned}
\text{fuel}_{sp} & \quad \rightarrow \quad \text{Steam Plant} \\
\text{fuel}_{cog} & \quad \rightarrow \quad \text{Co-Generation Plant}
\end{aligned}
```

```
\begin{aligned}
\text{Steam Bank} & \quad \rightarrow \quad \text{Steam Utilities} \\
\text{Electric Utilities} & \quad \rightarrow \quad \text{Electric Node}
\end{aligned}
```

$e_s^1, e_s^2, e_s^3, e_s^4, e_e^1, e_e^2, e_e^3, e_e^4$
Operation with Real-time Electric Prices

- **Electricity Price ($/GJ)**
- **Fuel to Steam Plant (GJ/s)**
- **Fuel to Co-Gen Plant (GJ/s)**
- **Power from Grid (GJ/s)**
Revenue with Real-time Electric Prices

Electricity Price ($/GJ)

Instantaneous Revenue ($/s)

Average revenue found to be $32.3/sec

A 22.8% increase wrt baseline $26.3/sec
Previous case required only RTO

With storage we need EMPC
Revenue with Real-time Pricing and Storage

EMPC average profit: $34.2/sec
A 30% increase wrt baseline
$26/sec
A 7.2% increase wrt RTO
$32.3/sec
AIChE Workshop on Smart Grid for the Chemical Process Industry
September 25-27, 2013 Chicago

Will focus on process related questions of operational flexibility and methods to evaluate and utilize this flexibility within Demand Response (DR).

- Do existing chemical plants possess sufficient flexibility for DR? How does one implement DR to exploit process flexibility?
- What is the cost saving one can expect from DR? Are there well developed assessment methods?
- Are there opportunities to create additional flexibility? Can one convert chemical processes from steam driven to electric driven? What are the economics of creating new flexibility? What is the payback period?
- What is the role of a steam/electric co-generation plant?
- Will cyclic operation impact equipment and maintenance costs?
- What are the experiences of others in the demand response community?

https://www.aiche.org/conferences/workshop-on-smart-grid-chemical-process-industry/2013
AIChe Workshop on Smart Grid for the Chemical Process Industry
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Conclusions

- EMPC provides desired economic performance
  - Many applications including smart grid
- EMPC has challenges:
  - Instability and inventory creep for small horizons
  - Bang-bang actuation and chattering
  - Large computational effort
- Infinite Horizon EMPC benefits:
  - Guaranteed stability and virtually no inventory creep
  - Reduces bang-bang and chattering
  - Small computational effort for small horizons