Physics 221 Equation Sheets

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Porter Johnson

"There is no royal road to geometry."

Euclid – 325 BCE

"Physics can only be learned by thinking, writing, and worrying."

David Atkinson and Porter Johnson – 2002 CE

Chapter 15: Oscillations

SHM: $x = A\cos(\omega t + \phi)$; ϕ (phase)
ω (angular frequency); A (amplitude)
Period: $T = 2\pi/\omega = 1/f$.
Frequency $f = 1/T = \omega/(2\pi)$
Velocity: $v = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi)$
Acceleration: $a = \frac{dv}{dt}$
$a = -\omega^2 A\cos(\omega t + \phi) = -\omega^2 x$
Ideal Spring: $m\ddot{x} = F = -kx$
Energy $E = 1/2mv^2 + 1/2kx^2$
Frequency $\omega^2 = k/m$
Simple Pendulum: $I\,\ddot{\theta} = m\ell^2\,\ddot{\theta} = \tau$
$= -mg\ell\sin\theta \approx -mg\ell\theta$
$\ddot{\theta} + g/\ell\,\theta = 0$; $\omega^2 = g/\ell$
Damped Harmonic Motion:
$m\ddot{x} + b\dot{x} + kx = 0$
$x = A\exp[-bt/(2m)]\sin(\omega' t + \phi)$;
$\omega'^2 = k/m - b^2/(4m^2)$.
Underdamped $\omega'^2 > 0$;
Overdamped $\omega'^2 < 0$;
Critically damped $\omega'^2 = 0$.
Resonant frequency: $\omega^2 = k/m$

Chapter 16: Waves I

transverse or longitudinal
$y(x,t) = A\sin(kx - \omega t + \phi)$
A: (amplitude); k (wave number);
ω (angular frequency); ϕ (phase).
$k = 2\pi/\lambda$; $\omega = 2\pi f$; $\omega/k = \lambda f = v$
(v = wave velocity).
Stretched string: $v = \sqrt{T/\mu}$.
T: (tension); μ (mass per unit length).
Superposition of waves:
$y(x,t) = y_1(x,t) + y_2(x,t)$
Interference of waves:
$A\sin(kx - \omega t) + A\sin(kx - \omega t + \phi)$
$= 2A\cos(\phi/2)\cos(kx - \omega t + \phi/2)$
$\phi = 0 \bmod 2\pi$: (constructive int)
$\phi = \pi \bmod 2\pi$: (destructive int)
Standing waves:$2A\sin kx\cos\omega t =$
$A\sin(kx - \omega t) + A\sin(kx + \omega t)$
$kL = n\pi$ (L is string length).
Number of internal nodes: $n - 1$.
$y(x,t) = B\sin(n\pi x/L)\sin(\omega t + \phi)$
$k = n\pi/L$; $\lambda = 2\pi/k = 2L/n$
$f = v/\lambda = nv/(2L)$

Chapter 17: Waves II

Speed of Sound: (longitudinal) $v = \sqrt{B/\rho}$
B: (bulk modulus); ρ: (density)
343 meters/sec at $20^\circ C$;
Speed increases with temperature.
Air displacement and pressure differential 90° out of phase,.
Interference phase $\phi = 2\pi(\Delta L)/\lambda$
ΔL: (path difference); λ (wave length).
Constructive Int: $(\Delta L)/\lambda = 0, 1, 2, \cdots$
Destructive Int: $(\Delta L)/\lambda = 1/2, 3/2, 5/2, \cdots$
I: Sound intensity: Power per unit area.
Threshold $I_{th} = 10^{-12}$ Watts/meter^2.
deciBels (dB): $\beta = 10\log_{10} I/I_{th}$
Standing waves in a Pipe of length L
Both ends open: $n\lambda = 2L$.
$f = v/\lambda = nv/(2L)$
One end open: $(n + 1/2)\lambda = 2L$.
$f = v/\lambda = (2n+1)v/(4L)$
Beats: $f_{beat} = |f_1 - f_2|$
Doppler Effect:
$f' = f \times (v \pm v_D)/(v \mp v_S)$
top sign: motion toward
bottom sign: motion away
v_D: velocity of detector
v_S: velocity of source
Shock waves, Mach cone:
$\sin\theta = v_{sound}/v_{projectile}$

Chapter 21: Electric Charge

$\leftarrow \cdot Q \quad Q\cdot \rightarrow$ (Like charges Repel)
$Q\cdot \rightarrow \quad \leftarrow \cdot -Q$ (Unlike charges attract)
Coulomb's Law: $F = (kq_1q_2)/r_{12}^2$
$k = 9 \times 10^9 (N\cdot m^2/C^2) = 1/(4\pi\varepsilon_0)$
$\varepsilon_0 = 8.85 \times 10^{-12} C^2/(N\cdot m^2)$
Charge: CONSERVED – QUANTIZED

Electron charge: $e_0 = 1.6 \times 10^{-19}$ C
Conductor: No free charges inside.
Uniformly charged spherical shell
(radius R; charge Q)
test charge q at distance $\vec{r}$ from center:
outside: $\vec{F} = (kqQ\hat{r})/r^2$. inside: $\vec{F} = \vec{0}$

Chapter 22: Electric Field

test charge q_0; $\vec{E} = \vec{F}/q_0 = kQ\hat{r}/r^2$
Electric field lines:
away from (+) charges: $\leftarrow (+) \rightarrow$
and toward (-) charges: $\rightarrow (-) \leftarrow$
Two opposite charges: lines from - to +
Two like charges: lines away from each
Dipole: $(-q) \rightarrow (+q)$: $\vec{p}$ from - to +
$p = qd$
along dipole axis (z): $E_z = (2kp)/z^3$
Continuous distribution of charge:
$\vec{E} = \int d\vec{E} = k \int dq\hat{r}/r^2$
Point charge in uniform electric field:
$\vec{F} = m\vec{a} = q\vec{E}$
Dipole in uniform electric field:
Torque: $\tau = \vec{p} \times \vec{E}$
Potential Energy: $U = -\vec{p} \cdot \vec{E}$

Chapter 23: Gauss's Law

$\varepsilon_0 \Phi_E = \varepsilon_0 \oint \vec{E} \cdot \vec{dS} = Q_{enc}$
$\varepsilon_0 = 8.85 \times 10^{-12}$ C^2 /(N m^2)
Any excess charge sits on outer surface of conductor; $\vec{E} = \vec{0}$ inside.
At surface, $E = \sigma/\varepsilon_0$ (outward)
$\sigma =$ charge per unit area on surface.
Infinite line charge: $\lambda =$ charge/length
small $E = \lambda/(2\pi\varepsilon_0 r)$ away from line.
Non-conducting sheet on either side:
$E = \sigma/(2\varepsilon_0)$
Spherical shell. electric field radially out
$E = 0$ inside; $E = Q/(4\pi\varepsilon_0 r^2)$ outside.
Spherical charge distribution:
$E = Q_{enc}/(4\pi\varepsilon_0 r^2)$ radially outward.
Q_{enc}: charge inside sphere of radius r.
Uniformly charged sphere:
$E = \rho r/(3\varepsilon_0)$ inside.
$E = kQ_{tot}/r^2$ outside: $Q_{tot} = 4\pi\rho R^3/3$.
Gauss's Law + symmetry:
Planar, cylindrical, or spherical

Chapter 24: Electric Potential

Work done by electric field $dW_E = -\delta U$.
$\Delta V = V_f - V_i = -W_E/Q$.
Electric potential: 1 Volt = 1 Joule/Coul
Equipotential surface (constant V):
Perpendicular to electric field lines.
$dV = -\vec{E} \cdot \vec{d\ell}$
$\Delta V = -\int_i^f \vec{E} \cdot \vec{d\ell}$
Potential of point charge: $V = kQ/r$.
Combination of charges (superposition)
$V = k \sum_i Q_i/r_i$.
Continuous distribution: $V = k \int dQ/r$.
$V(x,y,z) \rightarrow (E_x, E_y, E_z)$:
$E_x = -\partial V/\partial x; E_y = -\partial V/\partial y; E_z = -\partial V/\partial z$.
Electric potential energy of pair of charges:
$\mathcal{E} = kq_1q_2/r_{12}$
(superposition of pairs to get total)
Charged conductor: Potential constant inside.

Chapter 25: Capacitance

Two conductors; charges $+Q$ and $-Q$, respectively. Potential between conductors: ΔV: $Q = C\Delta V$.
C: capacitance (1 Farad = 1 Volt/Coul).
Parallel plate capacitor: $C = \varepsilon_0 A/d$.
plate area A; plate separation$d$.
Cylindrical capacitor (length ℓ; radii a, b):
$C = 2\pi\varepsilon_0 \ell/\ln(b/a)$.
Spherical capacitor (radii a, b):
$C = 4\pi\varepsilon_0\, ab/(b-a)$.
Capacitors in parallel:
$C_{tot} = C_1 + C_2 + C_3 + \cdots$
Capacitors in series:
$1/C_{tot} = 1/C_1 + 1/C_2 + 1/C_3 + \cdots$
Potential energy stored in capacitor:
$U = 1/2CV^2 = Q^2/(2C)$

Energy density stored in electric field: $u = 1/2\epsilon_0 E^2$.
Dielectric-filled capacitor:
$C \to \kappa C_0 = \kappa\epsilon_0 A/d$,
κ: dielectric constant.
Gauss's law for dielectrics:
$\epsilon_0 \oint \kappa\vec{E}\cdot\vec{dS} = Q_{enc}$

Chapter 26: Current and Resistance

Charge ΔQ passes by in time Δt:
$I = \Delta Q/\Delta t$. 1 Amp = 1 Coul/sec.
Current density $\vec{J}$: current per unit area
$I = \int \vec{J}\cdot d\vec{A}$
$\vec{J} = ne_0\vec{v}_d$; charge: e_0: drift speed: v_d.
n: num charge carriers per unit vol.
Electical resistance:
Current I flowing through circuit;
Voltage drop ΔV along circuit.
$R = \Delta V/I$. $1(\Omega)$ Ohm = 1 Volt/ Amp.
Resistivity ρ: $\vec{E} = \rho\vec{J}$.
Wire cross-section area A, length ℓ:
$R = \rho\ell/A$.
Metals: resistivity increases with temperature
$\rho = \rho_0(1+\alpha\Delta T)$.
$\alpha = (\Delta\rho/\Delta T)/\rho_0$
Ohm's Law:: $V = RI$.
Resistivity of metal (electron gas):
$\rho = m/(ne_0^2\tau)$: Collision time: τ.
Power dissipated inside resistor:
$P = I\Delta V = I^2R = \Delta V^2/R$.
Semiconductor ρ decreases with T.
Superconductor (low temp) $\rho = 0$.

Chapter 27: Circuits

Work done by EMF source (battery)
Chemical energy $\to$ Electrical energy
$dW = \mathcal{E}dQ$
Kirchhoff's Laws
Loop rule: sum of voltage changes around closed loop = 0.
Junction rule: net current into junction = 0.
Single loop circuit: Load resistor R.
Battery $\mathcal{E}$ with internal resistance r;
$I = \mathcal{E}/(R+r)$
Power provided by battery: $P = I\mathcal{E}$.
Thermal power in resistor $P = I^2R$.
Resistors in series:
$R_{tot} = R_1 + R_2 + R_3 + \cdots$
Resistors in parallel:
$1/R_{tot} = 1/R_1 + 1/R_2 + 1/R_3 + \cdots$
RC Circuit
Cbarging $\mathcal{E}, R, C$ in series:
$\mathcal{E} = RdQ/dt + Q/C$
$Q(t) = \mathcal{E}C(1-e^{-t/RC})$
$I = dQ/dt = \mathcal{E}/R\, e^{-t/RC}$
Discharging: R, C in series
$0 = RdQ/dt + Q/C$
$Q(t) = Q_0\, e^{-t/RC}$
$I(t) = dQ/dt = -Q_0/RCe^{-t/RC}$

Chapter 28: Magnetic Fields

Lorentz Force: $\vec{F} = q\vec{v}\times\vec{B}$
(1 Tesla = 1 N / (A m))
Hall effect: (determine sign of charge carriers)
$-------------------$
$\to I \cdots \to \vec{B}$ (out of paper) $\vec{F}$ (down)
$-------------------$
$J = I/A = ne_0v_d$.
$E = v_dB\ \Delta V = wE$.
Charged particle - uniform magnetic field:
planar motion. $qvB = mv^2/R$;
$R = mv/(qB)$.
$\omega = qB/m,\ f = \omega/(2\pi) = 1/T$
Three dimensional motion: helix
Cyclotron: Dees: voltage change produces acceleration in uniform magnetic field.
Current-carrying wire:
straight: $\vec{F} = I\vec{\ell}\times\vec{B}$
curved: $d\vec{F} = Id\vec{\ell}\times\vec{B}$
Magnetic moment of wire:
(Current I, area A, N turns)

M = N I A (right hand rule)
Torque: $\vec{\tau} = \vec{M} \times \vec{B}$
Potential Energy: $U = -\vec{M} \cdot \vec{B}$

Chapter 29: Magnetic Fields Due to Currents

Biot-Savart Law:
$d\vec{B} = \mu_0 I/(4\pi)\ d\vec{\ell} \times \vec{r}/r^3$
$\mu_0 = 4\pi \times 10^{-7} Tm/A$
Long Straight wire - current I:
$B_t = \mu_0 I/(2\pi r)$ right hand rule.
Circular arc center: $B = \mu_0 I/(2r)\theta/(2\pi)$
Force between wires: Like currents attract; unlike currents repel:
$F = \mu_0 I_1 I_2/(2\pi d)$
Ampère's Law: $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$
Ideal Solenoid: $B = \mu_0 NI/L$
N turns; current I; length L.

Chapter 30: Induction and Inductance

Magnetic Flux: $\Phi_B = \int \vec{B} \cdot d\vec{S}$
Faraday's Law:
(closed loop)-(open surface)
$\mathcal{E} = -d\Phi_B/dt = -d/dt(\int \vec{B} \cdot d\vec{S})$
Self-Inductance (N turns, current I):
$N\Phi_B = LI$; (1 Henry = 1 T m^2/A)
Long Solenoid (length ℓ, area A, N turns):
$L = \mu_0 N^2 A/\ell$
LR Circuit + battery: $\mathcal{E} = LdI/dt + RI$
$I(t) = \mathcal{E}/R \cdot (1 - e^{-Rt/L})$
No Battery: $LdI/dt + RI = 0$
$I(t) = I_0 e^{-Rt/L}$
Magnetic energy in inductor: $U_B = 1/2 \cdot LI^2$.
Magnetic energy density: $u = B^2/(2\mu_0)$
Mutual Inductance of two circuits: M
$\mathcal{E}_2 = -MdI_1/dt$; $\mathcal{E}_1 = -MdI_2/dt$

Chapter 31: Electromagnetic Oscillations

LC Circuit: $LdI/dt + Q/C = 0$;
$I = dQ/dt$.
$Q(t) = Q_0 \cos(\omega t + \phi)$.
$\omega^2 = 1/(LC)$
LRC Circuit: (Damped Oscillations)
$Ld^2Q/dt^2 + RdQ/dt + Q/C = 0$
$Q(t) = Ae^{-Rt/(2L)} \cos(\omega' t + \phi)$
$\omega'^2 = 1/(LC) - R^2/(4L^2)$
Forced Oscillations (steady state):
$Ld^2Q/dt^2 + RdQ/dt + Q/C = \mathcal{E}_0 \sin(\omega t)$
$I = \mathcal{E}_0/|Z| \sin(\omega t + \phi)$
$Z = R + \imath(\omega L - 1/\omega C))$
$\omega^2 < 1/(LC); \phi < 0$; I lags V
$\omega^2 > 1/(LC); \phi < 0$; I leads V
$\omega^2 = 1/(LC); \phi = 0$; resonance

Chapter 32: Magnetism; Maxwell's Equations

Gauss's Law for Magnetism:
$\Phi_B = \oint \vec{B} \cdot d\vec{S} = 0$.
Magnetic Dipole: $\vec{M}$ in field $\vec{B}$:
$U = -\vec{M} \cdot \vec{B}$; $\vec{\tau} = \vec{M} \times \vec{B}$.
Orbital Magnetic Moment $\vec{M}$;
angular momentum $\vec{L}$
$\vec{M}_L = q/(2m)\vec{L}$
Spin magnetic moment: $\vec{M}_S = q/(2m)\vec{S}$
Diamagnetism: $\vec{B}_{induced}$ opposite $\vec{B}_{ext}$.
Unpaired atomic spins are possible.
Paramagnetism: $\vec{B}_{induced} || \vec{B}_{ext}$.
Curie-Weiss Law: $M = CB_{ext}/T$
Ferromagnetism:
(Permanent magnetic moment in absence of field).
Displacement Current I_d
(extension of Ampère's Law)
$\oint \vec{B} \cdot d\vec{\ell} = \mu_0(I + I_d)$
$I_d = \varepsilon_0 d\Phi_E/dt = \varepsilon_0 d/dt(\int \vec{E} \cdot d\vec{S})$
Maxwell's Equations
Gauss: $\varepsilon_0 \oint \vec{E} \cdot d\vec{S} = Q_{enc}$
Magnetic Gauss: $\oint \vec{B} \cdot d\vec{S} = 0$
Faraday: $\oint \vec{E} \cdot d\vec{\ell} = -d/dt(\int \vec{B} \cdot d\vec{S})$
Ampère + Maxwell:
$\oint \vec{B} \cdot d\vec{\ell} = \mu_0[I_{enc} + \varepsilon_0 d/dt(\int \vec{E} \cdot d\vec{S}]$

Chapter 33: Electromagnetic Waves

$\vec{E} = \hat{i}E_0 \sin(kz - \omega t)$

$\vec{B} = \hat{j} B_0 \sin(kz - \omega t)$
$c = E_0/B_0 = \omega/k$
$c = 1/\sqrt{(\mu_0 \varepsilon_0)} = 3 \times 10^8$ m/sec.
Poynting Vector: (Watts/m^2)
energy flux per unit area per unit time.
$\vec{S} = \vec{E} \times \vec{B}/\mu_0$
Radiation pressure $P = F/A = |\vec{S}|/c$.
Polarization defined as direction of $\vec{E}$
Polarizer: light initially unpolarized.
$I_f = 1/2 I_0$
Malus Law: $I = I_0 \cos^2 \theta$
$\vec{E}$ makes angle θ with polarizer.
Reflection: $\theta_i = \theta_r$
angle of incidence = angle of reflection.
Refraction:
Snell's Law medium 1 - medium 2.
$n_1 \sin \theta_1 = n_2 \sin \theta_2$
n: index of refraction.
θ angle with surface normal.
Total internal feflection: $n_2 < n_1$:
$n_1 \sin \theta_1 > n_2$.
Polarization by reflection:
Brewster's angle: reflected ray polarized.
($\vec{E}$ perpendicular to plane of reflection.)
$\theta_1 + \theta_2 = 90^o$.