New Perspectives in Control System Design: Pseudo-Constrained to Market Responsive Control

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Outline

• Motivating Example
• Pseudo-Constrained Control
• Back-off and Profit Control
• Market Responsive Control
Motivating Example
(Non-isothermal Reactor)

\[ V \frac{dC_A}{dt} = F(C_{Ain} - C_A) + Vr_A \]

\[ V \frac{dT}{dt} = F(T_{in} - T) + (V\Delta H / \rho C_p)r_A \]

\[ r_A = -k(T)C_A \]

Increase \( F \) \( \Rightarrow \) Increased production rate
Motivating Example
(Non-isothermal Reactor)

\[
V \frac{dC_A}{dt} = F(C_{A_{in}} - C_A) + Vr_A
\]

\[
V \frac{dT}{dt} = F(T_{in} - T) + (V\Delta H / \rho C_p)r_A
\]

\[
r_A = -k(T)C_A
\]

Increase \( F \)  \( \rightarrow \)  Increased production rate

Decrease \( F \)  \( \rightarrow \)  Increase \( T \)  \( \rightarrow \)  Increase reaction rate  \( \rightarrow \)  Increase production
Limited Operating Region

Process Limitations:

\[ T(t) \leq T^{(\text{max})} \]  - Catalyst protection or onset of side reactions

\[ F(t) \leq F^{(\text{max})} \]  - Pump limit or limit on downstream unit
Limited Operating Region

Process Limitations:

\[ T(t) \leq T^{(\text{max})} \]
- Catalyst protection or onset of side reactions

\[ F(t) \leq F^{(\text{max})} \]
- Pump limit or limit on downstream unit

Possible Controller:

\[ F = K_c (T - T^{(sp)}) + F^{(sp)} \]
Performance in Time Series

$T(t)$

$F(t)$

$T^{(sp)}$

$T^{(max)}$

$F^{(sp)}$

$F^{(max)}$

$C_A, T$

$F$
Performance in Phase Plane

\[ T(t) \]

\[ F(t) \]
Dynamic Operating Region

\[ T(t) \]

\[ F(t) \]
Dynamic Operating Region

\[ T(t) \]

\[ F(t) \]
Controller:

\[ F = K_c (T - T^{(sp)}) + F^{(sp)} \]

Steady-State Relation:

\[ F^{(sp)} = f (T^{(sp)}) \]
Dynamic Operating Region

\[ T(t) \]

\[ F(t) \]
Steady-State Operating Line

$T(t)$

$F(t)$
Optimal Operating Point

Decrease $F$ → Increase $T$

→ Increase conversion

→ Increase production
Optimal Operating Point: Another Possibility

Increase $F$ → Increased production rate
Optimal Operating Point: Another Possibility

$T(t)$

Increase $F$ → Increased production rate

$F(t)$
Requires Different Controller Tuning

\[ T(t) \]

\[ F(t) \]
Less Aggressive Tuning

\[ T(t) \]

\[ F(t) \]

\[ T^{(max)} \]

\[ T^{(sp)} \]

\[ F(t) \]

\[ F^{(max)} \]

\[ F^{(sp)} \]
Need for Automated Tuning

\[ T(t) \]

\[ F(t) \]
Connect Controller Design to Plant Economics

$T(t)$

$F(t)$
Outline

• Motivating Example
• Pseudo-Constrained Control
• Back-off and Profit Control
• Market Responsive Control
Mass-Spring-Damper Example

$r$ is the mass position

$v$ is the velocity

$f$ is the input force (MV) and

$w$ is the disturbance force
Mass-Spring-Damper Example

$r$ is the mass position
$v$ is the velocity
$f$ is the input force (M V) and
$w$ is the disturbance force

System Model:

\[
\begin{bmatrix}
\dot{r} \\
\dot{v}
\end{bmatrix} = 
\begin{bmatrix}
0 & 1 \\
-2 & -3
\end{bmatrix}
\begin{bmatrix}
r \\
v
\end{bmatrix} + 
\begin{bmatrix}
0 \\
1
\end{bmatrix} f + 
\begin{bmatrix}
0 \\
1
\end{bmatrix} w
\]
**Mass-Spring-Damper Example**

$r$ is the mass position

$v$ is the velocity

$f$ is the input force (MV) and

$w$ is the disturbance force

**System Model:**

\[
\begin{bmatrix}
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r \\
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\end{bmatrix} + \begin{bmatrix}
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1
\end{bmatrix} f + \begin{bmatrix}
0 \\
1
\end{bmatrix} w
\]

**System Constraints:**

\[-1 \leq r \leq 1 \]

and

\[0 \leq f \leq 16\]
Mass-Spring-Damper Example

System Model:

\[
\begin{bmatrix}
\dot{r} \\
\dot{v} \\
z_1 \\
z_2
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-2 & -3 \\
1 & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
r \\
v \\
r \\
v
\end{bmatrix} +
\begin{bmatrix}
0 \\
1 \\
0 \\
0
\end{bmatrix} f +
\begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix} w
\]

System Constraints:

\(-1 \leq r \leq 1\)

and

\(0 \leq f \leq 16\)

\(\dot{x} = Ax + Bu + Gw\)

\(z = Dx x + Du u + Dw w\)

\(-\bar{z}_i \leq z_i(t) \leq \bar{z}_i\)

\(i = 1 \ldots n_z\)
Covariance Analysis
(Open-Loop Case)

Process Model:
\[
\dot{x} = Ax + Gw \\
z = Dx \\
w(t) \text{ Gaussian white noise with covariance } \Sigma_w
\]

Steady State Covariance:
\[
A \Sigma_x + \Sigma_x A^T + G \Sigma_w G^T = 0 \\
\Sigma_z = D \Sigma_x D^T
\]
Expected Dynamic Operating Region (EDOR)

EDOR defined by:

$$\sum_z = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 \end{bmatrix}$$
Closed-Loop Covariance Analysis 
(Full State Information Case)

**Process Model:**
\[
\dot{x} = Ax + Bu + Gw \\
z = Dx x + Du u + Dw w
\]

**Controller:**
\[
u(t) = Lx(t)
\]

**Steady-State Covariance:**
\[(A + BL) \Sigma_x + \Sigma_x (A + BL)^T + G \Sigma_w G^T = 0\]
\[
\Sigma_z = (D_x + Du L) \Sigma_x (D_x + Du L)^T + Dw \Sigma_w Dw^T
\]
Closed-Loop EDOR

EDOR's from different controllers

\[ u = L_1 x \]

\[ u = L_2 x \]
Constrained Closed-Loop EDOR

\[ \sigma_{zi} < \bar{z}_i \]
Constrained Closed-Loop EDOR

Constraints

\((\sigma_{zi} < \bar{z}_i)\)
Constrained Controller Existence

Does there exist $L$ such that:

$$(A + BL)\Sigma_x + \Sigma_x (A + BL)^T + G\Sigma_w G^T = 0$$

$$\Sigma_z = (D_x + D_u L)\Sigma_x (D_x + D_u L)^T + D_w\Sigma_w D_w^T$$

$$\xi_i = \phi_i\Sigma_z\phi_i^T < \bar{z}_i^2 \quad i = 1 \ldots n_z$$

$$\phi_i = \begin{bmatrix} 0 & 0 & \ldots & 1 & \ldots & 0 & 0 \end{bmatrix}$$

$\uparrow$ $i^{th}$ column
Constrained Controller Existence (Convex Condition)

If and only if there exist $X > 0$ and $Y$ such that:

$$(AX + BY) + (AX + BY)^T + G \Sigma_w G^T < 0$$

$$\begin{bmatrix}
\xi_i - \phi_i D_w \Sigma_w D_w^T \phi_i^T & \phi_i (D_x X + D_u Y) \\
(D_x X + D_u Y)^T \phi_i^T & X
\end{bmatrix} > 0$$

$$\xi_i < \bar{z}_i^2 \quad i = 1 \ldots n_z$$

And controller $u = LX$ is constructed as: $L = YX^{-1}$
Region of Controller Existence

Achievable Performance Levels

Unachievable

\[ \xi_1 \]

\[ \xi_2 \]
Region of Constrained Controller Existence

\[ \xi_1 < \overline{z}_1 \]

\[ \xi_2 < \overline{z}_2 \]
Pseudo-Constrained Control

\[ T(t) \]

\[ F(t) \]

\[ \xi_1 < z_1^2 \]

\[ \xi_2 < z_2^2 \]
Pseudo-Constrained Control

\[
\min_{X>0,Y,\xi} \sum_i d_i \xi_i
\]

such that:

\[
(AX + BY) + (AX + BY)^T + G\Sigma_w G^T < 0
\]

\[
\begin{bmatrix}
\xi - \phi_i D_w \Sigma_w D_w^T \phi_i^T \\
(D_x X + D_u Y)^T \phi_i^T
\end{bmatrix} > 0
\]

\[
\xi_i < z_i^2 \quad i = 1 \ldots n_z
\]
Pareto Frontier Interpretation

All Pseudo-Constrained Controllers are on the Pareto Frontier

Achievable Region

Unachievable Region
Controller Equivalence

Theorem 1 (Chmielewski & Manthanwar, 2004):

The controller generated by CMV

is coincident with

the controller generated by some
Unconstrained Model Predictive Controller.
Outline

- Model Predictive Controller Tuning
- Pseudo-Constrained Control
- Back-off and Profit Control
- Market Responsive Control
Constrained Operating Region

Steady-State Operating Point

EDOR

Constraints

CV’s

MV’s
Real-Time Optimization

Steady-State Operating Point

Constraints

Optimal Steady-State Operating Point (OSSOP)

EDOR

MV’s

CV’s
Real-Time Optimization

Original Nonlinear Process Model:

\[ \dot{s} = f(s,m,p) \quad q = h(s,m,p) \]

\( (s,m,p,q) \sim (\text{state, mv, dist, performance}) \sim (x,u,w,z) \)
Real-Time Optimization

Original Nonlinear Process Model:
\[ \dot{s} = f(s, m, p) \quad q = h(s, m, p) \]
\[(s, m, p, q) \sim \text{(state, mv, dist, performance)} \sim (x, u, w, z)\]

Real-Time Optimization (minimize profit loss):
\[ \min_{s, m, q} \{ g(q) \} \quad \text{s.t.} \]
\[ 0 = f(s, m, p) \quad q = h(s, m, p) \quad q_i^{\min} \leq \phi_i q \leq q_i^{\max} \]

RTO solution denoted as \((s^{\text{ossop}}, m^{\text{ossop}}, p^{\text{ossop}}, q^{\text{ossop}})\)
Backed-off Operating Point (BOP)

CV's

EDOR

Backed-off Operating Point (BOP)

Optimal Steady-State Operating Point (OSSOP)

MV's
Steady-State BOP Selection
(Bahri, Bandoni & Romagnoli, 1996)

Solve the following Semi-infinite Programming Problem

\[
\min_{s,m,q} \left\{ g(q) \right\} \quad \text{s.t.} \quad \max_{p \in [p^{\text{min}}, p^{\text{max}}]} \max_i \left\{ q_i - \bar{q}_i \right\}
\]

\[
\text{s.t.} \quad 0 = f(s, m, p) \quad q = h(s, m, p) \quad q_i - \bar{q}_i < 0
\]

Extensions:
# Linearized Perspective

<table>
<thead>
<tr>
<th>Nonlinear</th>
<th>Linear wrt OSSOP</th>
<th>Linear wrt BOP</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(q^{bop}) )</td>
<td>( g(q^{ossop}) + g_q q' )</td>
<td>( \dot{x} = A x + B u + G w )</td>
</tr>
<tr>
<td>( \dot{s} = f(s, m, p) )</td>
<td>( \dot{s}' = A s' + B m' + G p' )</td>
<td>( z = D_x x + D_u u + D_w w )</td>
</tr>
<tr>
<td>( q = h(s, m, p) )</td>
<td>( q' = D_x s' + D_u m' + D_w p' )</td>
<td>( z_i^{\text{min}} \leq z_i \leq z_i^{\text{max}} )</td>
</tr>
<tr>
<td>( q_i^{\text{min}} \leq q_i \leq q_i^{\text{max}} )</td>
<td>( q_i^{\text{min}} \leq q'_i \leq q_i^{\text{max}} )</td>
<td></td>
</tr>
</tbody>
</table>

**Deviation Variables w.r.t. OSSOP:**

\[
\begin{align*}
    s' &= s^{bop} - s^{ossop} \\
    m' &= m^{bop} - m^{ossop} \\
    p' &= p^{bop} - p^{ossop} \\
    q' &= q^{bop} - q^{ossop}
\end{align*}
\]

**Deviation Variables w.r.t. BOP:**

\[
\begin{align*}
    x &= s - s^{bop} \\
    u &= m - m^{bop} \\
    w &= p - p^{bop} \\
    z &= q - q^{bop}
\end{align*}
\]
Stochastic BOP Selection
(Loeblein & Perkins, 1999)
Stochastic BOP Selection
(Loeblein & Perkins, 1999)

Assume controller $L$ is given and calculate $\xi_i$:

$$(A + BL)\Sigma_x + \Sigma_x (A + BL)^T + G\Sigma_w G^T = 0$$

$$\Sigma_z = (D_x + D_u L)\Sigma_x (D_x + D_u L)^T + D_w \Sigma_w D_w^T$$

$$\xi_i = \phi_i \Sigma_z \phi_i^T \quad i = 1 \ldots n_z$$
Stochastic BOP Selection
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Assume controller $L$ is given and calculate $\xi_i$:

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$$\Sigma_z = (D_x + D_u L) \Sigma_x (D_x + D_u L)^T + D_w \Sigma_w D_w^T$$

$$\xi_i = \phi_i \Sigma_z \phi_i^T \quad i = 1 \ldots n_z$$

Solve the following Linear Program:

$$\min_{s',m',q'} \left\{ \begin{array}{c} g_q q' \end{array} \right\} \quad \text{s.t.} \quad 0 = As' + Bm'$$

$$q_i' = \phi_i (D_x s' + D_u m') \quad q_i^{\min} \leq q_i' \leq q_i^{\max}$$

$$\xi_i^{1/2} < q_i^{\max} - q_i' \quad \xi_i^{1/2} < q_i' - q_i^{\min}$$
Fixed Controller BOP Selection

Loeblein and Perkins (1999):

Controller is fixed $\iff$ EDOR has fixed size and shape.
Peng et al. (2005):

Variable Controller BOP Selection

Variable Controller $\iff$ EDOR has variable size and shape

EDOR

OSSOP
Profit Control
(Simultaneous BOP and Controller Selection)

EDOR’s due to different controller tunings

BOP with more profit

BOP with less profit

Max Profit

Beng et al. (2005)
Assume controller $L$ is given and calculate $\xi_i$:

\[
(A + BL) \Sigma_x + \Sigma_x (A + BL)^T + G \Sigma_w G^T = 0
\]

\[
\Sigma_z = (D_x + D_u L) \Sigma_x (D_x + D_u L)^T + D_w \Sigma_w D_w^T
\]

\[
\xi_i = \phi_i \Sigma_z \phi_i^T \quad i = 1 \ldots n_z
\]

Solve the following Linear Program:

\[
\min \left\{ g_q q' \right\} \quad \text{s.t.} \quad 0 = As' + Bm'
\]

\[
q_i' = \phi_i (D_x s' + D_u m') \quad q_i^{\min} \leq q_i' \leq q_i^{\max}
\]

\[
\xi_i^{1/2} < q_i^{\max} - q_i'
\]

\[
\xi_i^{1/2} < q_i' - q_i^{\min}
\]

\textbf{Stochastic BOP Selection} 
\textbf{(Loeblein & Perkins, 1999)}
Profit Control
(Simultaneous BOP and Controller Selection)

\[
\min \left\{ g_{q'q'} \right\} \quad \text{s.t.} \quad 0 = As' + Bm'
\]

\[
q_i' = \phi_i (D_x s' + D_u m') \quad q_i^{\min} \leq q_i' \leq q_i^{\max}
\]

\[
\xi_i^{1/2} < q_i^{\max} - q_i' \quad \xi_i^{1/2} < q_i' - q_i^{\min}
\]

\[
(AX + BY) + (AX + BY)^T + G\Sigma_wG^T < 0
\]

\[
\begin{bmatrix}
\xi_i - \phi_i D_w \Sigma_w D_w^T \phi_i^T & \phi_i (D_x X + D_u Y) \\
(D_x X + D_u Y)^T \phi_i^T & X
\end{bmatrix} > 0
\]

Peng et al. (2005)
Computational Aspects of Profit Control

\[
\min_{s', m', q', \xi_i, X, Y} \left\{ g_q q' \right\} \quad \text{s.t.} \quad 0 = As' + Bm'
\]

\[
q_i' = \phi_i (D_x s' + D_u m') \quad q_i^\text{min} \leq q_i' \leq q_i^\text{max}
\]

\[
\xi_i^{1/2} < q_i^\text{max} - q_i' \quad \xi_i^{1/2} < q_i' - q_i^\text{min}
\]

\[
(AX + BY) + (AX + BL)^T + G \Sigma_w G^T < 0
\]

\[
\begin{bmatrix}
\xi_i - \phi_i D_w \Sigma_w D_w^T \phi_i^T & \phi_i (D_x X + D_u Y) \\
(D_x X + D_u Y)^T \phi_i^T & X
\end{bmatrix} > 0
\]

Peng et al. (2005)
Reverse-Convex Constraints

\[ \xi_1 < (q_1^{\text{max}} - q_1')^2 \]

\[ \xi_1 < (q_1' - q_1^{\text{min}})^2 \]

Feasible Region
Global Solution

Based on Branch and Bound algorithm

![Graph with regions labeled Region 1 to Region 5, with axes labeled $q'_1$ and $\xi_1$.]
Profit Control Applications

• Mechanical Systems
• Chemical and Reaction Systems
• Hybrid Vehicle Design
• Inventory Control
• Electric Power System Design
• Building HVAC
• Water Resource Management
Profit Control Applications

- Mechanical Systems
- Chemical and Reaction Systems
- Hybrid Vehicle Design
- Inventory Control
- Electric Power System Design
- Building HVAC
- Water Resource Management
Fluidized Catalytic Cracker

Regenerator and Separator (dynamic):

\[
W \frac{dC_{rgc}}{dt} = F_{cat} (C_{st} - C_{rgc}) - R_{cb}
\]

\[
W_a \frac{dO_d}{dt} = \frac{F_{air}}{M_{air}} (O_{in} - O_d) \frac{1 + 1.5 \sigma R_{cb}}{1 + \sigma M_c}
\]

\[
W_{c_{pc}} \frac{dT_{reg}}{dt} = F_{cat} c_{pc} T_{st} + F_{air} C_{pair} T_{air} - (F_{cat} c_{pc} + F_{air} C_{pair}) T_{reg} - \left(\Delta H_{CO} + \frac{\sigma \Delta H_{CO_2}}{1 + \sigma} R_{cb}\right) \frac{R_{cb}}{M_c}
\]

\[
W_{st} \frac{dc_{at}}{dt} = F_{cat} (C_{sc} - C_{st})
\]

\[
W_{st} c_{pc} \frac{dT_{st}}{dt} = F_{cat} c_{pc} (T_{ro} - T_{st})
\]

\[T_{cy} = T_{reg} + 5.555O_d\]

Riser (pseudo steady state):

\[
\frac{dy_f}{dz} = -K_1 y_f [COR] \phi t_{cv}, \quad y_f(z = 0) = 1
\]

\[
\frac{dy_g}{dz} = (K_2 y_f^2 - K_3 y_g) [COR] \phi t_{c_v}, \quad y_g(z = 0) = 0
\]

\[
\frac{d\theta}{dz} = \frac{\Delta H_f F_{feed}}{T_{ri} (F_{cat} c_{cp} + F_{feed} c_{pf} + \lambda F_{feed} c_{pc})} \frac{dy_f}{dz}
\]

\[
\theta(z) = \frac{T(z) - T_{ri}}{T_{ri}}, \quad \theta(z = 0) = 0, \quad T_{ro} = T(z = 1)
\]

(adapted from Loeblein & Perkins, 1999)
**FCC Constraints and Economics**

**Process Constraints:**

\[
400 \, K \leq T_{st} \leq 1000 \, K \\
600 \, K \leq T_{reg} \leq 1000 \, K \\
T_{reg} \leq T_{cy} \leq 1000 \, K \\
100 \frac{kg}{s} \leq F_{cat} \leq 400 \frac{kg}{s} \\
0 \leq F_{air} \leq 60 \, kg/s
\]

**Profit Function:**

\[
\Phi = 86400 \left( P_{gs} F_{gs} + P_{gl} F_{gl} + P_{ugo} F_{ugo} - P_{uog} F_{Feed} \right)
\]

\(F_{gs}, F_{gl}\) and \(F_{ugo}\) are product flows (gasoline, light gas and unconverted oil).

(adapted from Loeblein & Perkins, 1999)
Profit Control vs. Fixed Controller Back-off

Graphs showing the comparison of profit control and fixed controller back-off in terms of regenerator temperature, coke fraction in separator, catalyst flow, and oxygen mass fraction.
# FCC Profit

<table>
<thead>
<tr>
<th></th>
<th>Gross Profit ($/day)</th>
<th>Diff from OSSOP ($/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OSSOP</td>
<td>$36,905</td>
<td>$0.0</td>
</tr>
<tr>
<td>Fixed Control</td>
<td>$34,631</td>
<td>- $2,274</td>
</tr>
<tr>
<td>Profit Control</td>
<td>$35,416</td>
<td>- $1,489</td>
</tr>
</tbody>
</table>

Improves profit by 2%
Hybrid Vehicle Design

Fuel Cell $E_{fc}$

$E_{bat}$

$E_{scap}$

Power Bus

$i_{fc}$

$i_{bat}$

$i_{scap}$

$i_{arm}$

$R_{bat}$

$R_{scap}$

$R_{arm}$

$L_{arm}$

$w_{arm}$

$E_{arm}$
Control of Components

Fuel Cell $E_{fc}$

DC-DC Converter

$R_{bat}$

$E_{bat}$

$\mathbf{k}_{fc}$

$i_{fc}$

$i_{afc}$

$i_{abat}$

$R_{bat}$

$E_{bat}$

$\mathbf{k}_{bat}$

$E_{arm}$

$i_{arm}$

Power Bus

DC-DC Converter

$R_{scap}$

$E_{scap}$

$\mathbf{k}_{scap}$

$w_{arm}$

$R_{arm}$

$L_{arm}$

Illinois Institute of Technology
Department of Chemical and Biological Engineering
Servo-Loops with PI Controllers
Supervisory Control

Vehicle Power System

High Level Controller

\[ P_{\text{mot}}^{(sp)} \]

\[ k_{fc} \]

\[ P_{fc} \]

\[ k_{bat} \]

\[ P_{bat} \]

\[ k_{scap} \]

\[ P_{scap} \]
Drive Cycle Data and Modeling
High Level Battery Model

\[ \dot{E}_{bat} = P_{bat} - P^{(loss)}_{bat} \]
High Level Battery Model

\[ \dot{E}_{bat} = P_{bat} - P_{bat}^{(loss)} \]

\[ E_{bat}^{\min} \leq E_{bat} \leq E_{bat}^{\max} \]

\[ E_{bat}^{\min} = 0 \]

\[ E_{bat}^{\max} = \hat{e}_{bat} m_{bat} \]
High Level Battery Model

\[ \dot{E}_{bat} = P_{bat} - P_{bat}^{(loss)} \]

\[ E_{bat}^{\min} \leq E_{bat} \leq E_{bat}^{\max} \]

\[ E_{bat}^{\min} = 0 \]

\[ E_{bat}^{\max} = \hat{e}_{bat} m_{bat} \]

\[ P_{bat}^{\min} \leq P_{bat} \leq P_{bat}^{\max} \]

\[ P_{bat}^{\min} = -\hat{C}_{bat}^{\text{rate},d} \hat{e}_{bat} m_{bat} \]

\[ P_{bat}^{\min} = \hat{C}_{bat}^{\text{rate},c} \hat{e}_{bat} m_{bat} \]
Power and Energy Constraints of the Battery

\[ P_{bat}^{max} = C_{bat}^{rate,c} E_{bat}^{max} \]

\[ E_{bat}^{max} = \hat{e}_{bat} m_{bat} \]

\[ E_{bat}^{min} = 0 \]

\[ P_{bat}^{min} = -C_{bat}^{rate,d} E_{bat}^{max} \]
Constraints a Function of the Mass

\[ P_{bat}^{\max} = C_{bat}^{rate,c} E_{bat}^{\max} \]

\[ E_{bat} = \hat{e}_{bat} m_{bat} \]
Aspect Ratio a Function of C-Rate

$$P_{bat}^{\text{max}} = C_{bat}^{\text{rate,c}} E_{bat}^{\text{max}}$$

$$E_{bat}^{\text{max}} = \hat{e}_{bat} m_{bat}$$
High Level Battery Model

\[ \dot{E}_{bat} = P_{bat} - P_{\text{loss}}^{(loss)} \]

\[ P_{\text{loss}}^{(loss)} = \frac{I_{bat}^2 \hat{R}_{bat}}{A_{bat}} \]

\[ = \frac{P_{bat}^2}{\hat{l}_{bat} m_{bat}} \]

\[ E_{\text{min}} \leq E_{bat} \leq E_{\text{max}} \]

\[ E_{bat} = 0 \]

\[ E_{\text{max}} = \hat{e}_{bat} m_{bat} \]

\[ P_{\text{min}} \leq P_{bat} \leq P_{\text{max}}^{\text{max}} \]

\[ P_{\text{min}} = -\hat{C}_{bat}^{\text{rate},d} \hat{e}_{bat} m_{bat} \]

\[ P_{\text{min}} = \hat{C}_{bat}^{\text{rate},c} \hat{e}_{bat} m_{bat} \]
Operating Region with Power Losses

\[ E_{bat} \]

\[ P_{bat} \]
High Level Super Cap Model

\[ \dot{E}_{sc} = P_{sc} - P_{sc}^{(loss)} \]

\[ P_{sc}^{(loss)} = \frac{I_{sc}^{2} \hat{R}_{sc}}{A_{sc}} \]

\[ = \frac{P_{sc}^{2}}{\hat{l}_{sc} m_{sc}} \]

\[ E_{sc}^{\text{min}} \leq E_{sc} \leq E_{sc}^{\text{max}} \]

\[ E_{sc}^{\text{min}} = 0 \]

\[ E_{sc}^{\text{max}} = \hat{e}_{sc} m_{sc} \]

\[ P_{sc}^{\text{min}} \leq P_{sc} \leq P_{sc}^{\text{max}} \]

\[ P_{sc}^{\text{min}} = -\hat{C}_{sc, rate,d} \hat{e}_{sc} m_{sc} \]

\[ P_{sc}^{\text{min}} = \hat{C}_{sc, rate,c} \hat{e}_{sc} m_{sc} \]
High Level Fuel Cell Model

\[ \dot{P}_{fc} = \Delta P_{fc} \]

\[ P_{fc}^{\text{min}} \leq P_{fc} \leq P_{fc}^{\text{max}} \]

\[ P_{fc}^{\text{min}} = 0 \]

\[ P_{fc}^{\text{max}} = \hat{p}_{fc} m_{fc} \]

\[ \Delta P_{fc}^{\text{min}} \leq \Delta P_{fc} \leq \Delta P_{fc}^{\text{max}} \]

\[ \Delta P_{fc}^{\text{min}} = -\Delta C_{bat}^{\text{rate,d}} \hat{p}_{fc} m_{fc} \]

\[ \Delta P_{fc}^{\text{max}} = \Delta C_{bat}^{\text{rate,c}} \hat{p}_{fc} m_{fc} \]
Power Losses Increases Average Fuel Cell Power

\[ P_{fc} \]

\[ \Delta P_{fc} \]

\[ E_{bat} \]

\[ P_{bat} \]
Hybrid Vehicle Optimization

\[
\begin{align*}
\min_{s.t.} & \left\{ \hat{c}_{fc} m_{fc} + \hat{c}_{bat} m_{bat} + \hat{c}_{sc} m_{sc} \right\} \\
\end{align*}
\]

\[
\begin{align*}
\overline{P}_{fc} &= P_o + \overline{P}_{bat} + \overline{P}_{sc} \\
m_v &= m_o + m_{fc} + m_{bat} + m_{sc} \\
\begin{bmatrix}
\overline{P}_{bat} & \sigma_{bat} & \overline{P}_{bat} \\
\sigma_{bat} & l_{bat} m_{bat} & 0 \\
\overline{P}_{bat} & 0 & l_{bat} m_{bat}
\end{bmatrix} > 0 \\
\begin{bmatrix}
\overline{P}_{sc} & \sigma_{sc} & \overline{P}_{sc} \\
\sigma_{sc} & l_{sc} m_{sc} & 0 \\
\overline{P}_{sc} & 0 & l_{sc} m_{sc}
\end{bmatrix} > 0 \\
- (AX + BY) - (AX + BY)^T & G_o + m_v G_1 & \Sigma_w^{-1} > 0 \\
\begin{bmatrix}
\xi_i & \phi_i \left( D_x X + D_u Y \right) \\
(D_x X + D_u Y)^T & X
\end{bmatrix} > 0 \\
i = 1...6
\end{align*}
\]

\[
\begin{align*}
\xi_i & < \sigma_i^2 \\
\sigma_i & > 0 \\
i = 1...6
\end{align*}
\]

\[
\begin{align*}
\sigma_1 & < P_{fc}^{\max} - \overline{P}_{fc} \quad \sigma_1 < \overline{P}_{fc} - P_{fc}^{\min} \\
\sigma_2 & < E_{bat}^{\max} - \overline{E}_{bat} \quad \sigma_2 < \overline{E}_{bat} - E_{bat}^{\min} \\
\sigma_3 & < E_{sc}^{\max} - \overline{E}_{sc} \quad \sigma_3 < \overline{E}_{sc} - E_{sc}^{\min} \\
\sigma_4 & < \Delta P_{fc}^{\max} - \Delta \overline{P}_{fc} \quad \sigma_4 < \Delta \overline{P}_{fc} - \Delta P_{fc}^{\min} \\
\sigma_5 & < P_{bat}^{\max} - \overline{P}_{bat} \quad \sigma_5 < \overline{P}_{bat} - P_{bat}^{\min} \\
\sigma_6 & < P_{sc}^{\max} - \overline{P}_{sc} \quad \sigma_6 < \overline{P}_{sc} - P_{sc}^{\min}
\end{align*}
\]
# Component Parameters

<table>
<thead>
<tr>
<th>Technology</th>
<th>Lithium Battery</th>
<th>Super-Capacitor</th>
<th>PEM Fuel Cell</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$59/kg</td>
<td>$93/kg</td>
<td>$300/kg</td>
</tr>
<tr>
<td>C-Rate</td>
<td>0.5 hr(^{-1})</td>
<td>360 hr(^{-1})</td>
<td>10 hr(^{-1})</td>
</tr>
<tr>
<td>Power Density</td>
<td>100 W/kg</td>
<td>110,000 W/kg</td>
<td>1 W/kg</td>
</tr>
</tbody>
</table>

Appetecchi & Prosini (2005)
Murphy, Cisar & Clarke (1998)
## Optimized Hybrid Vehicle

<table>
<thead>
<tr>
<th>Technology</th>
<th>Lithium Battery</th>
<th>Super-Capacitor</th>
<th>PEM Fuel Cell</th>
<th>Total Capital Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>30 kg</td>
<td>2.8 kg</td>
<td>3.5 kg</td>
<td></td>
</tr>
<tr>
<td>Nominal Power</td>
<td>0.1 kW</td>
<td>1.5 kW</td>
<td>1.8 kW</td>
<td></td>
</tr>
<tr>
<td>Cost</td>
<td>$1770</td>
<td>$260</td>
<td>$1050</td>
<td>$3,080</td>
</tr>
</tbody>
</table>
Optimal Fuel Cell Size and Operating Region

![Graph showing the optimal fuel cell size and operating region.](image)

- The x-axis represents the change in fuel cell power ($\Delta P_{fc}$) in kW/s.
- The y-axis represents the fuel cell power ($P_{fc}$) in kW.

The plot shows the optimal operating region for the fuel cell, with a star indicating the optimal point.

![Graph showing fuel cell power over time.](image)

- The x-axis represents time in hours (hr).
- The y-axis represents fuel cell power in kW.

The graph illustrates the fluctuation of fuel cell power over time, with peaks and troughs indicating the dynamic performance of the fuel cell system.
Optimal Super Cap Size and Operating Region

Super Capacitor

SuperCap Power, kW

time (hr)
Optimal Battery Size and Operating Region

![Diagram showing the optimal battery size and operating region with a plot of battery energy (E_{bat}) and battery power (P_{bat}) over time (hr).]
Building HVAC

Volume of Air (the Room) $T_{room}, C_{room}$

Contaminant Source: $S_c$

Heat Leakage ($T_{outside}$ measured)

Solid Material $T_{solid}$

Control Variables: $T_{room}$ and $C_{room}$

Manipulated Variables: $F_{rcy}$ and $F_{fresh}$

Disturbances: $T_{outside}$ and $S_c$

Air Processing Unit ($T_{cool} = 20^oC$)

Energy Usage

$F_{rcy}, T_{room}, C_{room}$

$F_{rcy}, T_{cool}, C_{room}$

$F_{fresh}, T_{room}, C_{room}$

$F_{fresh}, T_{cool}, C_{fresh}$

$F_{fresh}, T_{outside}, C_{fresh}$

($C_{fresh} = 0$)
HVAC Control

Energy Usage of Traditional Controller: 3.16 kW
Energy Usage of Energy Efficient Controller: 2.55 kW
(a reduction of almost 20%).
Outline

• Model Predictive Controller Tuning
• Pseudo-Constrained Control
• Back-off and Profit Control
• Market Responsive Control
Thermal Energy Storage (TES)

In HVAC systems TES is used for Load Leveling and to shift usage to Off-Peak Hours.
Energy Prices and Weather

Cyclical pattern with a phase shift of about 3 hours.
Operation of the TES

- Heat Leakage \( T_{outside} \)
- Volume of Air (the Room) \( T_{room} \)
- Heat from Room
- Heat to Cooler
- Energy Usage
- Cooling Unit
- TES Unit

Graph showing:
- Temperature (°C)
- Cents per k hr
- Time (days)

- Blue line: Electricity Price
- Green line: Outside Temperature
Response to Market Changes

- EDOR’s due to different controller tunings
- BOP with more profit
- BOP with less profit
- OSSOP

* Peng et al. (2005)
Electric Price Model

White Noise Input → Shaping Filter → Sequence with Electricity Price Characteristics
Electric Price Model

White Noise Input → Shaping Filter → Sequence with Electricity Price Characteristics

Measured Electricity Price → State Estimator and/or Predictor → Prediction of Electricity Price
Model Predictive Control

\[
\min_{\nu_u(t)} \left\{ \int_0^T p_e(t) \nu_u(t) \, dt \right\}
\]

where \( p_e(t) \sim \) the predicted price (or value)

\( \nu_u(t) \sim \) the velocity of usage

and \( S(t) \sim \) amount in storage

Constraints include:

\[
0 \leq \nu_u(t) \leq \nu_u^{\text{max}} \quad \text{and} \quad 0 \leq S(t) \leq S^{\text{max}}
\]
Model Predictive Control

\[
\min_{v_u(t)} \left\{ \int_0^T p_e(t) \cdot v_u(t) \, dt \right\} \approx E[ p_e \cdot v_u ] = \overline{C}_e
\]

where \( p_e(t) \) \sim the predicted price (or value)
\( v_u(t) \) \sim the velocity of usage

and \( S(t) \) \sim amount in storage

Constraints include:

\[ 0 \leq v_u(t) \leq v_u^{\text{max}} \quad \text{and} \quad 0 \leq S(t) \leq S^{\text{max}} \]
System Design

\[
\min_{v_u(t)} \left\{ \int_{0}^{T} p_e(t) * v_u(t) \, dt \right\} \equiv E[p_e * v_u] \equiv \bar{C}_e
\]

How does \( v_u^{\text{max}} \) and \( S^{\text{max}} \) impact \( \bar{C}_e \)?

\( 0 \leq v_u(t) \leq v_u^{\text{max}} \) and \( 0 \leq S(t) \leq S^{\text{max}} \)
System Design

\[
\min_{v_u(t)} \left\{ \int_{0}^{T} p_e(t) * v_u(t) \, dt \right\} \approx E[p_e * v_u] \equiv \overline{C}_e
\]

How does \( v_u^{\text{max}} \) and \( S^{\text{max}} \) impact \( \overline{C}_e \)?

\( 0 \leq v_u(t) \leq v_u^{\text{max}} \) and \( 0 \leq S(t) \leq S^{\text{max}} \)

MPC cannot answer this question!
Expected Cost of Electricity

White Noise Input

Shaping Filter

$\mathbf{p}_{e}(t)$

Process Model

$\mathbf{v}_{u}(t)$

$\mathbf{E}[\mathbf{p}_{e}^{*}\mathbf{v}_{u}]$

Manipulated Variables

(Controller is $u = Lx$)
Re-Scaling of Price

\[ p'_e \equiv \alpha \ p_e \]

\[ E[p'_e \nu_u] \]

\[ \alpha \ w(t) \rightarrow \text{Shaping Filter} \rightarrow p'_e(t) \rightarrow \nu_u(t) \]

Manipulated Variables

(Controller is \( u = Lx \))

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Correlating Price and Usage

If \( E[(p'_e - v_u)^2] < \varepsilon \approx 0 \) and \( p'_e \equiv \alpha \ p_e \)

then \( v_u(t) \approx \alpha \ p_e(t) \)
Correlating Price and Usage

If \( E[(p'_e - \nu_u)^2] < \varepsilon \cong 0 \) and \( p'_e \equiv \alpha \ p_e \)

then \( \nu_u(t) \cong \alpha \ p_e(t) \)

Also, \( E[\alpha^2 p_e^2] - 2E[\alpha \ p_e \ \nu_u] + E[\nu_u^2] \cong 0 \)

\( \Rightarrow \alpha^2 E[p_e^2] = \alpha E[p_e \ \nu_u] = E[\nu_u^2] \)

\( \Rightarrow \alpha \ E[p_e^2] = E[p_e \ \nu_u] \equiv C_e \)
Minimum Cost of Electricity

\[ \overline{C}_e = \min_{L, \alpha} \{c_R \alpha \} \]

\[ (c_R = E[p_e^2]) \]

\[ E[(p'_e - v_u)^2] < \varepsilon \approx 0 \]

\[ E[v_u^2] < (v_u^{\text{max}})^2 \]

\[ E[S^2] < (S^{\text{max}})^2 \]
Thermal Energy Storage
(Small Storage Unit)

Volume of Air (the Room)

Heat Leakage $T_{outside}$

Heat from Room

Heat to Cooler

Heat to TES Unit

Energy Usage

Cooling Unit

TES Unit

Heat from Room

Heat to Cooler

Heat to TES Unit

Time (days)

kW hr / day

59 60 61 62

-50 0 50 100 150 200 250

-50

0 50

100 150 200

250

Heat from Room

Heat to Cooler

Heat to TES Unit
Thermal Energy Storage
(Medium Storage Unit)

- Heat Leakage $T_{outside}$
- Volume of Air (the Room) $T_{room}$
- Heat from Room
- Heat to Cooler
- Heat to TES Unit
- Cooling Unit
- Energy Usage

Graph:
- Heat from Room
- Heat to Cooler
- Heat to TES Unit

KWh hr / day vs. Time (days)
Thermal Energy Storage
(Large Storage Unit)

Volume of Air (the Room)

Heat Leakage $T_{outside}$

Heat from Room

Heat to Cooler

Cooling Unit

Energy Usage

Heat to TES Unit

TES Unit

Heat from Room

Heat to Cooler

Heat to TES Unit

$kW \, hr / day$

Time (days)

Heat from Room

Heat to Cooler

Heat to TES Unit

59  60  61  62

-200
-100
0
100
200
300

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Department of Chemical and Biological Engineering
Thermal Energy Storage
(Comparison of Storage Cases)
Thermal Energy Storage
(Cost Comparisons)

Average Cooling Costs:
- One ton: $8 per day
- Five tons: $7 per day (14% savings)
- Ten tons: $6 per day (25% savings)
Minimum Levelized Cost

\[
\min_{L, \alpha, v_u^{\text{max}}, S^{\text{max}}} \left\{ c_R \alpha - c_{L,1} v_u^{\text{max}} - c_{L,2} S^{\text{max}} \right\}
\]

\[
E\left[(p'_e - v_u)^2\right] < \varepsilon \equiv 0
\]

\[
E\left[v_u^2\right] < (v_u^{\text{max}})^2
\]

\[
E\left[S^2\right] < (S^{\text{max}})^2
\]
Minimum Levelized Cost

\[
\min_{L, \alpha, v_u^{\text{max}}, S^{\text{max}}} \left\{ c_R \alpha - c_{L,1} v_u^{\text{max}} - c_{L,2} S^{\text{max}} \right\}
\]

\[
E \left[ (p'_e - v_u)^2 \right] < \epsilon \equiv 0
\]

\[
E \left[ v_u^2 \right] < (v_u^{\text{max}})^2
\]

\[
E \left[ S^2 \right] < (S^{\text{max}})^2
\]

Non-Convex Problem
(but global solution from branch and bound)
Conclusions

• Relationship between control system performance and plant profit quantified.
• Enables profit guided control system design.
• Broad set of applications from a variety of disciplines.
• Linear controller can be designed for market responsiveness.
• Non-convex, but global methods can be used to size and/or select equipment.
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  - Professor Demetrois Moschandreas (CAEE, IIT)

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  - National Science Foundation (CBET – 0967906)
  - Graduate and Armour Colleges, IIT
  - Chemical & Biological Engineering Department, IIT
Value of Electric Power Generation

![Graph showing the value of electric power generation over time. The x-axis represents time in days, ranging from 0 to 20. The y-axis represents converted gas value in cents per m$^3$. The graph shows a fluctuating pattern with a horizontal line indicating a threshold or average value.](image)
Synthesis Gas Storage

Coal, Oxygen and Steam → Gasification and Gas Cleaning Units → Gas Storage Unit → Energy Conversion Units (Gas Turbines and Electric Generators) → Electric Power
IGCC Example
(Small Storage Unit)

Coal, Oxygen and Steam → Gasification and Gas Cleaning Units → Energy Conversion Units (Gas Turbines and Electric Generators) → Electric Power → Gas Storage Unit

Graphs:
- Converted Gas Value (cents / m³)
- Gas Volume in Storage (million m³)
- Volumetric Flow (million m³ / day)
IGCC Example
(Larger Storage Unit)

Coal, Oxygen and Steam →
Gasification and Gas Cleaning Units →
Energy Conversion Units (Gas Turbines and Electric Generators) →
Electric Power →
Gas Storage Unit

Graphs:
- Converted Gas Value (cents / m$^3$)
- Gas Volume in Storage (million m$^3$)
- Volumetric Flow (million m$^3$ / day)
IGCC Example
(Changes in Revenue)

Average Revenue

- Nominal Case: $1.00 million per day (plot not depicted)
- Case 1: $1.04 million per day.
- Case 2: $1.15 million per day.
Electric Power System Design

Gas Turbine

PC Boiler

Renewable

Transmission Grid

Consumer Demand

Energy Storage
System Disturbances

Consumer Demand

Forcasted Data
Simulated Data

Renewable Power Generated

Power Load (GW)

Days

P_f (MW)

Days

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# Electric Power System Model

<table>
<thead>
<tr>
<th>Resource</th>
<th>Power Balance Equation</th>
<th>Power Limits</th>
<th>Rate Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC Boiler</td>
<td>( \dot{P}_C = r_C )</td>
<td>( P_C^{\text{min}} \leq P_C \leq P_C^{\text{max}} )</td>
<td>( r_C^{\text{min}} \leq r_C \leq r_C^{\text{max}} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( P_C^{\text{min}} = 0.8 \cdot P_C^{\text{max}} )</td>
<td>( r_C^{\text{max}} = 0.05 \cdot P_C^{\text{max}} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( P_C^{\text{max}} = 1200 )</td>
<td></td>
</tr>
<tr>
<td>Gas Turbine</td>
<td>( \dot{P}_T = r_T )</td>
<td>( P_T^{\text{min}} \leq P_T \leq P_T^{\text{max}} )</td>
<td>( r_T^{\text{min}} \leq r_T \leq r_T^{\text{max}} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( P_T^{\text{min}} = 0.2 \cdot P_T^{\text{max}} )</td>
<td>( r_T^{\text{max}} = 6 \cdot P_T^{\text{max}} )</td>
</tr>
<tr>
<td>Pumped Hydro</td>
<td>( \dot{E}_S = P_S )</td>
<td>( 0 \leq E_S \leq E_S^{\text{max}} )</td>
<td>( P_S^{\text{min}} \leq P_S \leq P_S^{\text{max}} )</td>
</tr>
</tbody>
</table>

**Energy Limits**

- Pumped Hydro: \( 0 \leq E_S \leq E_S^{\text{max}} \)
Manipulated Variables

PC Boiler
\[ \dot{P}_C = r_C \]

**Power Limits**
\[ P_{C}^{\text{min}} \leq P_C \leq P_C^{\text{max}} \]
\[ P_C^{\text{min}} = 0.8 \cdot P_C^{\text{max}} \]
\[ P_C^{\text{max}} = 1200 \]

**Rate Limits**
\[ r_C^{\text{min}} \leq r_C \leq r_C^{\text{max}} \]
\[ r_C^{\text{max}} = 0.05 \cdot P_C^{\text{max}} \]

Gas Turbine
\[ \dot{P}_T = r_T \]

**Power Limits**
\[ P_T^{\text{min}} \leq P_T \leq P_T^{\text{max}} \]
\[ P_T^{\text{min}} = 0.2 \cdot P_T^{\text{max}} \]
\[ P_T^{\text{max}} = 1000 \]

**Rate Limits**
\[ r_T^{\text{min}} \leq r_T \leq r_T^{\text{max}} \]
\[ r_T^{\text{max}} = 6 \cdot P_T^{\text{max}} \]

Pumped Hydro
\[ \dot{E}_S = P_S \]

**Energy Limits**
\[ 0 \leq E_S \leq E_S^{\text{max}} \]

**Power Limits**
\[ P_S^{\text{min}} \leq P_S \leq P_S^{\text{max}} \]
Case Study

Average of Power Generators

- 32% Gas Turbine
- 48% PC Boiler
- 20% Renewable

Pumped Hydro Equipment Costs

- Energy Storage: $55/kWh
- Power Rating: $1300/kW
Results Case 1

Gas Turbine

Coal

Storage
# Results Summary

<table>
<thead>
<tr>
<th>Case</th>
<th>Coal Power</th>
<th>Gas Turbine</th>
<th>Renewable</th>
<th>Storage Size</th>
<th>Storage Power</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>48%</td>
<td>32%</td>
<td>20%</td>
<td>12.9 GWh</td>
<td>948 MW</td>
</tr>
<tr>
<td>2</td>
<td>18%</td>
<td>32%</td>
<td>50%</td>
<td>26.8 GWh</td>
<td>1398 MW</td>
</tr>
<tr>
<td>3</td>
<td>75%</td>
<td>5%</td>
<td>20%</td>
<td>61.1 GWh</td>
<td>1188 MW</td>
</tr>
</tbody>
</table>