Outline

• Motivating Example
• Pseudo-Constrained Control
• Profit Control
• Market Responsive Control
Motivating Example 
(Non-isothermal Reactor)

\[ V \frac{dC_A}{dt} = F(C_{Ain} - C_A) + Vr_A \]
\[ V \frac{dT}{dt} = F(T_{in} - T) + \left( \frac{V\Delta H}{\rho C_p} \right) r_A \]
\[ r_A = -k(T)C_A \]

Increase \( F \) \( \rightarrow \) Increased production rate
Motivating Example
(Non-isothermal Reactor)

\[ V \frac{dC_A}{dt} = F(C_{Ain} - C_A) + Vr_A \]
\[ V \frac{dT}{dt} = F(T_{in} - T) + \frac{V \Delta H}{\rho C_p} r_A \]
\[ r_A = -k(T)C_A \]

Increase F \rightarrow Increased production rate
Decrease F \rightarrow Increase T \rightarrow Increase reaction rate
\rightarrow Increase production
Limited Operating Region

Process Limitations:

\[ T(t) \leq T^{(\text{max})} \]
- Catalyst protection or onset of side reactions

\[ F(t) \leq F^{(\text{max})} \]
- Pump limit or limit on downstream unit
Limited Operating Region

Process Limitations:

\[ T(t) \leq T^{(\text{max})} \]
- Catalyst protection or onset of side reactions

\[ F(t) \leq F^{(\text{max})} \]
- Pump limit or limit on downstream unit

Possible Controller:

\[ F = K_c (T - T^{(sp)}) + F^{(sp)} \]
Performance in Time Series
Performance in Phase Plane

$T(t)$

$F(t)$
Dynamic Operating Region

\[ T(t) \]

\[ F(t) \]
Expected Dynamic Operating Region

\[ T(t) \]

\[ F(t) \]
Steady-State Relation

Controller:

\[ F = K_c (T - T^{(sp)}) + F^{(sp)} \]

Steady-State Relation:

\[ F^{(sp)} = f (T^{(sp)}) \]
Dynamic Operating Region

\[ T(t) \]

\[ F(t) \]
Steady-State Operating Line

\[ T(t) \]

\[ F(t) \]
Optimal Operating Point

- Decrease $F$ → Increase $T$
- Increase conversion
- Increase production

Graph showing $T(t)$ and $F(t)$, with a star indicating the optimal operating point.
Optimal Operating Point: Another Possibility

Increase $F$ \rightarrow Increased production rate
Optimal Operating Point: Another Possibility

$T(t)$

Increase $F$ → Increased production rate

$F(t)$
Requires Different Controller Tuning

\[ T(t) \]

\[ F(t) \]
Less Aggressive Tuning

\[
T(t) \quad F(t)
\]

- \( T(t) \)
- \( F(t) \)

\[
F^{(sp)}(t) \quad F^{(max)}(t)
\]

\[
T^{(sp)}(t) \quad T^{(max)}(t)
\]

Time

\[
T(t) \quad F(t)
\]

\[
T^{(sp)}(t) \quad T^{(max)}(t)
\]

Time
Need for Automated Tuning

\[ F = K_c (T - T^{(sp)}) + F^{(sp)} \]
Fluidized Catalytic Cracker

Regenerator and Separator (dynamic):

\[
\begin{align*}
W \frac{dC_{rgc}}{dt} &= F_{cat}(C_{st} - C_{rgc}) - R_{cb} \\
W_a \frac{dO_d}{dt} &= F_{air} \left( O_{in} - O_d \right) - \frac{1 + 1.5 \sigma R_{cb}}{1 + \sigma} \frac{M_c}{M_{air}} \\
W_{c_{pc}} \frac{dT_{reg}}{dt} &= F_{cat} c_{pc} T_{st} + F_{air} c_{pair} T_{air} \\
&- (F_{cat} c_{pc} + F_{air} c_{pair}) T_{reg} - \left( \Delta H_{CO} + \frac{\sigma}{1 + \sigma} \Delta H_{CO_2} \right) \frac{R_{cb}}{M_c} \\
W_{st} \frac{dc_{at}}{dt} &= F_{cat} (C_{sc} - C_{st}) \\
W_{st} c_{pc} \frac{dT_{st}}{dt} &= F_{cat} c_{pc} (T_{ro} - T_{st})
\end{align*}
\]

Riser (pseudo steady state):

\[
\begin{align*}
\frac{dy_f}{dz} &= -K_1 y_f [COR] \phi t_c, \quad y_f(z = 0) = 1 \\
\frac{dy_g}{dz} &= (K_2 y_f^2 - K_3 y_g) [COR] \phi t_c, \quad y_g(z = 0) = 0 \\
\frac{d\theta}{dz} &= \frac{\Delta H_f F_{feed}}{T_{ri} \left( F_{cat} c_{cp} + F_{feed} c_{pf} + \lambda F_{feed} c_{pc} \right)} \frac{dy_f}{dz} \\
\theta(z) &= \frac{T(z) - T_{ri}}{T_{ri}}, \quad \theta(z = 0) = 0, \quad T_{ro} = T(z = 1)
\end{align*}
\]

(adapted from Loeblein & Perkins, 1999)
FCC Constraints and Economics

Process Constraints:

- $400 \, K \leq T_{st} \leq 1000 \, K$
- $600 \, K \leq T_{reg} \leq 1000 \, K$
- $T_{reg} \leq T_{cy} \leq 1000 \, K$
- $100 \frac{kg}{s} \leq F_{cat} \leq 400 \frac{kg}{s}$
- $0 \leq F_{air} \leq 60 \, kg/s$

Profit Function:

$$\Phi = 86400 \left( P_{gs}F_{gs} + P_{gl}F_{gl} + P_{ugo}F_{ugo} - P_{uog}F_{Feed} \right)$$

$F_{gs}$, $F_{gl}$ and $F_{ugo}$ are product flows (gasoline, light gas and unconverted oil).

(adapted from Loeblein & Perkins, 1999)
Model Predictive Control

\[
\min_{x,u} \left\{ \int_0^\infty (x^T Q x + u^T R u) dt \right\}
\]

s.t. \( \dot{x} = A x + B u + G w \)

\( z = D_x x + D_u u + D_w w \)

\( z_{i_{\text{min}}} \leq z_i(t) \leq z_{i_{\text{max}}} \quad i = 1 \ldots n_z \)
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• Market Responsive Control
Model Predictive Control

\[
\min_{x,u} \left\{ \int_{0}^{\infty} (x^T Q x + u^T R u) dt \right\}
\]

s.t. \( \dot{x} = Ax + Bu + Gw \)

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\( z_{i}^{\text{min}} \leq z_i(t) \leq z_i^{\text{max}} \quad i = 1\ldots n_z \)
Surrogate Controller

\[
\min_{x,u} \left\{ \int_0^\infty (x^T Q x + u^T R u) dt \right\}
\]

s.t. \[ \dot{x} = Ax + Bu + Gw \]
\[ z = D_x x + D_u u + D_w w \]
\[ \sigma_{z_i} \leq \min\{ z_i^{\text{min}}, z_i^{\text{max}} \} \quad i = 1 \ldots n_z \]
Covariance Analysis  
(Open-Loop Case)

**Process Model:**

\[
\dot{x} = Ax + Gw \\
z = Dx \\
w(t) \text{ Gaussian white noise with covariance } \Sigma_w
\]

**Steady State Covariance:**

\[
A \Sigma_x + \Sigma_x A^T + G \Sigma_w G^T = 0 \\
\Sigma_z = D \Sigma_x D^T
\]
Covariance Analysis
(Open-Loop Case)

Process Model:
\[
\dot{x} = Ax + Gw \\
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\]

Steady State Covariance:
\[
A \Sigma_x + \Sigma_x A^T + G \Sigma_w G^T = 0 \\
\Sigma_z = D \Sigma_x D^T
\]
Expected Dynamic Operating Region (EDOR)

EDOR defined by:

\[ \Sigma_z = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 \end{bmatrix} \]
Closed-Loop Covariance Analysis
(Full State Information Case)

Process Model:
\[
\begin{align*}
\dot{x} &= Ax + Bu + Gw \\
z &= Dx x + Du u + Dw w
\end{align*}
\]

Controller:
\[
u(t) = Lx(t)
\]

Steady-State Covariance:
\[
(A + BL) \Sigma_x + \Sigma_x (A + BL)^T + G \Sigma_w G^T = 0
\]
\[
\Sigma_z = (D_x + Du L) \Sigma_x (D_x + Du L)^T + Dw \Sigma_w Dw^T
\]
Closed-Loop Covariance Analysis
(Full State Information Case)

Process Model:
\[ \dot{x} = Ax + Bu + Gw \]
\[ z = D_x x + D_u u + D_w w \]

Controller:
\[ u(t) = Lx(t) \]

Steady-State Covariance:
\[ (A + BL) \Sigma_x + \Sigma_x (A + BL)^T + G \Sigma_w G^T = 0 \]
\[ \Sigma_z = (D_x + D_u L) \Sigma_x (D_x + D_u L)^T + D_w \Sigma_w D_w^T \]
Closed-Loop Covariance Analysis  
(Full State Information Case)

**Process Model:**

\[
\dot{x} = Ax + Bu + Gw \\
z = Dx x + Du u + Dw w
\]

**Controller:**

\[u(t) = Lx(t)\]

**Steady-State Covariance:**

\[
(A + BL) \Sigma_x + \Sigma_x (A + BL)^T + G \Sigma_w G^T = 0
\]

\[
\Sigma_z = (D_x + Du L) \Sigma_x (D_x + Du L)^T + Dw \Sigma_w Dw^T
\]
Closed-Loop EDOR

EDOR’s from different controllers

\[ u = L_1 x \]

\[ u = L_2 x \]
Pseudo-Constrained Control

\[ \sigma_{zi} < \bar{z}_i \]
Pseudo-Constrained Control

\[ z_1 \]

Constraints
\[ \sigma_{z_i} < \bar{z}_i \]
PC Controller Existence

Does there exist $L$ such that:

$$(A + BL)\Sigma_x + \Sigma_x (A + BL)^T + G\Sigma_w G^T = 0$$

$$\Sigma_z = (D_x + D_u L)\Sigma_x (D_x + D_u L)^T + D_w \Sigma_w D_w^T$$

$$\xi_i = \phi_i \Sigma \phi_i^T < \bar{z_i}^2 \quad i = 1 \ldots n_z$$

$$\phi_i = \begin{bmatrix} 0 & 0 & \ldots & 1 & \ldots & 0 & 0 \end{bmatrix}^T$$
PC Controller Existence
(Convex Condition)

If and only if there exist $X > 0$ and $Y$ such that:

$$(AX + BY) + (AX + BY)^T + G\Sigma_w G^T < 0$$

$$\begin{bmatrix}
\xi_i - \phi_i D_w \Sigma_w D_w^T \phi_i^T & \phi_i (D_x X + D_u Y) \\
(D_x X + D_u Y)^T \phi_i^T & X
\end{bmatrix} > 0$$

$$\xi_i < \bar{z}_i^2 \quad i = 1 \ldots n_z$$

And controller $u = Lx$ is constructed as: $L = YX^{-1}$
Region of PC Controller Existence

Achievable Performance Levels

Unachievable

\( \xi_1 \)

\( \xi_2 \)
Minimum Variance Pseudo-Constrained Control

$$\min_{X>0,Y,\xi} \sum_i d_i \xi_i$$

such that:

$$(AX + BY) + (AX + BY)^T + G\Sigma_w G^T < 0$$

$$\left[\begin{array}{ccc}
\xi_i - \phi_i D_w \Sigma_w D_w^T \phi_i^T & \phi_i (D_x X + D_u Y) \\
(D_x X + D_u Y)^T \phi_i^T & X
\end{array}\right] > 0$$

$$\xi_i < \bar{z}_i^2 \quad i = 1 \ldots n_z$$
Pareto Frontier Interpretation

All MV Pseudo-Constrained Controllers are on the Pareto Frontier

Achievable Region

Unachievable Region
MPC Equivalence

Theorem 1 (Chmielewski & Manthanwar, 2004):

All controllers generated by MV Pseudo-Constrained Control (PCC) are coincident with a controller generated by some Unconstrained Model Predictive Controller.
MV Pseudo-Constrained Control

\[
\min_{X>0, Y, \xi, \eta} \sum_i d_i \xi_i
\]

such that:

\[
(AX + BY) + (AX + BY)^T + G \Sigma_w G^T < 0
\]

\[
\left[ \xi_i - \phi_i D_w \Sigma_w D_w^T \phi_i^T \phi_i (D_x X + D_u Y) \right] > 0
\]

\[
\xi_i < \bar{z}_i^2 \quad i = 1 \ldots n_z
\]

And controller \( u = Lx \) is constructed as: \( L = YX^{-1} \)
Model Predictive Control

\[
\min_{x,u} \left\{ \int_{0}^{\infty} \left( x^T Q x + u^T R u \right) dt \right\}
\]

s.t. \( \dot{x} = A x + B u + G w \)

\[
z = D_x x + D_u u + D_w w
\]

\[
z_{i}^{\min} \leq z_{i}(t) \leq z_{i}^{\max} \quad i = 1 \ldots n_z
\]
Inverse Optimality

Theorem 2 (Chmielewski & Manthanwar, 2004):

If there exists $P > 0$ and $R > 0$ such that

$$\begin{bmatrix}
L^T RL - A^T P - PA & -(L^T R + PB) \\
-(L^T R + PB)^T & R
\end{bmatrix} > 0$$

then $M = -(L^T R + PB)$ and $Q = L^T RL - A^T P + PA$ are such that

$$\begin{bmatrix}
Q & M \\
M^T & R
\end{bmatrix} > 0$$

and $P$ and $L$ satisfy

$$A^T P + PA + Q - (PB + M)R^{-1}(PB + M)^T = 0$$

$$L = -R^{-1}(PB + M)^T$$
Model Predictive Control

\[
\min_{x,u} \left\{ \int_{0}^{\infty} \left( x^T Q x + 2u^T M x + u^T R u \right) dt \right\}
\]

s.t. \quad \dot{x} = A x + B u + G w

\[
z = D_x x + D_u u + D_w w
\]

\[
z_i^{\text{min}} \leq z_i(t) \leq z_i^{\text{max}} \quad i = 1 \ldots n_z
\]
Outline

• Model Predictive Controller Tuning
• Pseudo-Constrained Control
• Profit Control
• Market Responsive Control
Constrained Operating Region

CV’s

MV’s

Constraints
Real-Time Optimization

Original Nonlinear Process Model:

\[ \dot{s} = f(s,m,p) \quad q = h(s,m,p) \]

\[(s,m,p,q) \sim (\text{state, mv, dist, performance}) \sim (x,u,w,z)\]
Real-Time Optimization

Original Nonlinear Process Model:

\[
\dot{s} = f(s, m, p) \quad q = h(s, m, p)
\]

\((s, m, p, q) \sim \text{(state, mv, dist, performance)} \sim (x, u, w, z)\)

Real-Time Optimization (minimize profit loss):

\[
\min_{s, m, p} \left\{ g(q) \right\} \quad \text{s.t.}
\]

\[
0 = f(s, m, p) \quad q = h(s, m, p) \quad q_i^{\min} \leq \phi_i q \leq q_i^{\max}
\]

RTO solution denoted as \((s^{\text{ossop}}, m^{\text{ossop}}, p^{\text{ossop}}, q^{\text{ossop}})\)
Constrained Operating Region

CV’s

MV’s

Constraints
Real-Time Optimization

CV’s

Constraints

Optimal Steady-State Operating Point (OSSOP)

MV’s
Backed-off Operating Point (BOP)

Optimal Steady-State Operating Point (OSSOP)

MV’s

CV’s

EDOR
Steady-State BOP Selection  
(Bahri, Bandoni & Romagnoli, 1996)

Solve the following Semi-infinite Programming Problem

\[
\begin{align*}
\min_{s,m,q} \{ g(q) \} & \quad \text{s.t.} \quad \max_{p \in [p^{\min}, p^{\max}]} \max_i \left\{ q_i - \bar{q}_i \right\} \\
\text{s.t.} \quad 0 &= f(s,m,p) \\
q &= h(s,m,p) \\
q_i - \bar{q}_i &< 0
\end{align*}
\]

Extensions:

Assume controller $L$ is given and calculate $\xi_i$:

$$(A + BL) \Sigma_x + \sum_x (A + BL)^T + G \Sigma_w G^T = 0$$

$$\Sigma_z = (D_x + D_u L) \Sigma_x (D_x + D_u L)^T + D_w \Sigma_w D_w^T$$

$$\xi_i = \phi_i \Sigma_z \phi_i^T \quad i = 1 \ldots n_z$$
Assume controller $L$ is given and calculate $\xi_i$:

$$(A + BL) \Sigma_x + \Sigma_x (A + BL)^T + G \Sigma_w G^T = 0$$

$$\Sigma_z = (D_x + D_u L) \Sigma_x (D_x + D_u L)^T + D_w \Sigma_w D_w^T$$

$$\xi_i = \phi_i \Sigma_z \phi_i^T \quad i = 1 \ldots n_z$$

Solve the following Linear Program:

$$\min_{s',m',q'} \left\{ \begin{array}{l} g_q q' \end{array} \right\} \quad \text{s.t.} \quad 0 = As' + Bm'$$

$$q_i' = \phi_i (D_x s' + D_u m') \quad q_i^{\min} \leq q_i' \leq q_i^{\max}$$

$$\xi_i^{1/2} < q_i^{\max} - q_i'$$

$$\xi_i^{1/2} < q_i' - q_i^{\min}$$
Fixed Controller BOP Selection

Loeblein and Perkins (1999):

Controller is fixed $\iff$ EDOR has fixed size and shape

\[ \chi \]

\[ \mathcal{u} \]

EDOR

OSSOP
Variable Controller BOP Selection

Peng, Manthanwar & Chmielewski (2005):

Variable Controller $\iff$ EDOR has variable size and shape
Profit Control
(Simultaneous BOP and Controller Selection)

- EDOR’s due to different controller tunings
- BOP with more profit
- BOP with less profit
- Max Profit

* 

Illinois Institute of Technology
Department of Chemical and Biological Engineering
Stochastic BOP Selection
(Loeblein & Perkins, 1999)

Assume controller $L$ is given and calculate $\xi_i$:

$$(A + BL)\Sigma_x + \Sigma_x (A + BL)^T + G\Sigma_w G^T = 0$$

$$\Sigma_z = (D_x + D_u L)\Sigma_x (D_x + D_u L)^T + D_w \Sigma_w D_w^T$$

$$\xi_i = \phi_i \Sigma_z \phi_i^T \quad i = 1 \ldots n_z$$

Solve the following Linear Program:

$$\min \left\{ g_q q' \right\} \quad \text{s.t.} \quad 0 = As' + Bm'$$

$$q_i' = \phi_i (D_x s' + D_u m')$$

$$q_i^{\min} \leq q_i' \leq q_i^{\max}$$

$$\xi_i^{1/2} < q_i^{\max} - q_i'$$

$$\xi_i^{1/2} < q_i' - q_i^{\min}$$
Profit Control
(Simultaneous BOP and Controller Selection)

\[
\begin{align*}
\min_{s',m',q'} \left\{ g_{q' q'} \right\} & \quad \text{s.t.} \quad 0 = A s' + B m' \\
q'_i &= \phi_i (D_x s' + D_u m') \quad q_i^{\min} \leq q'_i \leq q_i^{\max} \\
\xi_i^{1/2} &< q_i^{\max} - q'_i \quad \xi_i^{1/2} < q'_i - q_i^{\min} \\
(AX + BY) + (AX + BY)^T + G \Sigma_w G^T &< 0 \\
\\
\begin{bmatrix}
\xi_i - \phi_i D_w \Sigma_w D_i^T & \phi_i (D_x X + D_u Y) \\
(D_x X + D_u Y)^T & X
\end{bmatrix} &> 0
\end{align*}
\]

Peng et al. (2005)
Computational Aspects of Profit Control

\[
\min_{\xi_i, X, Y} \left\{ g_q q' \right\} \quad \text{s.t.} \quad 0 = A s' + B m'
\]

\[
q_i' = \phi_i (D_x s' + D_u m') \quad q_i^{\min} \leq q_i' \leq q_i^{\max}
\]

\[
(AX + BY) + (AX + BL)^T + G \Sigma_w G^T < 0
\]

\[
\begin{bmatrix}
\xi_i - \phi_i D_w \Sigma_w D_w^T \phi_i^T & \phi_i (D_x X + D_u Y) \\
(D_x X + D_u Y)^T \phi_i^T & X
\end{bmatrix} > 0
\]

Peng et al. (2005)
Reverse-Convex Constraints

Feasible Region

\[ \xi_1 < (q_1^{\text{max}} - q_1')^2 \]

\[ \xi_1 < (q_1' - q_1^{\text{min}})^2 \]
Global Solution

Based on Branch and Bound algorithm
Profit Control Applications

- Mechanical Systems
- Chemical and Reaction Systems
- Hybrid Vehicle Design
- Inventory Control
- Electric Power System Design
- Building HVAC
- Water Resource Management
Profit Control Applications

- Mechanical Systems
- Chemical and Reaction Systems
- Hybrid Vehicle Design
- Inventory Control
- Electric Power System Design
- Building HVAC
- Water Resource Management
Fluidized Catalytic Cracker

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- (F_{cat} c_{pc} + F_{air} c_{pair}) T_{reg} - \left( \Delta H_{CO} + \frac{\sigma}{1 + \sigma} \Delta H_{CO_2} \right) \frac{R_{cb}}{M_c} \\
W_{st} \frac{dC_{st}}{dt} = F_{cat} (C_{sc} - C_{st}) \\
W_{st} c_{pc} \frac{dT_{st}}{dt} = F_{cat} c_{pc} (T_{ro} - T_{st}) \\
T_{cy} = T_{reg} + 5.555 O_a
\]

Riser (pseudo steady state):

\[
\frac{dy_f}{dz} = -K_1 y_f [COR] \phi t_c, \quad y_f(z = 0) = 1 \\
\frac{dy_g}{dz} = (K_2 y_f^2 - K_3 y_g) [COR] \phi t_c, \quad y_g(z = 0) = 0 \\
\frac{d\theta}{dz} = \frac{\Delta H_f F_{feed}}{T_{ri} (F_{cat} c_{cp} + F_{feed} c_{p f} + \lambda F_{feed} c_{pc})} \frac{dy_f}{dz} \\
\theta(z) = \frac{T(z) - T_{ri}}{T_{ri}}, \quad \theta(z = 0) = 0, \quad T_{ro} = T(z = 1)
\]
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Process Constraints:

\[400 \, K \leq T_{st} \leq 1000 \, K\]
\[600 \, K \leq T_{reg} \leq 1000 \, K\]
\[T_{reg} \leq T_{cy} \leq 1000 \, K\]
\[100 \frac{kg}{s} \leq F_{cat} \leq 400 \frac{kg}{s}\]
\[0 \leq F_{air} \leq 60 \, kg/s\]

Profit Function:

\[\Phi = 86400 \left( P_{gs} F_{gs} + P_{gl} F_{gl} + P_{ugo} F_{ugo} - P_{uog} F_{Feed} \right)\]

\(F_{gs}, F_{gl}\) and \(F_{ugo}\) are product flows (gasoline, light gas and unconverted oil).

(adapted from Loeblein & Perkins, 1999)
Profit Control vs. Fixed Controller Back-off

- Regenerator Temp (K) vs. Coke Fraction in Separator
- Cyclone Temperature (K) vs. Separator Temperature (K)
- Catalyst Flow (kg/s) vs. Fraction of Coke in Regenerator
- Inlet Air (kg/s) vs. Oxygen Mass Fraction
## FCC Profit

<table>
<thead>
<tr>
<th></th>
<th>Gross Profit ($/day)</th>
<th>Diff from OSSOP ($/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OSSOP</td>
<td>$36,905</td>
<td>$0.0</td>
</tr>
<tr>
<td>Fixed Control</td>
<td>$34,631</td>
<td>- $2,274</td>
</tr>
<tr>
<td>Profit Control</td>
<td>$35,416</td>
<td>- $1,489</td>
</tr>
</tbody>
</table>

Improves profit by 2%
Hybrid Vehicle Design
Servo-Loops with PI Controllers

Vehicle Power System

\[ \begin{align*}
&\text{Vehicle Power System} \\
&\text{Vehicle Power System} \\
&P_{\text{fc}}^{(sp)} \quad P_{\text{fc}} \\
&P_{\text{scap}}^{(sp)} \quad P_{\text{scap}} \\
&P_{\text{bat}}^{(sp)} \quad P_{\text{bat}} \\
\end{align*} \]
Supervisory Control

\[ P_{mot}^{(sp)} \quad P_{fc}^{(sp)} \quad P_{bat}^{(sp)} \quad P_{scap}^{(sp)} \]

High Level Controller

Vehicle Power System

\[ k_{fc} \quad k_{bat} \quad k_{scap} \]
Drive Cycle Data and Modeling

![Graphs showing speed and power to motor over time.](image-url)
High Level Battery Model

\[ \dot{E}_{bat} = P_{bat} - P^{(loss)}_{bat} \]
High Level Battery Model

\[ \dot{E}_{bat} = P_{bat} - P_{bat}^{(loss)} \]

\[ E_{bat}^{\min} \leq E_{bat} \leq E_{bat}^{\max} \]

\[ E_{bat}^{\min} = 0 \]

\[ E_{bat}^{\max} = \hat{e}_{bat} m_{bat} \]
High Level Battery Model

\[ \dot{E}_{bat} = P_{bat} - P_{bat}^{(loss)} \]

\[ E_{min} \leq E_{bat} \leq E_{max} \]

\[ E_{bat} = 0 \]

\[ E_{max} = \hat{e}_{bat} m_{bat} \]

\[ P_{min} \leq P_{bat} \leq P_{max} \]

\[ P_{bat} = -\hat{C}_{rate,d} \hat{e}_{bat} m_{bat} \]

\[ P_{bat} = \hat{C}_{rate,c} \hat{e}_{bat} m_{bat} \]
Power and Energy Constraints of the Battery

\[ P_{bat}^{\text{max}} = C_{bat}^{\text{rate,c}} E_{bat}^{\text{max}} \]

\[ E_{bat}^{\text{max}} = \hat{e}_{bat} m_{bat} \]

\[ E_{bat}^{\text{min}} = 0 \]

\[ P_{bat}^{\text{min}} = -C_{bat}^{\text{rate,d}} E_{bat}^{\text{max}} \]
Constraints a Function of the Mass

\[ P_{bat}^{\text{max}} = C_{bat}^{\text{rate,c}} E_{bat}^{\text{max}} \]

\[ E_{bat} = \hat{e}_{bat} m_{bat} \]
Variable Constraint Set

$E_{bat}$

$P_{bat}$
High Level Battery Model

\[ \dot{E}_{bat} = P_{bat} - P^{(loss)}_{bat} \]

\[ E^{\min}_{bat} \leq E_{bat} \leq E^{\max}_{bat} \]

\[ E^{\min}_{bat} = 0 \]

\[ E^{\max}_{bat} = \hat{e}_{bat} m_{bat} \]

\[ P^{(loss)}_{bat} = \frac{P^2_{bat}}{\hat{l}_{bat} m_{bat}} \]

\[ P^{\min}_{bat} \leq P_{bat} \leq P^{\max}_{bat} \]

\[ P^{\min}_{bat} = -\hat{C}_{rate,d} \hat{e}_{bat} m_{bat} \]

\[ P^{\min}_{bat} = \hat{C}_{rate,c} \hat{e}_{bat} m_{bat} \]
High Level Super Cap Model

\[
\dot{E}_{sc} = P_{sc} - P_{sc}^{(loss)}
\]

\[
P_{sc}^{(loss)} = \frac{P_{sc}^2}{\hat{l}_{sc} m_{sc}}
\]

\[
E_{sc}^{\min} \leq E_{sc} \leq E_{sc}^{\max}
\]

\[
E_{sc}^{\min} = 0
\]

\[
E_{sc}^{\max} = \hat{e}_{sc} m_{sc}
\]

\[
P_{sc}^{\min} \leq P_{sc} \leq P_{sc}^{\max}
\]

\[
P_{sc}^{\min} = \hat{C}_{rate,d} \hat{e}_{sc} m_{sc}
\]

\[
P_{sc}^{\min} = \hat{C}_{rate,c} \hat{e}_{sc} m_{sc}
\]
Fuel Cell Model

Anode
In
($H_2$, $H_2O$)

Anode
Exhaust

Cathode
Air in

Cathode
Exhaust

MEA

Solid Material

Current Collector

$H_2$

$H_2O$

$O_2$

$N_2$

$H_2O$
High Level Fuel Cell Model

\[ \dot{P}_{fc} = \Delta P_{fc} \]

\[ P_{fc}^{\text{min}} \leq P_{fc} \leq P_{fc}^{\text{max}} \]

\[ P_{fc}^{\text{min}} = 0 \]

\[ P_{fc}^{\text{max}} = \hat{P}_{fc} m_{fc} \]

\[ \Delta P_{fc}^{\text{min}} \leq \Delta P_{fc} \leq \Delta P_{fc}^{\text{max}} \]

\[ \Delta P_{fc}^{\text{min}} = -\Delta C_{bat}^{\text{rate,d}} \hat{P}_{fc} m_{fc} \]

\[ \Delta P_{fc}^{\text{max}} = \Delta C_{bat}^{\text{rate,c}} \hat{P}_{fc} m_{fc} \]
Power Losses Increases Average Fuel Cell Power

\[ P_{fc} \]
\[ \Delta P_{fc} \]

\[ E_{bat} \]
\[ P_{bat} \]
Design Problem Formulation

\[
\min \{\hat{c}_{fc} m_{fc} + \hat{c}_{bat} m_{bat} + \hat{c}_{sc} m_{sc}\} \\
\text{s.t.} \quad \begin{bmatrix}
\bar{P}_{v} & \sigma_{v} & \bar{P}_{v} \\
\sigma_{v} & l_{bat} m_{bat} & 0 \\
\bar{P}_{bat} & 0 & l_{bat} m_{bat}
\end{bmatrix} > 0 \\
\begin{bmatrix}
- (AX + BY)^T G_o + m_y G_1 \\
\xi_i \\
(D_x X + D_u Y)^T \phi_i^T X
\end{bmatrix} > 0 \\
i = 1...6
\]

\[
\begin{align*}
\zeta_i & < \sigma_i^2 & \quad \sigma_i > 0 & \quad i = 1...6 \\
\sigma_1 & < P_{fc}^{\text{max}} - \bar{P}_{fc} & \quad \sigma_1 < \bar{P}_{fc} - P_{fc}^{\text{min}} \\
\sigma_2 & < E_{bat}^{\text{max}} - \bar{E}_{bat} & \quad \sigma_2 < \bar{E}_{bat} - E_{bat}^{\text{min}} \\
\sigma_3 & < E_{sc}^{\text{max}} - \bar{E}_{sc} & \quad \sigma_3 < \bar{E}_{sc} - E_{sc}^{\text{min}} \\
\sigma_4 & < \Delta P_{fc}^{\text{max}} - \Delta \bar{P}_{fc} & \quad \sigma_4 < \Delta \bar{P}_{fc} - \Delta P_{fc}^{\text{min}} \\
\sigma_5 & < P_{bat}^{\text{max}} - \bar{P}_{bat} & \quad \sigma_5 < \bar{P}_{bat} - P_{bat}^{\text{min}} \\
\sigma_6 & < P_{sc}^{\text{max}} - \bar{P}_{sc} & \quad \sigma_6 < \bar{P}_{sc} - P_{sc}^{\text{min}}
\end{align*}
\]

Illinois Institute of Technology
Department of Chemical and Biological Engineering
# Case Study Data

<table>
<thead>
<tr>
<th>Technology</th>
<th>Lithium Battery</th>
<th>Super-Capacitor</th>
<th>PEM Fuel Cell</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$59/kg</td>
<td>$93/kg</td>
<td>$300/kg</td>
</tr>
<tr>
<td>C-Rate</td>
<td>0.5 hr⁻¹</td>
<td>360 hr⁻¹</td>
<td>10 hr⁻¹</td>
</tr>
<tr>
<td>Power Density</td>
<td>100 W/kg</td>
<td>110,000 W/kg</td>
<td>1 W/kg</td>
</tr>
</tbody>
</table>

Appetecchi & Prosini (2005)
Murphy, Cisar & Clarke (1998)
### Case Study Solution

<table>
<thead>
<tr>
<th>Technology</th>
<th>Lithium Battery</th>
<th>Super-Capacitor</th>
<th>PEM Fuel Cell</th>
<th>Total Capital Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>30 kg</td>
<td>2.8 kg</td>
<td>3.5 kg</td>
<td></td>
</tr>
<tr>
<td>Nominal Power</td>
<td>0.1 kW</td>
<td>1.5 kW</td>
<td>1.8 kW</td>
<td></td>
</tr>
<tr>
<td>Cost</td>
<td>$1770</td>
<td>$260</td>
<td>$1050</td>
<td>$3,080</td>
</tr>
</tbody>
</table>
Optimal Fuel Cell Size and Operating Region

![Graph showing optimal fuel cell size and operating region. The graph plots fuel cell power (P_{fc} (kW)) against the change in fuel cell power (\Delta P_{fc} (kW/s)). The operating region is indicated by a star on the graph, and a line chart shows the fuel cell power over time (hr).]
Optimal Super Cap Size and Operating Region

Super Capacitor

SuperCap Power, kW

time(hr)

E_{sc} (kJ)
P_{sc} (kW)
Optimal Battery Size and Operating Region

![Battery Diagram](image)

- $E_{bat}$ (kJ)
- $P_{bat}$ (kW)

![Battery Power Chart](chart)
Outline

• Model Predictive Controller Tuning
• Pseudo-Constrained Control
• Back-off and Profit Control
• Market Responsive Control
In HVAC systems TES is used for load-leveling and to shift usage to off-peak hours.
Energy Prices and Weather

Cyclical pattern with a phase shift of about 3 hours.
Operation of the TES

Volume of Air (the Room) $T_{room}$

Heat from Room

Heat to Cooler

Cooling Unit

Energy Usage

Heat Leakage $T_{outside}$

Heat to TES Unit

TES Unit

Graph showing:
- Cents per k hr
- Temperature (°C)
- Time (days)

Graph lines:
- Electricity Price
- Outside Temperature
Response to Market Changes

EDOR’s due to different controller tunings

BOP with more profit

BOP with less profit

OSSOP

Peng et al. (2005)
Electric Price Model

White Noise Input → Shaping Filter → Sequence with Electricity Price Characteristics
Electric Price Model

- White Noise Input
- Measured Electricity Price
- Shaping Filter
- State Estimator and/or Predictor
- Sequence with Electricity Price Characteristics
- Prediction of Electricity Price
Model Predictive Control

\[
\min_{v_u(t)} \left\{ \int_0^T p_e(t) \cdot v_u(t) \; dt \right\}
\]

where \( p_e(t) \sim \) the predicted price (or value)

\( v_u(t) \sim \) the velocity of usage

and \( S(t) \sim \) amount in storage

Constraints include:

\[ 0 \leq v_u(t) \leq v_u^{\text{max}} \quad \text{and} \quad 0 \leq S(t) \leq S^{\text{max}} \]
Model Predictive Control

\[
\min_{v_u(t)} \left\{ \int_0^T p_e(t) * v_u(t) \, dt \right\} \approx E[p_e * v_u] = \bar{C}_e
\]

where \( p_e(t) \sim \) the predicted price (or value)
\( v_u(t) \sim \) the velocity of usage

and \( S(t) \sim \) amount in storage

Constraints include:

\[ 0 \leq v_u(t) \leq v_{u_{\text{max}}} \quad \text{and} \quad 0 \leq S(t) \leq S_{\text{max}} \]
System Design

\[
\begin{align*}
\min_{v_u(t)} & \left\{ \int_0^T p_e(t) * v_u(t) \, dt \right\} \\
\cong & \quad E[p_e * v_u] \equiv \overline{C_e}
\end{align*}
\]

How does \( v_u^{\max} \) and \( S^{\max} \) impact \( \overline{C_e} \)?

\[
(0 \leq v_u(t) \leq v_u^{\max} \text{ and } 0 \leq S(t) \leq S^{\max})
\]
System Design

\[
\min_{v_u(t)} \left\{ \int_0^T p_e(t) * v_u(t) \, dt \right\} \equiv E[p_e * v_u] \equiv \overline{C}_e
\]

How does \( v_u^{\text{max}} \) and \( S^{\text{max}} \) impact \( \overline{C}_e \)?

\[(0 \leq v_u(t) \leq v_u^{\text{max}} \text{ and } 0 \leq S(t) \leq S^{\text{max}})\]

MPC cannot answer this question!
Expected Cost of Electricity

White Noise Input → Shaping Filter → $p_e(t)$

$E[p_e v_u]$

Manipulated Variables → Process Model → $v_u(t)$

(Controller is $u = Lx$)
Re-Scaling of Price

\[ \alpha w(t) \rightarrow \text{Shaping Filter} \rightarrow p'(t) \]

Manipulated Variables

(Controller is \( u = Lx \))

\[ E[p'_{e*}v_u] \]

\[ (p'_{e} \equiv \alpha p_e) \]
Correlating Price and Usage

If \( E[(p'_e - v_u)^2] < \varepsilon \leq 0 \) and \( p'_e = \alpha p_e \)

then \( v_u(t) \cong \alpha p_e(t) \)
Correlating Price and Usage

If \( E[(p_e' - v_u)^2] < \varepsilon \equiv 0 \) and \( p_e' \equiv \alpha \ p_e \)

then \( v_u(t) \equiv \alpha \ p_e(t) \)

\[
E[\alpha^2 p_e^2] - 2E[\alpha p_e v_u] + E[v_u^2] \approx 0
\]

\[
\Rightarrow \alpha^2 E[p_e^2] = \alpha E[p_e v_u] = E[v_u^2]
\]

\[
\Rightarrow \alpha E[p_e^2] = E[p_e v_u] \equiv C_e
\]
Minimum Cost of Electricity

\[
\bar{C}_e = \min_{L, \alpha} \left\{ c_R \alpha \right\} \quad (c_R = E[p_e^2])
\]

\[
E\left[(p_e' - v_u)^2\right] < \varepsilon \approx 0
\]

\[
E\left[v_u^2\right] < (v_u^{\text{max}})^2
\]

\[
E\left[S^2\right] < (S^{\text{max}})^2
\]
Thermal Energy Storage
(Small Storage Unit)

Volume of Air (the Room)

Heat from Room

Heat to Cooler

Cooling Unit

Energy Usage

Heat to TES Unit

TES Unit

Heat Leakage $T_{\text{outside}}$

$T_{\text{room}}$

Energy Usage

Heat from Room

Heat to Cooler

Heat to TES Unit

-kW hr/day

Time (days)

59 60 61 62

0 50 100 150 200 250

Heat from Room

Heat to Cooler

Heat to TES Unit
Thermal Energy Storage (Medium Storage Unit)

- Volume of Air (the Room)
- Heat Leakage $T_{outside}$
- Heat from Room
- Heat to Cooler
- Cooling Unit
- Energy Usage
- Heat to TES Unit
- TES Unit

Graph:
- Heat from Room
- Heat to Cooler
- Heat to TES Unit

$kW\, hr / day$

Time (days)
Thermal Energy Storage
(Large Storage Unit)

Volume of Air (the Room) $V_{room}$

Heat from Room $Q_{in}$

Heat to Cooler $Q_{out}$

Cooling Unit

Energy Usage

Heat to TES Unit $Q_{DS}

TES Unit

Heat Leakage $T_{outside}$

Energy Usage $E_{usage}$

Graph:

- Red line: Heat from Room $Q_{in}$
- Green line: Heat to Cooler $Q_{out}$
- Blue line: Heat to TES Unit $Q_{DS}$

$kW \cdot hr / day$

Time (days) 59 60 61 62

Heat from Room
Heat to Cooler
Heat to TES Unit
Thermal Energy Storage
(Comparison of Storage Cases)
Average Cooling Costs:

One ton: $8 per day
Five tons: $7 per day (14% savings)
Ten tons: $6 per day (25% savings)
Minimum Levelized Cost

\[
\min_{L,\alpha,\nu_u^{\text{max}},S^{\text{max}}} \left\{ c_R \alpha + c_L, \nu_u^{\text{max}} + c_L, S^{\text{max}} \right\}
\]

\[
E\left[ (p'_e - \nu_u)^2 \right] < \varepsilon \approx 0
\]

\[
E\left[ \nu_u^2 \right] < (\nu_u^{\text{max}})^2
\]

\[
E\left[ S^2 \right] < (S^{\text{max}})^2
\]
Minimum Levelized Cost

\[
\min_{L, \alpha, \nu_u^\text{max}, S^\text{max}} \left\{ c_R \alpha + c_{L,1} \nu_u^\text{max} + c_{L,2} S^\text{max} \right\}
\]

\[
E[(p'_e - \nu_u)^2] < \varepsilon \approx 0
\]

\[
E[\nu_u^2] < (\nu_u^\text{max})^2
\]

\[
E[S^2] < (S^\text{max})^2
\]

Non-Convex Problem
(but global solution from branch and bound)
Integrated Gasification Combined Cycle (IGCC)
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  - Chemical & Biological Engineering Department, IIT
Conclusions

• Relationship between control system performance and plant profit quantified.
• Enables profit guided control system design.
• Broad set of applications from a variety of disciplines.
• Linear controller can be designed for market responsiveness.
• Non-convex, but global methods can be used to size and/or select equipment.