New Perspectives in Control System Design: Pseudo-Constrained to Market Responsive Control

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EDOR’s due to different controller tunings
BOP with less profit

BOP with more profit
OSSOP

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Outline

• Motivating Example
• Pseudo-Constrained Control
• Profit Control
• Market Responsive Control
Motivating Example  
(Non-isothermal Reactor)

\[ \begin{align*}
V \frac{dC_A}{dt} &= F(C_{A\text{in}} - C_A) + Vr_A \\
V \frac{dT}{dt} &= F(T_{\text{in}} - T) + (V\Delta H / \rho C_p) r_A \\
r_A &= -k(T)C_A
\end{align*} \]

Increase \( F \) \( \rightarrow \) Increased production rate
Motivating Example
(Non-isothermal Reactor)

\[ V \frac{dC_A}{dt} = F(C_{Ain} - C_A) + Vr_A \]
\[ V \frac{dT}{dt} = F(T_{in} - T) + \left( \frac{V\Delta H}{\rho C_p} \right) r_A \]
\[ r_A = -k(T)C_A \]

Increase F \rightarrow Increased production rate
Decrease F \rightarrow Increase T \rightarrow Increase reaction rate
\rightarrow Increase production
Limited Operating Region

Process Limitations:

\[ T(t) \leq T^{(\text{max})} \quad - \text{Catalyst protection or onset of side reactions} \]

\[ F(t) \leq F^{(\text{max})} \quad - \text{Pump limit or limit on downstream unit} \]
Limited Operating Region

Process Limitations:

\[ T(t) \leq T^{(\text{max})} \]
- Catalyst protection or onset of side reactions

\[ F(t) \leq F^{(\text{max})} \]
- Pump limit or limit on downstream unit

Possible Controller:

\[ F = K_c (T - T^{(sp)}) + F^{(sp)} \]
Performance in Time Series

\[ F(t) \]

\[ F^{(sp)} \]

\[ T(t) \]

\[ T^{(sp)} \]

\[ T^{(max)} \]

\[ F^{(max)} \]

\[ C_A, T \]
Performance in Phase Plane
Dynamic Operating Region

\[ T(t) \]

\[ F(t) \]
Expected Dynamic Operating Region

\[ T(t) \]

\[ F(t) \]
Steady-State Relation

Controller:

\[ F = K_c (T - T^{(sp)}) + F^{(sp)} \]

Steady-State Relation:

\[ F^{(sp)} = f (T^{(sp)}) \]
Dynamic Operating Region

\[ T(t) \]

\[ F(t) \]

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Steady-State Operating Line

\[ T(t) \]

\[ F(t) \]
Optimal Operating Point

$F(t)$

$T(t)$

Decrease $F$ → Increase $T$ → Increase conversion → Increase production
Optimal Operating Point: Another Possibility

Increase $F$ → Increased production rate
Optimal Operating Point: Another Possibility

Increase $F$ → Increased production rate
Requires Different Controller Tuning

\[ T(t) \]

\[ F(t) \]
Less Aggressive Tuning

\[ T(t) \]

\[ F(t) \]
Need for Automated Tuning

\[ T(t) \quad \quad \quad F(t) \]
Outline

• Motivating Example

• Pseudo-Constrained Control

• Profit Control

• Market Responsive Control
Mass-Spring-Damper Example

$r$ is the mass position
$v$ is the velocity
$f$ is the input force (MV) and
$w$ is the disturbance force
Mass-Spring-Damper Example

System Model:

\[
\begin{bmatrix}
\dot{r} \\
\dot{v}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-2 & -3
\end{bmatrix}
\begin{bmatrix}
r \\
v
\end{bmatrix} +
\begin{bmatrix}
0 \\
1
\end{bmatrix} f +
\begin{bmatrix}
0 \\
1
\end{bmatrix} w
\]

- \( r \) is the mass position
- \( v \) is the velocity
- \( f \) is the input force (M V) and
- \( w \) is the disturbance force
Mass-Spring-Damper Example

System Model:

\[
\begin{bmatrix}
\dot{r} \\
\dot{v}
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
-2 & -3
\end{bmatrix} \begin{bmatrix}
r \\
v
\end{bmatrix} + \begin{bmatrix}
0 \\
1
\end{bmatrix} f + \begin{bmatrix}
0 \\
1
\end{bmatrix} w
\]

System Constraints:

\[-1 \leq r \leq 1\]

and

\[0 \leq f \leq 16\]

\(r\) is the mass position
\(v\) is the velocity
\(f\) is the input force (MV) and
\(w\) is the disturbance force
Mass-Spring-Damper Example

System Model:

\[
\begin{align*}
\dot{r} &= \begin{bmatrix} 0 & 1 \end{bmatrix} r + \begin{bmatrix} 0 \\ 0 \end{bmatrix} f + \begin{bmatrix} 0 \\ 0 \end{bmatrix} w \\
\dot{v} &= \begin{bmatrix} -2 & -3 \end{bmatrix} v + \begin{bmatrix} 1 \\ 0 \end{bmatrix} f + \begin{bmatrix} 0 \\ 0 \end{bmatrix} w \\
\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} [r \ v] + \begin{bmatrix} 0 \\ 0 \end{bmatrix} f + \begin{bmatrix} 0 \\ 0 \end{bmatrix} w
\end{align*}
\]

System Constraints:

\[-1 \leq r \leq 1 \quad \text{and} \quad 0 \leq f \leq 16\]

\[
\begin{align*}
\dot{x} &= Ax + Bu + Gw \\
z &= Dx x + Du u + Dw w \\
-\bar{z}_i &\leq z_i(t) \leq \bar{z}_i \\
i = 1 \ldots n_z
\end{align*}
\]
Covariance Analysis
(Open-Loop Case)

**Process Model:**

\[ \dot{x} = Ax + Gw \]
\[ z = Dx \]
\[ w(t) \text{ Gaussian white noise with covariance } \Sigma_w \]

**Steady State Covariance:**

\[ A \Sigma_x + \Sigma_x A^T + G \Sigma_w G^T = 0 \]
\[ \Sigma_z = D \Sigma_x D^T \]
Expected Dynamic Operating Region (EDOR)

EDOR defined by:

\[
\Sigma_z = \begin{bmatrix}
\sigma_{11}^2 & \sigma_{12}^2 \\
\sigma_{21}^2 & \sigma_{22}^2
\end{bmatrix}
\]
Closed-Loop Covariance Analysis
(Full State Information Case)

Process Model:
\[
\begin{align*}
\dot{x} &= Ax + Bu + Gw \\
z &= Dx x + Du u + Dw w
\end{align*}
\]

Controller:
\[
u(t) = Lx(t)
\]

Steady-State Covariance:
\[
\begin{align*}
(A + BL) \Sigma_x + \Sigma_x (A + BL)^T + G \Sigma_w G^T &= 0 \\
\Sigma_z &= (D_x + Du L) \Sigma_x (D_x + Du L)^T + Dw \Sigma_w Dw^T
\end{align*}
\]
Closed-Loop EDOR

$u = L_1 x$

$u = L_2 x$

EDOR’s from different controllers
Constrained Closed-Loop EDOR

Constraints

\( \sigma_{zi} < \bar{z}_i \)
Constrained Closed-Loop EDOR

Constraints

\( \sigma_{zi} < \bar{z}_i \)
Constrained Controller Existence

Does there exist $L$ such that:

$$(A + BL) \Sigma_x + \Sigma_x (A + BL)^T + G \Sigma_w G^T = 0$$

$$\Sigma_z = (D_x + D_u L) \Sigma_x (D_x + D_u L)^T + D_w \Sigma_w D_w^T$$

$$\xi_i = \phi_i \Sigma_z \phi_i^T < \bar{z}_i^2 \quad i = 1 \ldots n_z$$

$$\phi_i = \begin{bmatrix} 0 & 0 & \ldots & 1 & \ldots & 0 & 0 \end{bmatrix}$$

$\uparrow \ i^{th}$ column
Constrained Controller Existence  
(Convex Condition)

If and only if there exist $X > 0$ and $Y$ such that:

$$(AX + BY) + (AX + BY)^T + G \Sigma_w G^T < 0$$

$$
\begin{bmatrix}
\xi_i - \phi_i D_w \Sigma_w D_w^T \phi_i^T & \phi_i (D_x X + D_u Y) \\
(D_x X + D_u Y)^T \phi_i^T & X
\end{bmatrix} > 0
$$

$$\xi_i < \bar{z}_i^2 \quad i = 1 \ldots n_z$$

And controller $u = Lx$ is constructed as: $L = YX^{-1}$

$\phi_i = \begin{bmatrix} 0 & 0 & \ldots & 1 & \ldots & 0 & 0 \end{bmatrix}$
Region of Unconstrained Controller Existence

\[ \xi_1 \]

\[ \xi_2 \]

Achievable Performance Levels

Unachievable
Region of Constrained Controller Existence

\[ \xi_1 < \bar{z}_1 \]

\[ \xi_2 < \bar{z}_2 \]
Constrained Controller Existence

\[ T(t) \]

\[ F(t) \]
Pseudo-Constrained Control

\[ \min_{X > 0, Y, \xi_i} \sum_i d_i \xi_i \]

such that:

\[ (AX + BY) + (AX + BY)^T + G \Sigma_w G^T < 0 \]

\[
\begin{bmatrix}
\xi_i - \phi_i D_w \Sigma_w D_w^T \phi_i^T & \phi_i (D_x X + D_u Y) \\
(D_x X + D_u Y)^T \phi_i^T & X
\end{bmatrix} > 0
\]

\[ \xi_i < \bar{z}_i^2 \quad i = 1 \ldots n_z \]
Pareto Frontier Interpretation

All Pseudo-Constrained Controllers are on the Pareto Frontier

Achievable Region

Unachievable Region

\( \xi_1 \)

\( \xi_2 \)
MPC Equivalence

Theorem 1 (Chmielewski & Manthanwar, 2004):

All controllers generated by Pseudo-Constrained Control (PCC)

are coincident with

a controller generated by some Unconstrained Model Predictive Controller.
Outline

- Model Predictive Controller Tuning
- Pseudo-Constrained Control
- Profit Control
- Market Responsive Control
Constrained Operating Region

Steady-State Operating Point

MV’s

CV’s

Constraints

EDOR
Real-Time Optimization

Original Nonlinear Process Model:

\[ \dot{s} = f(s,m,p) \quad q = h(s,m,p) \]

\((s,m,p,q) \sim \text{(state, mv, dist, performance)} \sim (x,u,w,z)\)
Real-Time Optimization

Original Nonlinear Process Model:

\[
\dot{s} = f(s, m, p) \quad q = h(s, m, p)
\]

\((s, m, p, q) \sim \text{(state, mv, dist, performance)} \sim (x, u, w, z)\)

Real-Time Optimization (minimize profit loss):

\[
\min_{s, m, q} \left\{ g(q) \right\} \quad \text{s.t.}
\]

\[
0 = f(s, m, p) \quad q = h(s, m, p) \quad q_i^{\text{min}} \leq \phi_i q \leq q_i^{\text{max}}
\]

RTO solution denoted as \((s^{\text{ossop}}, m^{\text{ossop}}, p^{\text{ossop}}, q^{\text{ossop}})\)
Real-Time Optimization

Steady-State Operating Point

Constraints

Optimal Steady-State Operating Point (OSSOP)

EDOR

CV’s

MV’s
Backed-off Operating Point (BOP)

EDOR

Optimal Steady-State Operating Point (OSSOP)

CV’s

MV’s
Steady-State BOP Selection
(Bahri, Bandoni & Romagnoli, 1996)

Solve the following Semi-infinite Programming Problem

\[
\begin{align*}
\min_{s,m,q} \{ \ g(q) \ \} \quad \text{s.t.} \quad & \ \max_{p \in [p^{\min}, p^{\max}]} \max_i \{ q_i - \bar{q}_i \} \\
\quad \text{s.t.} \quad & \ 0 = f(s, m, p) \\
\quad & \ q = h(s, m, p) \\
\quad & \ q_i - \bar{q}_i < 0 
\end{align*}
\]

Extensions:
Natural Variables

Nonlinear

\[ g(q^{bop}) \]
\[ \dot{s} = f(s, m, p) \]
\[ q = h(s, m, p) \]
\[ q_i^{\text{min}} \leq q_i \leq q_i^{\text{max}} \]
Deviation Variables

Nonlinear

\[ g(q^{bop}) \]
\[ \dot{s} = f(s, m, p) \]
\[ q = h(s, m, p) \]
\[ q_i^{\text{min}} \leq q_i \leq q_i^{\text{max}} \]

Linear w.r.t OSSOP

\[ g(q^{ossop}) + g_q q' \]
\[ \dot{s}' = A s' + B m' + G p' \]
\[ q' = D_x s' + D_u m' + D_w p' \]
\[ q_i'^{\text{min}} \leq q_i' \leq q_i'^{\text{max}} \]

Deviation Variables w.r.t. OSSOP:

\[ s' = s^{bop} - s^{ossop} \]
\[ m' = m^{bop} - m^{ossop} \]
\[ p' = p^{bop} - p^{ossop} \]
\[ q' = q^{bop} - q^{ossop} \]
# More Deviation Variables

## Nonlinear

\[ g(q^{bop}) \]

\[ \dot{s} = f(s, m, p) \]

\[ q = h(s, m, p) \]

\[ q_i^{\min} \leq q_i \leq q_i^{\max} \]

## Linear wrt OSSOP

\[ g(q^{ossop}) + g_q q' \]

\[ \dot{s}' = As' + Bm' + Gp' \]

\[ q' = D_x s' + D_u m' + D_w p' \]

\[ q_i'^{\min} \leq q_i' \leq q_i'^{\max} \]

## Linear wrt BOP

\[ \dot{x} = Ax + Bu + Gw \]

\[ z = D_x x + D_u u + D_w w \]

\[ z_i^{\min} \leq z_i \leq z_i^{\max} \]

## Deviation Variables w.r.t. OSSOP:

\[ s' = s^{bop} - s^{ossop} \]

\[ m' = m^{bop} - m^{ossop} \]

\[ p' = p^{bop} - p^{ossop} \]

\[ q' = q^{bop} - q^{ossop} \]

## Deviation Variables w.r.t. BOP:

\[ x = s - s^{bop} \]

\[ u = m - m^{bop} \]

\[ w = p - p^{bop} \]

\[ z = q - q^{bop} \]
Stochastic BOP Selection
(Loeblein & Perkins, 1999)
Stochastic BOP Selection
(Loeblein & Perkins, 1999)

Assume controller $L$ is given and calculate $\xi_i$:

$$(A + BL) \Sigma_x + \Sigma_x (A + BL)^T + G \Sigma_w G^T = 0$$

$$\Sigma_z = (D_x + D_u L) \Sigma_x (D_x + D_u L)^T + D_w \Sigma_w D_w^T$$

$$\xi_i = \phi_i \Sigma_z \phi_i^T \quad i = 1 \ldots n_z$$
Assume controller $L$ is given and calculate $\xi_i$:

$$
(A + BL) \Sigma_x + \Sigma_x (A + BL)^T + G \Sigma_w G^T = 0
$$

$$
\Sigma_z = (D_x + D_u L) \Sigma_x (D_x + D_u L)^T + D_w \Sigma_w D_w^T
$$

$$
\xi_i = \phi_i \Sigma_z \phi_i^T \quad i = 1 \ldots n_z
$$

Solve the following Linear Program:

$$
\min_{s', m', q'} \{ g_q q' \} \quad \text{s.t.} \quad 0 = As' + Bm'
$$

$$
q_i' = \phi_i (D_x s' + D_u m') \quad q_i^{\min} \leq q_i' \leq q_i^{\max}
$$

$$
\xi_i^{1/2} < q_i^{\max} - q_i' \quad \xi_i^{1/2} < q_i' - q_i^{\min}
$$
Fixed Controller BOP Selection

Loeblein and Perkins (1999):

Controller is fixed $\implies$ EDOR has fixed size and shape
Peng et al. (2005):

Variable Controller BOP Selection

Variable Controller $\iff$ EDOR has variable size and shape

EDOR

OSSOP
Profit Control
(Simultaneous BOP and Controller Selection)

EDOR’s due to different controller tunings

BOP with more profit

BOP with less profit

Max Profit

Peng et al. (2005)
Stochastic BOP Selection
(Loeblein & Perkins, 1999)

Assume controller $L$ is given and calculate $\xi_i$:

$$(A + BL)\Sigma_x + \Sigma_x (A + BL)^T + G\Sigma_w G^T = 0$$

$$\Sigma_z = (D_x + D_u L)\Sigma_x (D_x + D_u L)^T + D_w \Sigma_w D_w^T$$

$$\xi_i = \phi_i \Sigma_z \phi_i^T \quad i = 1 \ldots n_z$$

Solve the following Linear Program:

$$\min_{s',m',q'} \{ g_q q' \} \quad \text{s.t.} \quad 0 = As' + Bm'$$

$$q_i' = \phi_i (D_x s' + D_u m')$$

$$q_i^\min \leq q_i' \leq q_i^\max$$

$$\xi_i^{1/2} < q_i'^\max - q_i'$$

$$\xi_i^{1/2} < q_i' - q_i'^\min$$
Profit Control
(Simultaneous BOP and Controller Selection)

\[
\min_{s',m',q'} \left\{ g_q q' \right\} \quad \text{s.t.} \quad 0 = As' + Bm' \\
\xi_i, X, Y
\]

\[
q_i' = \phi_i (D_x s' + D_u m') \quad q_i^{\min} \leq q_i' \leq q_i^{\max}
\]

\[
\xi_i^{1/2} < q_i^{\max} - q_i' \quad \xi_i^{1/2} < q_i' - q_i^{\min}
\]

\[
(AX + BY) + (AX + BY)^T + G \Sigma_w G^T < 0
\]

\[
\left[
\begin{array}{ccc}
\xi_i - \phi_i D_w \Sigma_w D_w^T & \phi_i^T & \phi_i (D_x X + D_u Y) \\
(D_x X + D_u Y)^T & \phi_i^T & X
\end{array}
\right] > 0
\]
Computational Aspects of Profit Control

\[
\min_{s', m', q', \xi_i, X, Y} \left\{ g_{q, q'} \right\} \quad \text{s.t.} \quad 0 = As' + Bm' \\
q_i' = \phi_i (D_x s' + D_u m') \quad q_i^{\min} \leq q_i' \leq q_i^{\max} \\
{\xi_i}^{1/2} < q_i^{\max} - q_i' \\
{\xi_i}^{1/2} < q_i' - q_i^{\min} \\
(AX + BY) + (AX + BL)^T + G \Sigma_w G^T < 0 \\
\left[ \xi_i - \phi_i D_w \Sigma_w D_w^T \phi_i^T \phi_i (D_x X + D_u Y) \right] > 0 \\
(\begin{array}{cc}
\xi_i & (D_x X + D_u Y)^T \\
(D_x X + D_u Y) & X
\end{array}) > 0
\]

Peng et al. (2005)
Reverse-Convex Constraints

\[ \xi_1 < (q_1^{\text{max}} - q_1')^2 \]

\[ \xi_1 < (q_1' - q_1^{\text{min}})^2 \]
Global Solution

Based on Branch and Bound algorithm

![Graph showing regions and variables](image)
Profit Control Applications

- Mechanical Systems
- Chemical and Reaction Systems
- Hybrid Vehicle Design
- Inventory Control
- Electric Power System Design
- Building HVAC
- Water Resource Management
Profit Control Applications

- Mechanical Systems
- Chemical and Reaction Systems
- Hybrid Vehicle Design
- Inventory Control
- Electric Power System Design
- Building HVAC
- Water Resource Management
**Fluidized Catalytic Cracker**

**Regenerator and Separator (dynamic):**

\[
W \frac{dC_{reg}}{dt} = F_{cat} (C_{st} - C_{reg}) - R_{cb}
\]
\[
W_a \frac{dO_d}{dt} = \frac{F_{air}}{M_{air}} (O_{in} - O_d) - \frac{1 + 1.5 \sigma R_{cb}}{1 + \sigma} \frac{R_{cb}}{M_c}
\]
\[
W_{c_{pc}} \frac{dT_{reg}}{dt} = F_{cat} c_{pc} T_{st} + F_{air} c_{pair} T_{air}
- (F_{cat} c_{pc} + F_{air} c_{pair}) T_{reg} - \left( \Delta H_{CO} + \frac{\sigma}{1 + \sigma} \Delta H_{CO_2} \right) \frac{R_{cb}}{M_c}
\]
\[
W_{st} \frac{dC_{at}}{dt} = F_{cat} (C_{sc} - C_{st})
\]
\[
W_{st} c_{pc} \frac{dT_{st}}{dt} = F_{cat} c_{pc} (T_{ro} - T_{st})
\]
\[
T_{cy} = T_{reg} + 5.555 O_d
\]

**Riser (pseudo steady state):**

\[
\frac{dy_f}{dz} = -K_1 y_f \left[ COR \right] \phi T_c, \quad y_f(z = 0) = 1
\]
\[
\frac{dy_g}{dz} = (K_2 y_f^2 - K_3 y_g) \left[ COR \right] \phi T_c, \quad y_g(z = 0) = 0
\]
\[
\frac{d\theta}{dz} = \frac{\Delta H_f F_{feed}}{T_{ri} \left( F_{cat} c_{cp} + F_{feed} c_{pf} + \lambda F_{feed} c_{pc} \right)} \frac{dy_f}{dz}
\]
\[
\theta(z) = \frac{T(z) - T_{ri}}{T_{ri}}, \quad \theta(z = 0) = 0, \quad T_{ro} = T(z = 1)
\]

(adapted from Loeblein & Perkins, 1999)
FCC Constraints and Economics

Process Constraints:

\[ 400 \, K \leq T_{st} \leq 1000 \, K \]
\[ 600 \, K \leq T_{reg} \leq 1000 \, K \]
\[ T_{reg} \leq T_{cy} \leq 1000 \, K \]
\[ 100 \frac{kg}{s} \leq F_{cat} \leq 400 \frac{kg}{s} \]
\[ 0 \leq F_{air} \leq 60 \, kg/s \]

Profit Function:

\[ \Phi = 86400 \left( P_{gs} F_{gs} + P_{gl} F_{gl} + P_{ugo} F_{ugo} - P_{uog} F_{Feed} \right) \]

\( F_{gs}, F_{gl}, \) and \( F_{ugo} \) are product flows (gasoline, light gas and unconverted oil).

(adapted from Loeblein & Perkins, 1999)
Profit Control vs. Fixed Controller Back-off
## FCC Profit

<table>
<thead>
<tr>
<th></th>
<th>Gross Profit ($/day)</th>
<th>Diff from OSSOP ($/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OSSOP</td>
<td>$36,905</td>
<td>$0.0</td>
</tr>
<tr>
<td>Fixed Control</td>
<td>$34,631</td>
<td>- $2,274</td>
</tr>
<tr>
<td>Profit Control</td>
<td>$35,416</td>
<td>- $1,489</td>
</tr>
</tbody>
</table>

Improves profit by 2%
Hybrid Vehicle Design

Fuel Cell $E_{fc}$

$i_{fc}$

$R_{bat}$ $E_{bat}$

$i_{bat}$

$R_{scap}$ $E_{scap}$

$i_{scap}$

$E_{arm}$ $R_{arm}$ $L_{arm}$ $w_{arm}$

Power Bus

$i_{arm}$
DC-DC Converters

Fuel Cell $E_{fc}$

DC-DC Converter $i_{fc} \rightarrow i_{afc}$

DC-DC Converter $i_{abat} \rightarrow i_{bat}$

DC-DC Converter $i_{scap} \rightarrow i_{ascap}$

Power Bus $E_{arm}$

$k_{fc}$

$k_{bat}$

$k_{scap}$

$R_{bat}$

$R_{scap}$

$L_{arm}$

$w_{arm}$

Illinois Institute of Technology
Department of Chemical and Biological Engineering
Servo-Loops with PI Controllers

Vehicle Power System

\[ P_{scap} \]  \quad \rightarrow \quad k_{scap} \quad \rightarrow \quad P_{scap} \]

\[ P_{bat} \]  \quad \rightarrow \quad k_{bat} \quad \rightarrow \quad P_{bat} \]

\[ P_{fc} \]  \quad \rightarrow \quad k_{fc} \quad \rightarrow \quad P_{fc} \]
Supervisory Control

\[ P_{mot}^{(sp)} \]

\[ P_{fc}^{(sp)} \]

\[ P_{bat}^{(sp)} \]

\[ P_{scap}^{(sp)} \]

\[ k_{fc} \]

\[ k_{bat} \]

\[ k_{scap} \]

Vehicle Power System
Drive Cycle Data and Modeling
High Level Battery Model

\[ \dot{E}_{bat} = P_{bat} - P_{bat}^{(loss)} \]
High Level Battery Model

\[ \dot{E}_{bat} = P_{bat} - P_{bat}^{(loss)} \]

\[ E_{\text{bat}}^{\text{min}} \leq E_{\text{bat}} \leq E_{\text{bat}}^{\text{max}} \]

\[ E_{\text{bat}}^{\text{min}} = 0 \]

\[ E_{\text{bat}}^{\text{max}} = \hat{e}_{\text{bat}} m_{\text{bat}} \]
High Level Battery Model

\[ \dot{E}_{bat} = P_{bat} - P^{(\text{loss})}_{bat} \]

\[ E_{\text{min}} \leq E_{bat} \leq E_{\text{max}} \]
\[ E_{bat} = 0 \]
\[ E_{\text{max}} = \hat{e}_{bat} m_{bat} \]

\[ P_{\text{min}} \leq P_{bat} \leq P_{\text{max}} \]
\[ P_{bat} = -\hat{C}_{bat} \hat{e}_{bat} m_{bat} \]
\[ P_{\text{min}} = \hat{C}_{bat} \hat{e}_{bat} m_{bat} \]
Power and Energy Constraints of the Battery

\[ P_{bat}^{\max} = C_{bat}^{\text{rate},c} E_{bat}^{\max} \]

\[ E_{bat}^{\max} = \hat{e}_{bat} m_{bat} \]

\[ E_{bat}^{\min} = 0 \]

\[ P_{bat}^{\min} = -C_{bat}^{\text{rate},d} E_{bat}^{\max} \]
Constraints a Function of the Mass

\[ P_{bat}^{\text{max}} = C_{bat}^{\text{rate},c} E_{bat}^{\text{max}} \]

\[ E_{bat} = \hat{e}_{bat} m_{bat} \]
Aspect Ratio a Function of C-Rate

\[
P_{bat}^{\text{max}} = C_{rate, bat}^{\text{c}} E_{bat}^{\text{max}}
\]

\[
E_{bat}^{\text{max}} = \hat{c}_{bat} m_{bat}
\]
High Level Battery Model

\[ \dot{E}_{bat} = P_{bat} - P_{bat}^{(loss)} \]

\[ P_{bat}^{(loss)} = \frac{P_{bat}^2}{\hat{l}_{bat} m_{bat}} \]

\[ E_{bat}^{\min} \leq E_{bat} \leq E_{bat}^{\max} \]

\[ E_{bat}^{\min} = 0 \]

\[ E_{bat}^{\max} = \hat{e}_{bat} m_{bat} \]

\[ P_{bat}^{\min} \leq P_{bat} \leq P_{bat}^{\max} \]

\[ P_{bat}^{\min} = -\hat{C}_{bat} \hat{e}_{bat} m_{bat} \]

\[ P_{bat}^{\min} = \hat{C}_{bat} \hat{e}_{bat} m_{bat} \]
Operating Region with Power Losses

\[ E_{bat} \]
\[ P_{bat} \]
High Level Super Cap Model

\[ \dot{E}_{sc} = P_{sc} - P_{sc}^{(loss)} \]

\[ P_{sc}^{(loss)} = \frac{P_{sc}^2}{\hat{l}_{sc} m_{sc}} \]

\[ E_{sc}^{\text{min}} \leq E_{sc} \leq E_{sc}^{\text{max}} \]

\[ E_{sc}^{\text{min}} = 0 \]

\[ E_{sc}^{\text{max}} = \hat{e}_{sc} m_{sc} \]

\[ P_{sc}^{\text{min}} \leq P_{sc} \leq P_{sc}^{\text{max}} \]

\[ P_{sc}^{\text{min}} = -\hat{C}_{sc}^{\text{rate,d}} \hat{e}_{sc} m_{sc} \]

\[ P_{sc}^{\text{min}} = \hat{C}_{sc}^{\text{rate,c}} \hat{e}_{sc} m_{sc} \]
High Level Fuel Cell Model

\[ \dot{P}_{fc} = \Delta P_{fc} \]

\[ P_{fc}^{\text{min}} \leq P_{fc} \leq P_{fc}^{\text{max}} \]

\[ P_{fc}^{\text{min}} = 0 \]

\[ P_{fc}^{\text{max}} = \hat{p}_{fc} m_{fc} \]

\[ \Delta P_{fc}^{\text{min}} \leq \Delta P_{fc} \leq \Delta P_{fc}^{\text{max}} \]

\[ \Delta P_{fc}^{\text{min}} = -\Delta C_{bat}^{\text{rate,d}} \hat{p}_{fc} m_{fc} \]

\[ \Delta P_{fc}^{\text{max}} = \Delta C_{bat}^{\text{rate,c}} \hat{p}_{fc} m_{fc} \]
Power Losses Increases Average Fuel Cell Power

\[ P_{fc} \]
\[ \Delta P_{fc} \]
\[ E_{bat} \]
\[ P_{bat} \]
Design Problem Formulation

\[
\min_{s.t.}\{\hat{c}_{fc} m_{fc} + \hat{c}_{bat} m_{bat} + \hat{c}_{sc} m_{sc}\}
\]

\[
\begin{align*}
\bar{P}_{fc} &= P_o + \bar{P}_{bat} + \bar{P}_{sc} \\
m_v &= m_o + m_{fc} + m_{bat} + m_{sc} \\
[P_{bat} \quad \sigma_{bat} \quad \bar{P}_{bat} ] &> 0 \\
[\sigma_{bat} \quad l_{bat} m_{bat} \quad 0 ] &> 0 \\
[\bar{P}_{bat} \quad 0 \quad l_{bat} m_{bat} ] &> 0 \\
[\bar{P}_{sc} \quad \sigma_{sc} \quad \bar{P}_{sc} ] &> 0 \\
[\sigma_{sc} \quad l_{sc} m_{sc} \quad 0 ] &> 0 \\
[\bar{P}_{sc} \quad 0 \quad l_{sc} m_{sc} ] &> 0
\end{align*}
\]

\[
\begin{bmatrix}
-(AX + BY) - (AX + BY)^T \\
G_o + m_v G_1 \\
\sum_w^{-1}
\end{bmatrix}
> 0
\]

\[
[\xi_i \quad \phi_i (D_x X + D_u Y) ] > 0
\]

\[
i = 1...6
\]

\[
\begin{align*}
\xi_i &< \sigma_i^2 \\
\sigma_i &> 0 \\
i &= 1...6
\end{align*}
\]

\[
\begin{align*}
\sigma_1 &< P_{fc}^{\max} - \bar{P}_{fc} \\
\sigma_1 &> \bar{P}_{fc} - P_{fc}^{\min}
\end{align*}
\]

\[
\begin{align*}
\sigma_2 &< E_{bat}^{\max} - \bar{E}_{bat} \\
\sigma_2 &> \bar{E}_{bat} - E_{bat}^{\min}
\end{align*}
\]

\[
\begin{align*}
\sigma_3 &< E_{sc}^{\max} - \bar{E}_{sc} \\
\sigma_3 &> \bar{E}_{sc} - E_{sc}^{\min}
\end{align*}
\]

\[
\begin{align*}
\sigma_4 &< \Delta P_{fc}^{\max} - \Delta \bar{P}_{fc} \\
\sigma_4 &> \Delta \bar{P}_{fc} - \Delta P_{fc}^{\min}
\end{align*}
\]

\[
\begin{align*}
\sigma_5 &< P_{bat}^{\max} - \bar{P}_{bat} \\
\sigma_5 &> \bar{P}_{bat} - P_{bat}^{\min}
\end{align*}
\]

\[
\begin{align*}
\sigma_6 &< P_{sc}^{\max} - \bar{P}_{sc} \\
\sigma_6 &> \bar{P}_{sc} - P_{sc}^{\min}
\end{align*}
\]
# Case Study Data

<table>
<thead>
<tr>
<th>Technology</th>
<th>Lithium Battery</th>
<th>Super-Capacitor</th>
<th>PEM Fuel Cell</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$59/kg</td>
<td>$93/kg</td>
<td>$300/kg</td>
</tr>
<tr>
<td>C-Rate</td>
<td>0.5 hr⁻¹</td>
<td>360 hr⁻¹</td>
<td>10 hr⁻¹</td>
</tr>
<tr>
<td>Power Density</td>
<td>100 W/kg</td>
<td>110,000 W/kg</td>
<td>1 W/kg</td>
</tr>
</tbody>
</table>

Appetecchi & Prosini (2005)
Murphy, Cisar & Clarke (1998)
## Case Study Solution

<table>
<thead>
<tr>
<th>Technology</th>
<th>Lithium Battery</th>
<th>Super-Capacitor</th>
<th>PEM Fuel Cell</th>
<th>Total Capital Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>30 kg</td>
<td>2.8 kg</td>
<td>3.5 kg</td>
<td></td>
</tr>
<tr>
<td>Nominal Power</td>
<td>0.1 kW</td>
<td>1.5 kW</td>
<td>1.8 kW</td>
<td></td>
</tr>
<tr>
<td>Cost</td>
<td>$1770</td>
<td>$260</td>
<td>$1050</td>
<td>$3,080</td>
</tr>
</tbody>
</table>
Optimal Fuel Cell Size and Operating Region

![Graph showing Fuel Cell Power (kW) over time (hr)]

Fuel Cell

- $P_{fc}$ (kW)
- $\Delta P_{fc}$ (kW/s)

![Graph showing Fuel Cell Power (kW) over time (hr)]

Fuel Cell Power (kW)
Optimal Super Cap Size and Operating Region

Super Capacitor

SuperCap Power, kW

E_{sc} (kJ) vs. P_{sc} (kW)
Optimal Battery Size and Operating Region
Building HVAC

Heat Leakage ($T_{outside}$ measured)

Volume of Air (the Room) $T_{room}, C_{room}$

Solid Material $T_{solid}$

Contaminant Source: $S_c$

Control Variables: $T_{room}$ and $C_{room}$

Manipulated Variables: $F_{rcy}$ and $F_{fresh}$

Disturbances: $T_{outside}$ and $S_c$

Air Processing Unit ($T_{cool} = 20^\circ C$)

$F_{rcy}, T_{room}, C_{room}$

$F_{rcy}, T_{cool}, C_{room}$

$F_{fresh}, T_{room}, C_{room}$

$F_{fresh}, T_{cool}, C_{fresh}$

$F_{fresh}, T_{outside}, C_{fresh}$ ($C_{fresh} = 0$)

Energy Usage
HVAC Control

Energy Usage of Traditional Controller: 3.16 kW
Energy Usage of Energy Efficient Controller: 2.55 kW
(a reduction of almost 20%).
Outline

• Model Predictive Controller Tuning
• Pseudo-Constrained Control
• Back-off and Profit Control
• Market Responsive Control
Thermal Energy Storage (TES)

In HVAC systems TES is used for Load Leveling and to shift usage to Off-Peak Hours
Energy Prices and Weather

Cyclical pattern with a phase shift of about 3 hours.
Operation of the TES

- Heat Leakage $T_{outside}$
- Volume of Air (the Room) $T_{room}$
- Heat from Room
- Heat to Cooler
- Heat to TES Unit
- Cooling Unit
- TES Unit
- Energy Usage

Graph:
- Cents per k hr
- Temperature (°C)
- Time (days)
- Electricity Price
- Outside Temperature

59 60 61 62

0 10 20 30 40

0 59 60 61 62
Response to Market Changes

EDOR’s due to different controller tunings

BOP with more profit

BOP with less profit

OSSOP

Peng et al. (2005)
Electric Price Model

White Noise Input → Shaping Filter → Sequence with Electricity Price Characteristics
Electric Price Model

- White Noise Input
- Measured Electricity Price

**Shaping Filter**

- Sequence with Electricity Price Characteristics

**State Estimator and/or Predictor**

- Prediction of Electricity Price
Model Predictive Control

\[
\min_{v_u(t)} \left\{ \int_0^T p_e(t) \ast v_u(t) \, dt \right\}
\]

where \( p_e(t) \sim \) the predicted price (or value)
\( v_u(t) \sim \) the velocity of usage
and \( S(t) \sim \) amount in storage

Constraints include:

\[
0 \leq v_u(t) \leq v_u^{\max} \quad \text{and} \quad 0 \leq S(t) \leq S^{\max}
\]
Model Predictive Control

\[
\min_{v_u(t)} \left\{ \int_0^T p_e(t) \cdot v_u(t) \, dt \right\} \approx E[p_e \cdot v_u] = \bar{C}_e
\]

where \( p_e(t) \sim \) the predicted price (or value)
\( v_u(t) \sim \) the velocity of usage
and \( S(t) \sim \) amount in storage

Constraints include:

\[
0 \leq v_u(t) \leq v_u^{\text{max}} \quad \text{and} \quad 0 \leq S(t) \leq S^{\text{max}}
\]
System Design

\[
\min_{\nu_u(t)} \left\{ \int_0^T p_e(t) \ast \nu_u(t) \, dt \right\} \equiv E[p_e \ast \nu_u] \equiv \bar{C}_e
\]

How does \( \nu_u^{\text{max}} \) and \( S^{\text{max}} \) impact \( \bar{C}_e \)?

\( 0 \leq \nu_u(t) \leq \nu_u^{\text{max}} \) and \( 0 \leq S(t) \leq S^{\text{max}} \)
System Design

\[
\min_{v_u(t)} \left\{ \int_0^T p_e(t) * v_u(t) \, dt \right\} \approx E[p_e * v_u] \equiv \bar{C}_e
\]

How does \( v_u^{\text{max}} \) and \( S^{\text{max}} \) impact \( \bar{C}_e \)?

\( (0 \leq v_u(t) \leq v_u^{\text{max}} \) and \( 0 \leq S(t) \leq S^{\text{max}} )\)

MPC cannot answer this question!
Expected Cost of Electricity

White Noise Input

Shaping Filter

$pe(t)$

Process Model

$vu(t)$

$E[pe*vu]$
Re-Scaling of Price

\[ \alpha w(t) \rightarrow \text{Shaping Filter} \rightarrow p'(t) \rightarrow \text{Process Model} \rightarrow \nu_u(t) \]

\( (p'_e \equiv \alpha p_e) \)

\[ E[p'_e \nu_u] \]

Manipulated Variables

(Controller is \( u = Lx \))
Correlating Price and Usage

If $E\left[\left(p'_e - v_u\right)^2\right] < \varepsilon \equiv 0$ and $p'_e \equiv \alpha \ p_e$

then $v_u(t) \equiv \alpha \ p_e(t)$
Correlating Price and Usage

If \( E[(p'_e - v_u)^2] < \varepsilon \equiv 0 \) and \( p'_e \equiv \alpha \ p_e \)

then \( v_u(t) \equiv \alpha \ p_e(t) \)

\[
E[\alpha^2 p_e^2] - 2E[\alpha p_e v_u] + E[v_u^2] \equiv 0
\]

\[
\Rightarrow \alpha^2 E[p_e^2] = \alpha E[p_e v_u] = E[v_u^2]
\]

\[
\Rightarrow \alpha \ E[p_e^2] = E[p_e v_u] \equiv \bar{C}_e
\]
Minimum Cost of Electricity

\[ \bar{C}_e = \min_{L, \alpha} \{ c_R \alpha \} \]

\[ (c_R = E[p_e^2]) \]

\[ E[(p'_e - v_u)^2] < \varepsilon \approx 0 \]

\[ E[v_u^2] < (v_u^{\text{max}})^2 \]

\[ E[S^2] < (S^{\text{max}})^2 \]
Thermal Energy Storage
(Small Storage Unit)

Volume of Air (the Room) → Heat from Room → Heat to Cooler → Cooling Unit → Energy Usage

Heat Leakage $T_{outside}$ → $T_{room}$ → Heat to TES Unit → TES Unit

Graph:
- Heat from Room
- Heat to Cooler
- Heat to TES Unit

Time (days) vs. kW/hr/day

Illinois Institute of Technology
Department of Chemical and Biological Engineering
Thermal Energy Storage
(Medium Storage Unit)

Volume of Air (the Room) $V_{room}$

Heat Leakage $T_{outside}$

Heat from Room

Heat to Cooler

Cooling Unit

Energy Usage

Heat to TES Unit

TES Unit

Graph showing energy usage:
- Red line: Heat from Room
- Green line: Heat to Cooler
- Blue line: Heat to TES Unit

$kW/hr/day$ vs. Time (days)
Thermal Energy Storage
(Large Storage Unit)

Volume of Air (the Room) $V_{room}$

Heat from Room

Heat to Cooler

Cooling Unit

Energy Usage

Heat to TES Unit

TES Unit

Heat Leakage $T_{outside}$

Heat from Room

Heat to Cooler

Heat to TES Unit

Heat Usage

Graph showing kW hr/day vs. Time (days) for Heat from Room, Heat to Cooler, and Heat to TES Unit.
Thermal Energy Storage
(Comparison of Storage Cases)
Thermal Energy Storage
(Cost Comparisons)

Average Cooling Costs:
One ton: $8 per day
Five tons: $7 per day (14% savings)
Ten tons: $6 per day (25% savings)
Minimum Levelized Cost

\[
\min_{L, \alpha, \nu_u^{\text{max}}, S^{\text{max}}} \left\{ c_R \alpha + c_L, \nu_u^{\text{max}} + c_L, S^{\text{max}} \right\}
\]

\[
E[ (p_e' - \nu_u)^2 ] < \varepsilon \approx 0
\]

\[
E[\nu_u^2] < (\nu_u^{\text{max}})^2
\]

\[
E[S^2] < (S^{\text{max}})^2
\]
**Minimum Levelized Cost**

\[
\min_{L, \alpha, v_u \text{ max}, S \text{ max}} \left\{ c_R \alpha + c_{L,1} v_u \text{ max} + c_{L,2} S \text{ max} \right\}
\]

\[
E\left[ (p_e' - v_u)^2 \right] < \varepsilon \approx 0
\]

\[
E\left[ v_u^2 \right] < (v_u \text{ max})^2
\]

\[
E\left[ S^2 \right] < (S \text{ max})^2
\]

**Non-Convex Problem**

(but global solution from branch and bound)
Integrated Gasification Combined Cycle (IGCC)
Acknowledgements

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  - Syed Amed

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  - Chemical & Biological Engineering Department, IIT
Conclusions

• Relationship between control system performance and plant profit quantified.
• Enables profit guided control system design.
• Broad set of applications from a variety of disciplines.
• Linear controller can be designed for market responsiveness.
• Non-convex, but global methods can be used to size and/or select equipment.