Profit Based Control System Design:
A Globally Optimal Approach

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EDOR’s due to different controller tunings
BOP with less profit

BOP with more profit

OSSOP
Current Research Topics

• Fuel Cells – Modeling, Design, and Control
  • PEMFC, SOFC and On-board Fuel Processors (ATR)
Current Research Topics

- Fuel Cells – Modeling, Design, and Control
  - PEMFC, SOFC and On-board Fuel Processors (ATR)
- Stationary Power Plants – Modeling and Control
  - Coal Fired Boilers with Oxygen Enrichment, IGCC
Current Research Topics

- Fuel Cells – Modeling, Design, and Control
  - PEMFC, SOFC and On-board Fuel Processors (ATR)

- Stationary Power Plants – Modeling and Control
  - Coal Fired Boilers with Oxygen Enrichment, IGCC

- Control Theory
  - Profit Based Controller Design
  - Sensor and Actuator Selection
Outline

• Motivating Example
• Controller Tuning
• Economic Based Tuning
• Robust Formulation
Motivating Example
(Non-isothermal Reactor)

\[ V \frac{dC_A}{dt} = F(C_{A_{in}} - C_A) + Vr_A \]

\[ V \frac{dT}{dt} = F(T_{in} - T) + \left( V\Delta H / \rho C_p \right) r_A \]

\[ r_A = -k(T)C_A \]

Increase \( F \) \( \rightarrow \) Increased production rate
Motivating Example
(Non-isothermal Reactor)

\[ V \frac{dC_A}{dt} = F (C_{Ain} - C_A) + Vr_A \]

\[ V \frac{dT}{dt} = F (T_{in} - T) + \left( \frac{V \Delta H}{\rho C_p} \right) r_A \]

\[ r_A = -k(T)C_A \]

Increase \( F \) \( \Rightarrow \) Increased production rate

Decrease \( F \) \( \Rightarrow \) Increase \( T \) \( \Rightarrow \) Increase reaction rate

\( \Rightarrow \) Increase production
Motivating Example
(Limited Operating Region)

Process Limitations:

\[ T(t) \leq T^{(\text{max})} \]  
- Catalyst protection or onset of side reactions

\[ F(t) \leq F^{(\text{max})} \]  
- Pump limit or limit on downstream unit
Motivating Example
(Limited Operating Region)

Process Limitations:

\[ T(t) \leq T^{(\text{max})} \]
- Catalyst protection or onset of side reactions

\[ F(t) \leq F^{(\text{max})} \]
- Pump limit or limit on downstream unit

Possible Controller:

\[ F = K_c (T - T^{(\text{sp})}) + F^{(\text{sp})} \]
Motivating Example
(Performance in Time Series)
Motivating Example
(Performance in Phase Plane)

\[ T(t) \]

\[ F(t) \]
Motivating Example
(Elliptical Operating Region)

\[ T(t) \]

\[ F(t) \]
Motivating Example
(Elliptical Operating Region)
Motivating Example
(Limited Operating Region)

Controller:

\[ F = K_c (T - T^{(sp)}) + F^{(sp)} \]

Steady-State Relation:

\[ F^{(sp)} = f (T^{(sp)}, \bar{w}) \]
Motivating Example
(Elliptical Operating Region)

\[ T(t) \]

\[ F(t) \]
Motivating Example
(Steady-State Operating Line)
Motivating Example
(Optimal Operating Point)

Decrease $F$ $\rightarrow$ Increase $T$
$\rightarrow$ Increase conversion
$\rightarrow$ Increase production
Motivating Example
(Optimal Operating Point: Another Possibility)

Increase $F$ → Increased production rate
Motivating Example
(Optimal Operating Point: Another Possibility)

$T(t)$

Increase $F$  
→ Increased production rate

$F(t)$
Motivating Example
(Suggests Different Controller Tuning)
Motivating Example
(Less Aggressive Tuning)

\[ F(t) \]
\[ T^{(sp)} \]
\[ T^{(max)} \]
\[ T(t) \]
\[ time \]
\[ F^{(sp)} \]
\[ F^{(max)} \]
\[ F(t) \]
\[ time \]
Motivating Example
(Need for Automated Tuning)
Motivating Example
(Need for Automated Tuning)
Outline

• Motivating Example
• Controller Tuning
• Economic Based Tuning
• Robust Formulation
Covariance Analysis
(Open-Loop Case)

Process Model:
\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Gw(t) \\
z(t) &= Dx(t)
\end{align*}
\]

\(w(t)\) Gaussian white noise with covariance \(\Sigma_w\)

Steady State Covariance:
\[
A \Sigma_x + \Sigma_x A^T + G \Sigma_w G^T = 0
\]
\[
\Sigma_z = D \Sigma_x D^T
\]
Expected Dynamic Operating Region (EDOR)

EDOR defined by:

\[
\sum_{z} = \begin{bmatrix}
\sigma_{11}^2 & \sigma_{12}^2 \\
\sigma_{21}^2 & \sigma_{22}^2
\end{bmatrix}
\]
Flexibility in EDOR Definition

\[ \alpha = 1 \rightarrow \text{constraint observance } \sim 84\% \text{ of time} \]

\[ \alpha = 2 \rightarrow \text{constraint observance } \sim 95\% \text{ of time} \]

\[ \alpha = 3 \rightarrow \text{constraint observance } \sim 99.5\% \text{ of time} \]
Closed-Loop Covariance Analysis
(Full State Information Case)

Process Model:
\[ \dot{x} = Ax + Bu + Gw \]
\[ z = D_x x + D_u u + D_w w \]

Controller:
\[ u(t) = Lx(t) \]

Steady-State Covariance:
\[ (A + BL) \Sigma_x + \Sigma_x (A + BL)^T + G \Sigma_w G^T = 0 \]
\[ \Sigma_z = (D_x + D_u L) \Sigma_x (D_x + D_u L)^T + D_w \Sigma_w D_w^T \]
Closed-Loop EDOR

EDOR’s from different controllers

\[ u = L_1 x \]

\[ u = L_2 x \]
Constrained Closed-Loop EDOR

Constraints

\[(\alpha \cdot \sigma_{zi} < \bar{z}_i)\]
Constrained Closed-Loop EDOR

Constraints

\[ \alpha \cdot \sigma_{zi} < \bar{z}_i \]
Constrained Controller Existence

Does there exist $L$ such that:

$$(A + BL)\Sigma_x + \Sigma_x (A + BL)^T + G \Sigma_w G^T = 0$$

$$\Sigma_z = (D_x + D_u L)\Sigma_x (D_x + D_u L)^T + D_w \Sigma_w D_w^T$$

$$\xi_i = \phi_i \Sigma_z \phi_i^T = \sigma_i^2 < \left(\bar{z}_i / \alpha\right)^2 \quad i = 1 \ldots n_z$$

$$\phi_i = \begin{bmatrix} 0 & 0 & \ldots & 1 & \ldots & 0 & 0 \end{bmatrix}^T$$
Constrained Controller Existence

Does there exist $L$ such that:

$$(A + BL) \sum_x + \sum_x (A + BL)^T + G \sum_w G^T = 0$$

$$\sum_z = (D_x + D_u L) \sum_x (D_x + D_u L)^T + D_w \sum_w D_w^T$$

$$\xi_i = \phi_i \sum_z \phi_i^T < \bar{z}_i^2 \quad i = 1 \ldots n_z$$

$$\phi_i = \begin{bmatrix} 0 & 0 & \ldots & 1 & \ldots & 0 & 0 \end{bmatrix}$$

$\uparrow \text{\textit{ith column}}$
Constrained Controller Existence
(Convex Condition)

If and only if there exist $X>0$ and $Y$ such that:

$$(AX + BY) + (AX + BY)^T + G \Sigma_w G^T < 0$$

\[
\left[ \begin{array}{cccc}
\xi_i - \phi_i D_w \Sigma_w D_w^T \phi_i^T & \phi_i (D_x X + D_u Y) \\
(D_x X + D_u Y)^T \phi_i^T & X
\end{array} \right] > 0
\]

$$\xi_i < \bar{z}_i^2 \quad i = 1 \ldots n_z$$

And controller $u = Lx$ is constructed as: $L = YX^{-1}$
Implementation of (Pseudo-) Constrained Controller

Constraints:

$$(\alpha \cdot \sigma_{z_i} < \bar{z}_i)$$

Controller:

$$u = Lx$$

EDOR based on controller.
Model Predictive Control

\[
\min_{x,u} \left\{ \int_0^\infty \left( x^T Q x + 2u^T M x + u^T R u \right) dt \right\}
\]

s.t. \quad \dot{x} = Ax + Bu + Gw

\[z(t) = D_x x + D_u u + D_w w\]

\[-\bar{z}_i \leq z_i(t) \leq \bar{z}_i \quad i = 1 \ldots n_z\]
\[ \min_{x,u} \left\{ \int_{0}^{\infty} (x^T Q x + 2u^T M x + u^T R u) dt \right\} \]

s.t. \[ \dot{x} = Ax + Bu + Gw \]
\[ z(t) = D_x x + D_u u + D_w w \]
\[ -\infty \leq z_i(t) \leq \infty \quad i = 1 \ldots n_z \]
LQG

\[
\min_{x,u} \left\{ \int_0^\infty \left( x^T Q x + 2u^T M x + u^T R u \right) dt \right\}
\]

s.t. \( \dot{x} = Ax + Bu + Gw \)

Given A, B, Q, R and M, then the unconstrained controller is

\[
A^T P + PA + Q - (PB + M) R^{-1} (PB + M)^T = 0
\]

\[
L = -R^{-1} (PB + M)^T
\]
Constrained LQG Controller Existence

Does there exist \( Q, R \) and \( M \) such that:

\[
A^T P + PA + Q - (PB + M)R^{-1}(PB + M)^T = 0
\]

\[
L = -R^{-1}(PB + M)^T
\]

\[
(A + BL)\Sigma_x + \Sigma_x (A + BL)^T + G\Sigma_w G^T = 0
\]

\[
\Sigma_z = (D_x + D_u L)\Sigma_x (D_x + D_u L)^T + D_w\Sigma_w D_w^T
\]

\[
\xi_i = \phi_i\Sigma_z \phi_i^T < \bar{z}_i^2 \quad i = 1 \ldots n_z
\]
Constrained LQR Controller Existence (Convex Condition)
Constrained Controller Existence

Does there exist $L$ such that:

$$(A + BL) \Sigma_x + \Sigma_x (A + BL)^T + G \Sigma_w G^T = 0$$

$$\Sigma_z = (D_x + D_u L) \Sigma_x (D_x + D_u L)^T + D_w \Sigma_w D_w^T$$

$$\xi_i = \phi_i \Sigma_z \phi_i^T < \bar{z}_i^2 \quad i = 1 \ldots n_z$$
Constrained Minimum Variance (CMV) Controller

$$\min_{\Sigma_x>0,L,\xi} \sum_i d_i \xi_i$$

such that:

$$(A + BL) \Sigma_x + \Sigma_x (A + BL)^T + G \Sigma_w G^T = 0$$

$$\Sigma_z = (D_x + D_u L) \Sigma_x (D_x + D_u L)^T + D_w \Sigma_w D_w^T$$

$$\xi_i = \phi_i \Sigma_z \phi_i^T < \bar{z}_i^2 \quad i = 1 \ldots n_z$$
Controller Equivalence

Theorem 1 (Chmielewski & Manthanwar, 2004):

The controller generated by CMV

is coincident with

the controller generated by some
LQG problem.
Pareto Frontier Interpretation of Minimum Variance Control

\[
\min_{\Sigma_x > 0, L, \xi_i} \sum_i d_i \xi_i
\]

such that:

\[
(A + BL) \Sigma_x + \Sigma_x (A + BL)^T + G \Sigma_w G^T = 0
\]

\[
\Sigma_z = (D_x + D_u L) \Sigma_x (D_x + D_u L)^T + D_w \Sigma_w D_w^T
\]

\[
\xi_i = \phi_i \Sigma_z \phi_i^T \quad i = 1 \ldots n_z
\]
Pareto Frontier Interpretation of CMV Control

\[
\min_{\sum x > 0, L, \xi_i} \sum_i d_i \xi_i
\]

such that:

\[
(A + BL) \Sigma_x + \Sigma_x (A + BL)^T + G \Sigma_w G^T = 0
\]

\[
\Sigma_z = (D_x + D_u L) \Sigma_x (D_x + D_u L)^T + D_w \Sigma_w D_w^T
\]

\[
\xi_i = \phi_i \Sigma_z \phi_i^T < \bar{z}_{i}^2 \quad i = 1 \ldots n_z
\]
Example 1: Surge Tanks

State Variables: \( x = [V_1 \ V_2]^T \)

Manipulated Variables: \( u = [q_1 \ q_2]^T \)

Disturbance \( w = [q_0] \)

Performance Output: \( z = [V_1 \ V_2 \ q_1 \ q_2]^T \)

\[
A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}, \quad G = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad D_x = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad D_u = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad D_w = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\]
Example 1: Surge Tanks

Objective Weights: $d_1 = d_2 = d_3 = d_4 = 1$  Performance Output: $z = [V_1 \ V_2 \ q_1 \ q_2]^T$

Constraint: $\xi_4 < 2.25^2$
Example 1: Surge Tanks

Objective Weights: \( d_1 = d_2 = d_3 = d_4 = 1 \)

Performance Output: \( z = [V_1 \ V_2 \ q_1 \ q_2]^T \)

Constraints: \( \xi_4 < 2.25^2 \) and \( 1.5^2 \)
Example 1: Surge Tanks

Objective Weights: \( d_1 = d_2 = d_3 = d_4 = 1 \) 
Performance Output: \( z = [V_1 \ V_2 \ q_1 \ q_2]^T \)

Constraints: \( \xi_4 < 2.25^2, 1.5^2 \) and \( 0.75^2 \)
Example 1: Surge Tanks

Constraints: $\xi_4 < 2.25^2$ \rightarrow \quad L = \begin{bmatrix} 0.726 & -0.687 \\ 0.039 & 0.041 \end{bmatrix}$

Constraints: $\xi_4 < 1.5^2$ \rightarrow \quad L = \begin{bmatrix} 0.712 & -0.700 \\ 0.013 & 0.014 \end{bmatrix}$

Constraints: $\xi_4 < 0.75^2$ \rightarrow \quad L = \begin{bmatrix} 0.734 & -0.701 \\ 0.003 & 0.003 \end{bmatrix}$
Model Predictive Control

$$\min_{x,u} \left\{ \int_0^\infty \left( x^T Q x + 2u^T M x + u^T Ru \right) dt \right\}$$

s.t. \quad \dot{x} = Ax + Bu + Gw \\
\quad z(t) = D_x x + D_u u + D_w w \\
\quad -\bar{z}_i \leq z_i(t) \leq \bar{z}_i \quad i = 1 \ldots n_z$$
Inverse Optimality

Theorem 2 (Chmielewski & Manthanwar, 2004): If there exists $P > 0$ and $R > 0$ such that

$$\begin{bmatrix} L^T R L - A^T P - PA & -\left(L^T R + PB\right) \\ -\left(L^T R + PB\right)^T & R \end{bmatrix} > 0$$

Then $M = -(L^T R + PB)$ and $Q = L^T R L - A^T P + PA$ are such that

$$\begin{bmatrix} Q & M \\ M^T & R \end{bmatrix} > 0$$ and $P$ and $L$ satisfy

$$A^T P + PA + Q - (PB + M)R^{-1}(PB + M)^T = 0$$

$$L = -R^{-1}(PB + M)^T$$
Example 1: Surge Tanks

Constraints: $\xi_4 < 0.75^2 \rightarrow \begin{bmatrix} 0.734 & -0.701 \\ 0.003 & 0.003 \end{bmatrix}$

$Q = \begin{bmatrix} 161 & -160 \\ -160 & 160 \end{bmatrix}$, $R = \begin{bmatrix} 299 & -81.8 \\ -81.8 & 444 \end{bmatrix}$, $M = \begin{bmatrix} 20.2 & -60.8 \\ -20.8 & 58.8 \end{bmatrix}$

Thus, weights are available for MPC implementation.
CMV Control

\[
\min_{\Sigma_x > 0, L, \xi_i} \sum_i d_i \xi_i
\]

such that:

\[
(A + BL) \Sigma_x + \Sigma_x (A + BL)^T + G \Sigma_w G^T = 0
\]

\[
\Sigma_z = (D_x + D_u L) \Sigma_x (D_x + D_u L)^T + D_w \Sigma_w D_w^T
\]

\[
\xi_i = \phi_i \Sigma \phi_i^T < \bar{z}_i^2 \quad i = 1 \ldots n_z
\]

\[
\min_{x, u} \left\{ \int_0^\infty (x^T Q x + 2u^T M x + u^T R u) dt \right\}
\]

s.t. \quad \dot{x} = Ax + Bu + Gw

Achievable Performance Levels

Unachievable

Pareto frontier
Example 2: Inventory Control

Starts

Delivery delay

IC

Inventory

Demand

Inventory Set-point
Example 2: Inventory Control

Specific scenario:

Delivery Delay = 5 days

Demand Variance = $10^2$
Point A (the point of minimum inventory variance) is the basis of classic “safety stock” analysis.
Multi-Echelon Inventory Control

(from Seferlis & Giannelos, 2004)
2-Echelon Inventory Control
(Decentralized Control)

Starts

Delivery delay = 3

Inventory 1

Delivery delay = 5

Inventory 2

Demand

Inventory Set-point 1

Inventory Set-point 2
2-Echelon Inventory Control
(Centralized Control)

Starts

Delivery delay = 3

Inventory 1

Starts

Delivery delay = 5

Inventory 2

Demand

Inventory Set-point 1

Inventory Set-point 2

IC
2-Echelon Inventory Control
(Pareto Frontier)
2-Echelon Inventory Control (EDORs)
2-Echelon Inventory Control
(Pareto Frontier)

Small “safety stock” increase at $I_2$, leads to large “safety stock” decrease at $I_1$
2-Echelon Inventory Control (EDORs)

Small “safety stock” increase at I₂, leads to large “safety stock” decrease at I₁
2-Echelon Inventory Control
(Pareto Frontier)

Small “safety stock” increase at $I_2$, leads to large “safety stock” decrease at $I_1$
Small “safety stock” increase at $I_2$, leads to large “safety stock” decrease at $I_1$ but starts variance size trend is unexpected.
Example 3: Hybrid Fuel Cell Vehicle

\[
\begin{align*}
V_{fc} & \quad i_{fc} \\
K_{fc} & \quad i_{afc} \\
V_{b} & \quad i_{b} \\
E_{b} & \quad i_{ab} \\
R_{b} & \quad L_{a} \\
R_{a} & \quad V_{a} \\
\end{align*}
\]
Classic Control

Vehicle

$P_{mot}$

$P_{mot}(sp)$

$P_{fc}$

$V_{fc}(sp)$

$V_{fc}$

$P_{bat}$

$P_{bat}(sp)$

$P_{bat}$

$V_{fc}$

$FUEL CELL VOLTAGE CONTROLLER$

$PI$

$PI$

$Vehicle$

$K_{fc}$

$K_{b}$

$R_{b}$

$E_{b}$

$L_{a}$

$V_{a}$

$K_{fc}$

$P_{mot}$

$P_{bat}$

$V_{fc}$

$FUEL CELL VOLTAGE CONTROLLER$

$PI$

$PI$

$Vehicle$

$K_{fc}$

$P_{mot}(sp)$

$P_{bat}(sp)$

$P_{fc}$

$V_{fc}(sp)$

$V_{fc}$

$P_{mot}$

$P_{bat}$

$V_{fc}$

$FUEL CELL VOLTAGE CONTROLLER$

$PI$

$PI$

$Vehicle$

$K_{fc}$

$P_{mot}(sp)$

$P_{bat}(sp)$

$P_{fc}$

$V_{fc}(sp)$

$V_{fc}$

$P_{mot}$

$P_{bat}$

$V_{fc}$

$FUEL CELL VOLTAGE CONTROLLER$

$PI$

$PI$

$Vehicle$

$K_{fc}$

$P_{mot}(sp)$

$P_{bat}(sp)$

$P_{fc}$

$V_{fc}(sp)$

$V_{fc}$

$P_{mot}$

$P_{bat}$

$V_{fc}$

$FUEL CELL VOLTAGE CONTROLLER$

$PI$

$PI$

$Vehicle$

$K_{fc}$
Separation of Time-Scales

- Power Profiles [W]
- Time, sec
- P_{load}(sp)
- Battery
- Fuel Cell
- Armature
- P_{mot}(sp)
- P_{bat}(sp)
- V_{fc} (sp)
- FUEL CELL VOLTAGE CONTROLLER
- PI
- Vehicle
- P_{mot}
- P_{bat}
- P_{fc}

Diagram illustrating the separation of time-scales in a vehicular power system, showing the interactions between fuel cell, battery, and armature power outputs over time.
Dynamics of PEMFC
Dynamics of PEMFC

Anode In (H₂, H₂O) → H₂ → H₂O → H₂O → Anode Exhaust

Cathode In (air) → O₂ → N₂ → N₂ → Cathode Exhaust

Cooling Air In

Jacket Exhaust

Solid Material

Current Collector

Insulator

Gas Diffusion Layers (GDLs)

Catalyst Layers

Polymer Membrane

E_{cell}
Dynamics of PEMFC

Cooling Air In

Anode In (H₂, H₂O)

Solid Material

Current Collector

Insulator

Cathode In (air)

Gas Diffusion Layers (GDLs)

Polymer Membrane

Catalyst Layers

Anode Exhaust

Cathode Exhaust

H₂

H₂O

H₂O

O₂

N₂

E_{cell}
Hybrid Fuel Cell Vehicle
(Double Storage Configuration)
Supervisory Control

$P_{motor}$

Supervisory Controller

$P_{fc}^{(sp)}$

$P_{fc}$

$k_{fc}$

Vehicle

Power System

$P_{bat}^{(sp)}$

$P_{bat}$

$k_{bat}$

$P_{scap}^{(sp)}$

$P_{scap}$

$k_{scap}$
Hybrid Fuel Cell Vehicle
(Supervisory Model)

\[
\begin{align*}
\dot{P}_{fc} &= \Delta P_{fc} \\
\dot{E}_{bat} &= P_{bat} \\
\dot{E}_{scap} &= P_{scap} \\
\end{align*}
\]

\[
\begin{align*}
0 &\leq P_{fc} \leq P_{fc}^{\text{max}} \\
0 &\leq E_{bat} \leq E_{bat}^{\text{max}} \\
0 &\leq E_{scap} \leq E_{scap}^{\text{max}} \\
\end{align*}
\]

\[
\begin{align*}
\Delta P_{fc}^{\text{min}} &\leq \Delta P_{fc} \leq \Delta P_{fc}^{\text{max}} \\
P_{bat}^{\text{min}} &\leq P_{bat} \leq P_{bat}^{\text{max}} \\
P_{scap}^{\text{min}} &\leq P_{scap} \leq P_{scap}^{\text{max}} \\
\end{align*}
\]

\[
\begin{align*}
P_{mot} &= P_{fc} + P_{bat} + P_{scap} \\
\end{align*}
\]

\[
\begin{align*}
P_{bat}^{\text{max}} &= E_{bat}^{\text{max}} \cdot C_{rate}^{bat} \\
P_{sc}^{\text{max}} &= E_{sc}^{\text{max}} \cdot C_{rate}^{sc} \\
\Delta P_{fc}^{\text{max}} &= P_{fc}^{\text{max}} \cdot C_{rate}^{fc} \\
\end{align*}
\]

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Hybrid Fuel Cell Vehicle
(Supervisory Model)

\[ \dot{P}_{fc} = \Delta P_{fc} \quad 0 \leq P_{fc} \leq P_{fc}^{\max} \]
\[ \dot{E}_{bat} = P_{bat} \quad 0 \leq E_{bat} \leq E_{bat}^{\max} \]
\[ \dot{E}_{scap} = P_{scap} \quad 0 \leq E_{scap} \leq E_{scap}^{\max} \]

\[ P_{mot} = P_{fc} + P_{bat} + P_{scap} \]

\[ P_{bat}^{\max} = E_{bat}^{\max} \cdot C_{rate}^{bat} \]
\[ P_{sc}^{\max} = E_{sc}^{\max} \cdot C_{rate}^{sc} \]

\[ \Delta P_{fc}^{\min} \leq \Delta P_{fc} \leq \Delta P_{fc}^{\max} \]
\[ P_{bat}^{\min} \leq P_{bat} \leq P_{bat}^{\max} \]
\[ P_{scap}^{\min} \leq P_{scap} \leq P_{scap}^{\max} \]

\[ \Delta P_{fc}^{\max} = P_{fc}^{\max} \cdot C_{rate}^{fc} \]
Hybrid Fuel Cell Vehicle  
(Supervisory Model)

\[
\begin{align*}
\dot{P}_{fc} &= \Delta P_{fc} \\
0 &\leq P_{fc} \leq P_{fc}^{\max} \\
\dot{E}_{bat} &= P_{bat} \\
0 &\leq E_{bat} \leq E_{bat}^{\max} \\
\dot{E}_{scap} &= P_{scap} \\
0 &\leq E_{scap} \leq E_{scap}^{\max}
\end{align*}
\]

\[
\begin{align*}
\Delta P_{fc}^{\min} &\leq \Delta P_{fc} \leq \Delta P_{fc}^{\max} \\
P_{bat}^{\min} &\leq P_{bat} \leq P_{bat}^{\max} \\
P_{scap}^{\min} &\leq P_{scap} \leq P_{scap}^{\max}
\end{align*}
\]

\[
\begin{align*}
P_{mot} &= P_{fc} + P_{bat} + P_{scap}
\end{align*}
\]

\[
\begin{align*}
P_{bat}^{\max} &= E_{bat}^{\max} \cdot C_{rate}^{bat} \\
P_{sc}^{\max} &= E_{sc}^{\max} \cdot C_{rate}^{sc} \\
\Delta P_{fc}^{\max} &= P_{fc}^{\max} \cdot C_{rate}^{fc}
\end{align*}
\]

Illinois Institute of Technology  
Department of Chemical and Biological Engineering
Hybrid Fuel Cell Vehicle
(Supervisory Model)

\[
\dot{P}_{fc} = \Delta P_{fc} \quad 0 \leq P_{fc} \leq P_{fc}^{\text{max}} \\
\dot{E}_{bat} = P_{bat} \quad 0 \leq E_{bat} \leq E_{bat}^{\text{max}} \\
\dot{E}_{scap} = P_{scap} \quad 0 \leq E_{scap} \leq E_{scap}^{\text{max}}
\]

\[
\Delta P_{fc}^{\text{min}} \leq \Delta P_{fc} \leq \Delta P_{fc}^{\text{max}} \\
P_{bat}^{\text{min}} \leq P_{bat} \leq P_{bat}^{\text{max}} \\
P_{scap}^{\text{min}} \leq P_{scap} \leq P_{scap}^{\text{max}}
\]

\[P_{mot} = P_{fc} + P_{bat} + P_{scap}\]

\[
P_{bat}^{\text{max}} = E_{bat}^{\text{max}} \cdot C_{rate}^{bat} \quad P_{sc}^{\text{max}} = E_{sc}^{\text{max}} \cdot C_{rate}^{sc} \quad \Delta P_{fc}^{\text{max}} = P_{fc}^{\text{max}} \cdot C_{rate}^{fc}
\]

\leftarrow \text{Colored noise disturbance} \\
\text{(modeled from drive cycle data)}

Hybrid Fuel Cell Vehicle
(Drive Cycle Simulation 1)
Hybrid Fuel Cell Vehicle (EDOR Case 1)
Hybrid Fuel Cell Vehicle  
(EDOR Case 2)
Hybrid Fuel Cell Vehicle
(Drive Cycle Simulation 2)
Outline

• Motivating Example
• Controller Tuning
• Economic Based Tuning
• Robust Formulation
Motivating Example

\[ T(t) \quad \uparrow \]

\[ F(t) \]

\[ \star \star \star \]
Constrained Operating Region

Steady-State Operating Point

CV’s

Constraints

EDOR

MV’s
Real-Time Optimization

Steady-State Operating Point

Constraints

EDOR

Optimal Steady-State Operating Point (OSSOP)

CV’s

MV’s
Real-Time Optimization

Original Nonlinear Process Model:

\[ \dot{s} = f(s, m, p) \quad q = h(s, m, p) \]

\[(s, m, p, q) \sim \text{(state, mv, dist, performance)} \sim (x, u, w, z)\]
Real-Time Optimization

Original Nonlinear Process Model:

\[ \dot{s} = f(s, m, p) \quad q = h(s, m, p) \]

\((s, m, p, q) \sim (\text{state, mv, dist, performance}) \sim (x, u, w, z)\)

Real-Time Optimization (minimize profit loss):

\[ \min_{s, m, q} \{ g(q) \} \quad \text{s.t.} \]

\[ 0 = f(s, m, p) \quad q = h(s, m, p) \quad q_i^{\min} \leq \phi_i q \leq q_i^{\max} \]

RTO solution denoted as \((s^{ossop}, m^{ossop}, p^{ossop}, q^{ossop})\)
Backed-off Operating Point (BOP)

- CV’s
- MV’s

Backed-off Operating Point (BOP)
Optimal Steady-State Operating Point (OSSOP)

EDOR
Steady-State BOP Selection
(Bahri, Bandoni & Romagnoli, 1996)

Solve the following Semi-infinite Programming Problem

\[
\min_{s,m,q} \left\{ g(q) \right\} \quad \text{s.t.} \quad \max_{p \in [p^{\text{min}}, p^{\text{max}}]} \left( \max_{i} \left\{ q_i - \bar{q}_i \right\} \right)
\]

\[
\text{s.t.} \quad 0 = f(s,m,p)
\quad q = h(s,m,p)
\quad q_i - \bar{q}_i < 0
\]

**Extensions:**

# Linearized Perspective

**Nonlinear**

\[ g(q^{bop}) \]

\[ \dot{s} = f(s, m, p) \]

\[ q = h(s, m, p) \]

\[ q_{i}^{\text{min}} \leq q_{i} \leq q_{i}^{\text{max}} \]

**Linear wrt OSSOP**

\[ g(q^{ossop}) + g_{q}q' \]

\[ \dot{s}' = A s' + B m' + G p' \]

\[ q' = D_{x} s' + D_{u} m' + D_{w} p' \]

\[ q'_{i}^{\text{min}} \leq q'_{i} \leq q'_{i}^{\text{max}} \]

**Linear wrt BOP**

\[ \dot{x} = A x + B u + G w \]

\[ z = D_{x} x + D_{u} u + D_{w} w \]

\[ z_{i}^{\text{min}} \leq z_{i} \leq z_{i}^{\text{max}} \]

Deviation Variables w.r.t. OSSOP:

\[ s' = s^{bop} - s^{ossop} \]

\[ m' = m^{bop} - m^{ossop} \]

\[ p' = p^{bop} - p^{ossop} \]

\[ q' = q^{bop} - q^{ossop} \]

Deviation Variables w.r.t. BOP:

\[ x = s - s^{bop} \]

\[ u = m - m^{bop} \]

\[ w = p - p^{bop} \]

\[ z = q - q^{bop} \]
Stochastic BOP Selection
(Loeblein & Perkins, 1999)

Assume controller $L$ is given and calculate $\xi_i$:

$$(A + BL)\Sigma_x + \Sigma_x (A + BL)^T + G\Sigma_w G^T = 0$$

$$\Sigma_z = (D_x + D_u L)\Sigma_x (D_x + D_u L)^T + D_w \Sigma_w D_w^T$$

$$\xi_i = \phi_i \Sigma_z \phi_i^T \quad i = 1 \ldots n_z$$
Stochastic BOP Selection
(Loeblein & Perkins, 1999)

Assume controller $L$ is given and calculate $\xi_i$:

$$(A + BL)\Sigma_x + \Sigma_x (A + BL)^T + G\Sigma_w G^T = 0$$

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$$\xi_i = \phi_i \Sigma_z \phi_i^T \quad i = 1 \ldots n_z$$

Solve the following Linear Program:

$$\min_{s',m',q'} \left\{ g_q q' \right\} \quad \text{s.t.} \quad 0 = As' + Bm'$$

$$q_i' = \phi_i (D_x s' + D_u m')$$

$$q_i^{\min} \leq q_i' \leq q_i^{\max}$$

$$\xi_i^{1/2} < q_i^{\max} - q_i'$$

$$\xi_i^{1/2} < q_i' - q_i^{\min}$$
Stochastic BOP Selection
(Loeblein & Perkins, 1999)

Assume controller $L$ is given and calculate $\xi_i$:

$$(A + BL)\Sigma_x + \Sigma_x (A + BL)^T + G\Sigma_w G^T = 0$$

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Solve the following Linear Program:

$$\min \left\{ g_q q' \right\} \quad \text{s.t.} \quad 0 = As' + Bm'$$

$$q_i' = \phi_i (D_x s' + D_u m') \quad q_i^{\min} \leq q_i' \leq q_i^{\max}$$

$$\xi_i^{1/2} < q_i^{\max} - q_i' \quad \xi_i^{1/2} < q_i' - q_i^{\min}$$
Stochastic BOP Selection
(EDOR within Constraint Set)

\[ \xi_i^{1/2} < q_i' - q_i'^{\text{min}} \]

\[ \xi_i^{1/2} < q_i'^{\text{max}} - q_i' \]
Stochastic BOP Selection
(EDOR within Constraint Set)

\[ \sigma_i < q'_i - q'_{i \text{min}} \quad \text{and} \quad \sigma_i < q'_{i \text{max}} - q'_i \]
Stochastic BOP Selection
(EDOR within Constraint Set)

\[ \sigma_i < q'_i - q'^{\text{min}}_i \]
\[ \sigma_i < q'^{\text{max}}_i - q'_i \]

\( q'^{\text{max}}_1 \)
\( q'^{\text{min}}_1 \)
\( q'^{\text{max}}_2 - q'^{\text{min}}_2 \)
\( q'_2 \)
Fixed Controller BOP Selection

Loeblein and Perkins (1999):

Controller is fixed $\Rightarrow$ EDORs have fixed sizes and shapes
Variable Controller BOP Selection

Peng et al. (2005):

Variable Controller $\iff$ EDORs have variable sizes and shapes
Profit Control
(Simultaneous BOP and Controller Selection)

EDOR’s due to different controller tunings

BOP with more profit

BOP with less profit

OSSOP

* Peng et al. (2005)
Profit Control
(Simultaneous BOP and Controller Selection)

\[
\begin{align*}
\min \quad & \left\{ g \frac{q_1'}{g_2'} \right\} \\
\text{s.t.} \quad & 0 = As' + Bm' \\
q_i' &= \phi_i (D_x s' + D_u m') \\
q_i^\min & \leq q_i' \leq q_i^\max \\
\xi_i^{1/2} & < q_i^\max - q_i' \\
\xi_i^{1/2} & < q_i' - q_i^\min \\
(A + BL) \Sigma_x + \Sigma_x (A + BL)^T + G \Sigma_w G^T &= 0 \\
\Sigma_z &= (D_x + D_u L) \Sigma_x (D_x + D_u L)^T + D_w \Sigma_w D_w^T \\
\xi_i &= \phi_i \Sigma_z \phi_i^T \quad i = 1 \ldots n_z
\end{align*}
\]

Peng et al. (2005)
Profit Control
(Simultaneous BOP and Controller Selection)

\[
\min_{s', m', q', \xi_i, X, Y} \left\{ g_{q, q'} \right\} \quad \text{s.t.} \quad 0 = As' + Bm'
\]

\[
q_i' = \phi_i (D_x s' + D_u m') \quad q_i^{\min} \leq q_i' \leq q_i^{\max}
\]

\[
\xi_i^{1/2} < q_i^{\max} - q_i' \quad \xi_i^{1/2} < q_i' - q_i^{\min}
\]

\[
(AX + BY) + (AX + BL)^T + G \Sigma_w G^T < 0
\]

\[
\left[
\begin{array}{c}
\xi_i - \phi_i D_w \Sigma_w D_w^T \phi_i^T \\
(D_x X + D_u Y)^T \phi_i^T
\end{array}
\right]
\left[
\begin{array}{c}
\phi_i (D_x X + D_u Y) \\
X
\end{array}
\right] > 0
\]

Peng et al. (2005)
Computational Aspects of Profit Control

\[
\begin{align*}
\min_{s', m', q', \xi_i, X, Y} & \left\{ g_q q' \right\} \quad \text{s.t.} \quad 0 = As' + Bm' \\
q_i' &= \phi_i (D_x s' + D_u m') \quad q_i^{\text{min}} \leq q_i' \leq q_i^{\text{max}} \\
\xi_i^{1/2} &< q_i^{\text{max}} - q_i' \quad \xi_i^{1/2} < q_i' - q_i^{\text{min}} \\
(AX + BY) + (AX + BL)^T + G \Sigma_w G^T &< 0 \\
\begin{bmatrix}
\xi_i - \phi_i D_w \Sigma_w D_w^T \phi_i^T \\
(D_x X + D_u Y)^T \phi_i^T \\
(D_x X + D_u Y) \phi_i \\
X
\end{bmatrix} &> 0
\end{align*}
\]

Peng et al. (2005)
Reverse-Convex Constraints

\[ \xi_1 < (q_1^{\text{max}} - q_1')^2 \]

\[ \xi_1 < (q_1' - q_1^{\text{min}})^2 \]

Feasible Region
Global Solution

Based on Branch and Bound algorithm

- Region 1
- Region 2
- Region 3
- Region 4
- Region 5
Combinatorial Growth of Branch and Bound
Combinatorial Growth of Branch and Bound
Heuristic Scheme

Feasible Region

\[ \xi_i \]

\[ q'_i \]

\[ z_{ss,i} \]

\[ d_{min,i} \]

\[ d_{max,i} \]
Heuristic Scheme

Feasible Region

\[ z_{ss,i} \leq \frac{d_{min,i}}{2} \leq \frac{d_{max,i}}{2} \]
Heuristic Scheme

\[ \xi_i = \left( z_{ss,i} + d_{\text{min},i} \right)^2 \left( z_{ss,i} + d_{\text{max},i} \right)^2 \]

Feasible Region
Heuristic Scheme

\[
\xi_i = \frac{(z_{ss,i} + d_{min,i})^2}{(z_{ss,i} + d_{max,i})^2}
\]

Feasible Region
Heuristic Scheme

Feasible Region

\[ z_{ss,i} + d_{min,i} \leq \xi_i \leq z_{ss,i} + d_{max,i} \]
Heuristic Scheme

\[ \xi_i = \frac{(z_{ss,i} + d_{min,i})^2}{(z_{ss,i} + d_{max,i})^2} \]

Feasible Region
Heuristic Scheme

\[ \xi_i (z_{ss,i} + d_{\text{min},i})^2 \leq \left( z_{ss,i} + d_{\text{max},i} \right)^2 \]

Feasible Region

\( q_1' \)

\( z_{ss,i} \)
Combinatorial Growth??

Feasible Region

$z_{ss,i}$

$\xi_i$

$q'_1$

$q'_2$

$q'_3$

$q'_4$
Mass-Spring-Damper Example

System Model:

\[
\begin{bmatrix}
\dot{r} \\
\dot{v}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-2 & -3
\end{bmatrix}
\begin{bmatrix}
r \\
v
\end{bmatrix} +
\begin{bmatrix}
0 \\
1
\end{bmatrix} f +
\begin{bmatrix}
0 \\
1
\end{bmatrix} w
\]

where \( r \) is the mass position, \( v \) is the velocity,
\( f \) is the input force (MV) and
\( w \) is the disturbance force

System Constraints:

\(-1 \leq r \leq 1 \) and \( 0 \leq f \leq 16 \)
Mass-Spring-Damper Example

System Model:

\[
\begin{bmatrix}
\dot{r} \\
\dot{v}
\end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} r \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w
\]

where \( r \) is the mass position, \( v \) is the velocity, \( f \) is the input force (MV) and \( w \) is the disturbance force

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Department of Chemical and Biological Engineering
Illinois Institute of Technology
Mass-Spring-Damper Example

System Model:
\[
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\dot{v}
\end{bmatrix} =
\begin{bmatrix}
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\begin{bmatrix}
r \\
v
\end{bmatrix}
+ \begin{bmatrix}
0 \\
1
\end{bmatrix} f + \begin{bmatrix}
0 \\
1
\end{bmatrix} w
\]

where \( r \) is the mass position, \( v \) is the velocity, 
\( f \) is the input force (MV) and 
\( w \) is the disturbance force.

System Constraints:
\[-1 \leq r \leq 1 \quad \text{and} \quad 0 \leq f \leq 16\]
Mass-Spring-Damper Example
(Phase Plane)

- OSSOP
- Steady-State Line
- Constraints

BOP
Mass-Spring-Damper Example (Phase Plane Solution)

FSI Case:
Full State Information:
Controller is $u(t) = Lx(t)$

PSI Case:
Partial State Information:
One Velocity Sensor.
Controller is $u(t) = L\hat{x}(t)$
Mass-Spring-Damper Example
(Impact of Constraints)
Discrete-time Simulation
(Scatter Plot)
MPC and the EDOR

\[
\min_{x,u} \left\{ \int_0^\infty \left( x^T Q x + 2 u^T M x + u^T R u \right) dt \right\}
\]

s.t. \( \dot{x} = A x + B u + G w \)

\( z(t) = D_x x + D_u u + D_w w \)

\( z_i^{\min} \leq z_i(t) \leq z_i^{\max} \quad i = 1 \ldots n_z \)
Soft Constraints

$$\min_{x,u} \left\{ \int_{0}^{\infty} \left( x^T Q x + 2u^T M x + u^T R u \right) dt + s^T \Gamma s \right\}$$

s.t.  \( \dot{x} = Ax + Bu + Gw \)

\( z(t) = D_x x + D_u u + D_w w \)

\( z_i^{\text{min}} - s_i \leq z_i(t) \leq z_i^{\text{max}} + s_i \quad i = 1 \ldots n_z \)

\( s_i \geq 0 \)
Soft Constraints

\[
\min_{x,u} \left\{ \int_0^\infty (x^T Q x + 2u^T M x + u^T R u) \, dt + s^T \Gamma s \right\}
\]

s.t. \quad \dot{x} = A x + B u + G w

\[
z(t) = D_x x + D_u u + D_w w
\]

\[
z_i^{\min} - s_i \leq z_i(t) \leq z_i^{\max} + s_i \quad i = 1 \ldots n_z
\]

\[
s_i \geq 0
\]
MPC with Soft Constraints

\[ \gamma_m = 10^7 \quad \gamma_f = 10^3 \]

\[ \gamma_m = 10^3 \quad \gamma_f = 10^3 \]
Flexibility in EDOR Definition

\[ a = 1 \rightarrow \text{constraint observance } \sim 84\% \text{ of time} \]

\[ a = 2 \rightarrow \text{constraint observance } \sim 95\% \text{ of time} \]

\[ a = 3 \rightarrow \text{constraint observance } \sim 99.5\% \text{ of time} \]
Impact of EDOR Definition

\[ \alpha = 1 \quad \text{and} \quad \alpha = 2 \]
MPC with Soft Constraints

EDOR = 2 std dev’s

\[ \gamma_m = 10^7 \quad \gamma_f = 10^3 \]

\[ \gamma_m = 10^3 \quad \gamma_f = 10^3 \]
Impact of EDOR Definition
(Reduced Sensitivity to Soft Weights)

\[ \gamma_m = 10^7 \quad \gamma_f = 10^3 \quad \gamma_m = 10^3 \quad \gamma_f = 10^3 \]
Fluidized Catalytic Cracker (FCC) Example

Regenerator and Separator (dynamic):

\[ W \frac{dC_{rgc}}{dt} = F_{cat}(C_{st} - C_{rgc}) - R_{cb} \]
\[ W_a \frac{dO_d}{dt} = \frac{F_{air} (O_{in} - O_d)}{M_{air}} - \frac{1 + 1.5 \sigma R_{cb}}{1 + \sigma} \frac{M_c}{M_c} \]
\[ W_{c_{pc}} \frac{dT_{reg}}{dt} = F_{cat}c_{pc}T_{st} + F_{air}c_{pair}T_{air} - (F_{cat}c_{pc} + F_{air}c_{pair})T_{reg} - \left( \Delta H_{CO} + \frac{\sigma}{1 + \sigma} \Delta H_{CO_2} \right) \frac{R_{cb}}{M_c} \]
\[ W_{st} \frac{dC_{cat}}{dt} = F_{cat}(C_{sc} - C_{st}) \]
\[ W_{st}c_{pc} \frac{dT_{st}}{dt} = F_{cat}c_{pc}(T_{ro} - T_{st}) \]
\[ T_{cy} = T_{reg} + 5.555O_d \]

Riser (pseudo steady state):

\[ \frac{dy_f}{dz} = -K_1y_f[COR]\phi t_{c}, \quad y_f(z = 0) = 1 \]
\[ \frac{dy_g}{dz} = (K_2y_f^2 - K_3y_g)[COR]\phi t_{c}, \quad y_g(z = 0) = 0 \]
\[ \frac{d\theta}{dz} = \frac{\Delta H_f F_{feed}}{T_{ri}(F_{cat}c_{cp} + F_{feed}c_{pf} + \lambda F_{feed}c_{pc})} \frac{dy_f}{dz} \]
\[ \theta(z) = \frac{T(z) - T_{ri}}{T_{ri}}, \quad \theta(z = 0) = 0, \quad T_{ro} = T(z = 1) \]

(adapted from Loeblein & Perkins, 1999)
FCC Example

Process Constraints:

\[ 400 \, K \leq T_{st} \leq 1000 \, K \]
\[ 600 \, K \leq T_{reg} \leq 1000 \, K \]
\[ T_{reg} \leq T_{cy} \leq 1000 \, K \]
\[ 100 \frac{kg}{s} \leq F_{cat} \leq 400 \frac{kg}{s} \]
\[ 0 \leq F_{air} \leq 60 \, kg/s \]

Profit Function:

\[ \Phi = 86400 \left(P_{gs} F_{gs} + P_{gl} F_{gl} + P_{ugo} F_{ugo} - P_{uog} F_{Feed}\right) \]

\( F_{gs}, F_{gl}\) and \( F_{ugo}\) are product flows (gasoline, light gas and unconverted oil).

(adapted from Loeblein & Perkins, 1999)
Fixed LQG Controller

\[
\min_{x,u} \left\{ \int_0^\infty (z^T Dz) dt \right\}
\]

s.t. \quad \dot{x} = Ax + Bu + Gw \quad z = Dx x + Du u + Dw w

\[D = I\]

\[Q = D_x^T D D_x \quad R = D_u^T D D_u \quad M = D_x^T D D_u\]

\[L = Ric(Q, R, M)\]
Fixed Controller FCC
(Loeblein & Perkins, 1999)
Free Controller FCC
(Profit Control)
# FCC Profit

<table>
<thead>
<tr>
<th>Gross Profit ($/day)</th>
<th>Diff from OSSOP ($/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OSSOP</td>
<td>$36,905</td>
</tr>
<tr>
<td>Fixed Control</td>
<td>$35,768</td>
</tr>
</tbody>
</table>
## FCC Profit

<table>
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<tr>
<td>OSSOP</td>
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<td>-</td>
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<td>- $1,137</td>
</tr>
<tr>
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<td>$36,160</td>
<td>- $745</td>
</tr>
</tbody>
</table>
# FCC Profit

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</thead>
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<td><strong>OSSOP</strong></td>
<td>$36,905</td>
<td>-</td>
</tr>
<tr>
<td><strong>EDOR = 1 std. dev.</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fixed Control</td>
<td>$35,768</td>
<td>- $1,137</td>
</tr>
<tr>
<td>Profit Control</td>
<td>$36,160</td>
<td>- $745</td>
</tr>
<tr>
<td><strong>EDOR = 2 std. dev.</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fixed Control</td>
<td>$34,631</td>
<td>- $2,274</td>
</tr>
<tr>
<td>Profit Control</td>
<td>$35,416</td>
<td>- $1,489</td>
</tr>
</tbody>
</table>
CSTR’s in Series

(adapted from de Hennin & Perkins, 1991)
CSTR’s in Series

Manipulated Variables:

\[ Q_{F1}, \quad Q_M \quad Q_{c1} \quad Q_{c2} \]

Disturbances:

\[ w = \begin{bmatrix} T_{in} \\ C_{A,in} \end{bmatrix} \]

\[ \Sigma_w = \begin{bmatrix} 3^2 & 0 \\ 0 & 1^2 \end{bmatrix} \]

(adapted from de Hennin & Perkins, 1991)
CSTR’s in Series

Some Process Constraints:

\[ T_1 \leq 350K \quad T_2 \leq 350K \quad T_{c1,\text{out}} \leq 330K \quad T_{c2,\text{out}} \leq 300K \]

\[ Q_{F1} + Q_M \leq 0.8m^3/s \quad Q_{F1} \geq 0.05m^3/s \quad Q_M \geq 0.05m^3/s \]

Profit Function:

\[ \Phi = 10(Q_{F1} + Q_M)C_{B2} \]

\[ -0.1Q_{F1} - 0.1Q_M \]

\[ -0.01q_1 - q_2 \]

\[ q_1 = \frac{2U_a Q_{j,1}(T_1 - T_{c1,\text{in}})}{2Q_{j,1} + U_a} \quad q_2 = \frac{2U_a Q_{j,2}(T_2 - T_{c2,\text{in}})}{2Q_{j,2} + U_a} \]
CSTR’s in Series

1. Temperature from Jacket 1 (K) vs. Temperature from Jacket 2 (K)
2. Reactor 1 Temperature (K) vs. Reactor 2 Temperature (K)
3. Feed Flow (m³/s) vs. Makeup Flow (m³/s)
4. Jacket Flow 1 (m³/s) vs. Jacket Flow 2 (m³/s)
CSTR’s in Series

Disturbances:

\[
W = \begin{bmatrix}
T_{in} \\
C_{A,in} \\
U_a
\end{bmatrix}
\]

\[
\Sigma_w = \begin{bmatrix}
3^2 & 0 & 0 \\
0 & 1^2 & 0 \\
0 & 0 & 0.021^2
\end{bmatrix}
\]

(adapted from de Hennin & Perkins, 1991)
CSTR’s in Series

![Graphs showing temperature, flow, and fault conditions for CSTRs in series.](image)
CSTR’s in Series

Disturbances:

\[
W = \begin{bmatrix} T_{in} \\ C_{A,in} \\ U_a \end{bmatrix}
\]

\[
\Sigma_w = \begin{bmatrix} 3^2 & 0 & 0 \\ 0 & 1^2 & 0 \\ 0 & 0 & 0.021^2 \end{bmatrix}
\]

Assume \( U_a \) is not white noise, but highly correlated colored noise (slowly varying).
CSTR’s in Series

- **Temperature from Jacket 2 (K)** vs **Temperature from Jacket 1 (K)**
  - No HEX Fault
  - HEX Fault
  - Highly Correlated Noise
  - White Noise

- **Reactor 2 Temperature (K)** vs **Reactor 1 Temperature (K)**
  - Plots showing different temperature distributions and fault conditions.

- **Feed Flow (m^3/s)** vs **Makeup Flow (m^3/s)**
  - Demonstrates flow rate correlation.

- **Jacket Flow 1 (m^3/s)** vs **Jacket Flow 2 (m^3/s)**
  - Graphs showing flow rates in the jacket system.
## CSTR Profit

<table>
<thead>
<tr>
<th>OSSOP</th>
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<th>Diff from OSSOP ($/day)</th>
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<tbody>
<tr>
<td>$U_a = 0$</td>
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<td>-</td>
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# CSTR Profit

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<td>$U_a$ – slow varying</td>
<td>$2,115</td>
<td>- $371</td>
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Outline

• Motivating Example
• Controller Tuning
• Economic Based Tuning
• Robust Formulation
Uncertainty Characterization

Plant in the polytopic set:

\[ \Omega = \text{co}\{A_j, B_j, G_j, D_{xj}, D_{uj} \mid j = 0 \ldots N_\Omega \} \]
Uncertainty and Stability

Plant in the polytopic set:

\[ \Omega = \text{co}\{A_j, B_j, G_j, D_{xj}, D_{uj} \mid j = 0 \ldots N_\Omega \} \]

Sufficient condition (quadratic stability):

\[ \exists \ P > 0 \ \text{s.t.} \ A_j P + P A_j^T < 0 \quad j = 0 \ldots N_\Omega \]
Profit Control
(Simultaneous BOP and Controller Selection)

\[
\begin{align*}
\min_{s',m',q',\xi_i,X,Y} & \left\{ g_q q' \right\} \quad \text{s.t.} \quad 0 = A s' + B m' \\
q_i' = \phi_i(D_x s' + D_u m') & \quad q_i^{\min} \leq q_i' \leq q_i^{\max} \\
\xi_i < (q_i^{\max} - q_i')^2 & \quad \xi_i < (q_i' - q_i^{\min})^2 \\
(AX + BY) + (AX + BL)^T + G \Sigma_w G^T & < 0 \\
\begin{bmatrix}
\xi_i & \phi_i(D_x X + D_u Y) \\
(D_x X + D_u Y)^T \phi_i^T & X
\end{bmatrix} & > 0
\end{align*}
\]
Profit Control
with Robust Performance Conditions

\[
\min \left\{ g_q q' \right\} \quad \text{s.t.} \quad 0 = A_0 s' + B_0 m'
\]

\[
q_i' = \phi_i (D_{x0} s' + D_{u0} m') \quad q_i^{\min} \leq q_i' \leq q_i^{\max} \quad i = 1 \ldots n_z
\]

\[
\xi_{ij} < (q_i^{\max} - q_i')^2 \quad \xi_{ij} < (q_i' - q_i^{\min})^2 \quad i = 1 \ldots n_z \quad j = 1 \ldots N_{\Omega}
\]

\[
\begin{bmatrix}
(A_j X + B_j Y) + (A_j X + B_j Y)^T & G_j \\
G_j^T & -\Sigma_w^{-1}
\end{bmatrix} < 0 \quad j = 1 \ldots N_{\Omega}
\]

\[
\begin{bmatrix}
\xi_{ij} & \phi_i (D_{xj} X + D_{uj} Y) \\
(D_{xj} X + D_{uj} Y)^T \phi_i^T & X
\end{bmatrix} > 0 \quad i = 1 \ldots n_z \quad j = 1 \ldots N_{\Omega}
\]
Profit Control with Robust Performance Conditions

EDOR’s due to uncertain plant model

Robust BOP

OSSOP
Specific scenario:

Delivery Delay = 5 days

Demand Variance = $10^2$

Yield Rate $\in [0.8, 1.0]$
Inventory Control with Yield Uncertainty

\[ \text{Yield} = 1.0 \]
\[ \text{Yield} = 0.8 \]

Robust

Std. Dev. Inventory

Yield vs. Std. Dev. Inventory

Yield = 0.8

Yield = 1.0

Robust
2-Echelon Inventory Control
(Delivery Delay Uncertainty)

Starts

Delivery delay = 3

Inventory 1

IC

Inventory Set-point 1

Inventory Set-point 2

Demand

Delivery delay ∈ [4, 5]

Inventory 2

Inventory Set
2-Echelon Inventory Control
(Delivery Delay Uncertainty)

Starts

Delivery delay = 3

Inventory 1

Starts

Delivery delay ∈ [4, 5]

Inventory 2

IC

Inventory Set-point 1

Inventory Set-point 2

Demand
Inventory Control with Delivery Delay Uncertainty

![Graph showing the relationship between Alpha and the standard deviation of inventory tank 2 for robust and non-robust cases. The graph compares Alpha = 0 and Alpha = 1.]
Inventory Control with Delivery Delay Uncertainty

![Graph showing inventory control with delivery delay uncertainty]

- **Robust**
- **Alpha = 0**
- **Alpha = 1**

- **Centralized A**
- **Alpha = 0**
- **Alpha = 1**
Profit Control
(Peak-to-Peak Formulation)

\[
\begin{align*}
\text{min } & \{ g \_q \_q' \} \quad \text{s.t.} \quad 0 = A \_s' + B \_m' \\
& q_i' = \phi_i (D_x s' + D_u m') \quad q_i^{\text{min}} \leq q_i' \leq q_i^{\text{max}} \\
& \xi_i < (q_i^{\text{max}} - q_i')^2 \quad \xi_i < (q_i' - q_i^{\text{min}})^2 \\
\begin{bmatrix}
(AX + BY) + (AX + BY)^T + \alpha X & G \\
G^T & -\alpha I
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\end{align*}
\]
Profit Control
(Peak-to-Peak Formulation)

\[
\min_{s',m',q', \xi_i, X, Y, \alpha} \left\{ g_q q' \right\} \quad \text{s.t.} \quad 0 = As' + Bm'
\]

\[
q_i' = \phi_i(D_x s' + D_u m') \quad q_i^\text{min} \leq q_i' \leq q_i^\text{max}
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\xi_i < (q_i^\text{max} - q_i')^2 \quad \xi_i < (q_i' - q_i^\text{min})^2
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\[
\begin{bmatrix}
(AX + BY) + (AX + BY)^T & + \alpha X & G \\
G^T & -\alpha I
\end{bmatrix} < 0
\]

\[
\begin{bmatrix}
\xi_i \\
(D_x X + D_u Y)^T & \phi_i(D_x X + D_u Y)
\end{bmatrix} > 0
\]

\[
X
\]
Conclusions

• Relationship between control system performance and plant profit quantified.
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• Relationship between control system performance and plant profit quantified.
• Enables profit guided control system design.
• Globally optimal search algorithm as well as heuristic scheme proposed.
• Applicable to a broad set of applications from a variety of disciplines.
• Extendable it robust and peak-to-peak framework, but conservatism a concern.
Acknowledgements

• **Students and Collaborators:**
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  - Jui-Kun Peng
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