Math 252 - Sample Exam II – C. Tier 2008

1. Compute

   (a) $\mathcal{L}\{te^{-2t}\mathcal{U}(t-7)\}$

   (b) $\mathcal{L}^{-1}\left\{\dfrac{se^{-2s}}{s^2+16}\right\}$

   (c) $\mathcal{L}\{t\cos 3t\}$

2. Solve using Laplace transforms: $y' - 5y = te^{-t}, \;\; y(0) = 2.$

3. Solve the IVP for $x(t)$ (you need not find $y(t)$):

$$\begin{aligned} x' &= 3x - 2y + \delta(t), \;\; x(0) = 0 \\ y' &= 2x - y, \;\; y(0) = 0. \end{aligned}$$

4. Find the first two nonzero terms in the two linearly independent power series solutions of:

$$y'' + 4xy' - 2y = 0.$$

5. Solve: $y' + y = \begin{cases} 0, & 0 \le t < 1, \\ 3, & t \ge 1. \end{cases} \qquad y(0) = 4.$

6. Find $y(t)$ using Laplace transforms:

$$y'' + 9y = g(t), \;\; y(0) = 0, \;\; y'(0) = 1.$$

   The function $g(t)$ has Laplace transform $G(s)$ and your solution should be a function of t.

7. Determine whether the statement is true or false. If true, explain why. If it is false, explain why or give an example that disproves the statement.

   (a) $\mathcal{L}\{f(t) * g(t)\} = \mathcal{L}\{f(t)\} * \mathcal{L}\{g(t)\}$ **True** **False**

   (b) If $\lim_{s\to\infty} F(s) \neq 0$ then the inverse is a continuous function **True** **False**

   (c) $x = 0$ is a regular singular point of $x^2y'' + xy' + (x^2 - 9)y = 0.$ **True** **False**

   (d) Every function has a Laplace transform. **True** **False**

   (e) $\displaystyle\int_0^\infty \delta(t+5)dt = 1.$ **True** **False**

Answers

1. (a) $e^{-7(s+2)}\left[\frac{1}{(s+2)^2}+\frac{7}{s+2}\right]$

   (b) $\mathcal{U}(t-2)\cos 4(t-2)$

   (c) $\frac{s^2-9}{(s^2+9)^2}$

2. $y(t) = 2e^{5t} - \frac{1}{6}te^{-t} - \frac{1}{36}e^{-t} + \frac{1}{36}e^{5t}$

3. $x(t) = e^t + 2te^t, \quad y(t) = 2te^t$

4. $y = c_0(1 + x^2 + \dots) + c_1(x - \frac{x^3}{3} + \dots)$

5. $y(t) = 4e^{-t} + 3\left[1 - e^{-(t-1)}\right]\mathcal{U}(t-1)$

6. $y(t) = \frac{1}{3}\sin 3t + \frac{1}{3}\int_0^t \sin(3\tau)g(t-\tau)d\tau$

7. F. F, T, F, F