

**Directions:** Answer all questions (15 pts. each) and show *all* work. **Box** your final answer.

1. Solve the IVP:  $y \frac{dy}{dx} = e^{y^2} \sin x$ ,  $y(\pi) = 0$ .

2. Solve the IVP for  $y(t)$ :

$$y'' + 2y' + 17y = 0, \quad y(0) = 0, \quad y'(0) = 2$$

3. Consider the undamped, forced motion described by:

$$y'' + y = -4 \sin t$$

(a) Find the general solution.

(b) What is the behavior of the solution as  $t \rightarrow \infty$ . What phenomenon is occurring?

4. Consider:  $y'' + 4y' + 4y = 10 + 7e^{-2t} + t \cos 2t$

(a) Find two linearly independent solutions to the corresponding homogeneous equation.

(b) Find the *form* of the particular solution using the method of undetermined coefficients. (DO NOT EVALUATE THE UNKNOWN CONSTANTS).

5. An 800L tank initially contains only fresh water. At time  $t = 0$ , a polluted solution begins to enter the tank from two separate input pipes. Polluted solution enters from the first input pipe with concentration 2 Kg/L at the rate of 3 L/min. The concentration of pollutant entering the tank from the second input pipe varies in time and is given by  $e^{-t/100}$  Kg/L at the rate of 5 L/min. The well-stirred solution is removed from the tank at the rate of 8 L/min. *Solve* for  $A(t)$  = the amount of pollutant in the tank at time  $t$ .

6. Use variation of parameters to find a particular solution to:  $y'' + 2y' = 6e^{-2x}$ .

7. Determine whether the statement is true or false. If true, explain why. If it is false, explain why or give an example that disproves the statement.

(a) To use variation of parameters to solve a linear differential equation, we must first find the complementary solutions. **True False**

(b) Undetermined coefficients can be used find a particular solution to  $x^2y'' + xy' + 4y = \sin x$ . **True False**

(c) The functions  $e^{-x}$  and  $7e^{-x}$  form a fundamental set of solutions to  $y'' + 2y' + y = 0$ . **True False**

(d) All first order ODE's are separable. **True False**

(e) The ODE:  $y' = \frac{x - xy}{x^2 + 1}$  is linear. **True False**