Physics 123 Equation Sheets

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Porter Johnson

"There is no royal road to geometry."

Euclid – 325 BCE

"Physics can only be learned by thinking, writing, and worrying."

David Atkinson and Porter Johnson – 2002 CE

Chapter 1: Measurement
SI Units: *kg*, *m*, *sec*
Unit Conversion; Dimensional consistency
Chapter 2: One Dimensional Motion
Displacement: $\Delta x = x_2 - x_1$
Average Velocity: $\bar{v} = \frac{\Delta x}{\Delta t}$
Instantaneous Velocity $v = \frac{dx}{dt}$
Average Acceleration $\bar{a} = \frac{\Delta v}{\Delta t}$
Instanteous Acceleration $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$
Constant Acceleration
$v = v_0 + at$
$x = x_0 + v_0 t + at^2/2 = x_0 + (v + v_0)t/2$
$v^2 = v_0^2 + 2g(x - x_0)$
Chapter 3: Vectors
Magnitude; Direction;
Components: a_x; Unit vectors $\hat{i}$
$\vec{a} = a_x\hat{i} + a_y\hat{j}$
$a_x = a\cos\theta$; $a_y = a\sin\theta$
$a^2 = a_x^2 + a_y^2$; $\tan\theta = a_y/a_x$
Addition of vectors: $\vec{b} = b_x\hat{i} + b_y\hat{j}$
$\vec{a} + \vec{b} = (a_x + b_x)\hat{i} + (a_y + b_y)\hat{j}$
$\vec{c} = \vec{a} + \vec{b}$ triangle
Multiplication by Scalar Parameter λ:
$\lambda\vec{a} = \lambda a_x\hat{i} + \lambda a_y\hat{j}$
ϕ – angle between $\vec{a}$ and $\vec{B}$
Scalar Product: $\vec{a}\cdot\vec{b} = |\vec{a}|\ |\vec{b}|\ \cos\phi$
$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z$
Vector Product: right hand rule
$|\vec{a}\times\vec{b}| = |\vec{a}|\ |\vec{b}|\ \sin\phi$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

Chapter 4: Three-Dimensional Motion
$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$
$\vec{v} = d\vec{r}/dt = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$
$\vec{a} = d\vec{v}/dt = d^2\vec{r}/dt^2 = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$
Average Velocity: $\vec{v}_{avg} = \Delta\vec{r}/\Delta t$
Average Acceleration: $\vec{a}_{avg} = \Delta\vec{v}/\Delta t$
Projectile Motion: uniform acc in $-y$-dir
$g = 9.8\ m/sec^2$, downward
$x = v_0 t\cos\theta$; $v_x = v_0\cos\theta$
$y = v_0 t\sin\theta - 1/2gt^2$; $v_y = v_0\sin\theta - gt$
Trajectory: $y = x\tan\theta - gx^2/(2v_0^2\cos^2\theta)$
Height: $h = v_0^2\sin^2\theta/(2g)$
Range: $R = v_0^2\sin 2\theta/g$
Uniform Circular Motion
r: radius of circular arc
Centripetal Acceleration: $a_c = v^2/r$ inward
Period: $T = 2\pi r/v$
Relative Motion: add velocity vectors
$\vec{v}_{ac} = \vec{v}_{ab} + \vec{v}_{bc}$
Chapter 5: Forces and Motion I
$\vec{F}$ Force (Newtons): Vector
m Mass (kilograms) scalar
Newton's Laws I, II, III
I: An isolated body in an inertial frame remains at rest or in uniform motion
II; $\vec{F} = m\vec{a}$
III. Action-reaction force: $\vec{F}_{21} = -\vec{F}_{12}$
Weight $\vec{W} = m\vec{g}$
Normal Force: $\vec{N}$
perpendicular to plane of contact
Friction force: $\vec{f}$
Parallel to plane of contact
Tension $\vec{T}$ in flexible string: along string
Chapter 6: Force and Motion II
I. Coefficient of kinetic friction μ_k
Kinetic friction opposes motion between moving surfaces
$f_k = \mu_k N$
II. Static friction: Whatever is necessary to preserve equilibrium
Coefficient of Static Friction μ_s
(bonding force) $|f_s| \leq \mu_s N$.
Uniform Circular Motion: $\vec{F} = m\vec{a}_c$
$a_c = v^2/R$; $F = mv^2/R$ inward
Chapter 7: Kinetic Energy and Work
Kinetic Energy of particle: $K = mv^2/2$

Work done by constant force $\vec{F}$ through displacement $\vec{d}$: $W = \vec{F} \cdot \vec{d}$
Gravity: $W = m\vec{g}$ downward
Work done by gravity $W_g = -mg(h_f - h_i)$
Variable force: $W_{12} = \int_1^2 \vec{F} \cdot \vec{dx}$
Ideal Spring: $\vec{F}_s = -k\vec{x}$ (Hooke's Law)
$W_s = -\frac{1}{2}k(x_f^2 - x_1^2)$
Power: (instantaneous; average)
$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$

Chapter 8: Potential Energy; Energy Conservation

Conservative Field: Work is path-independent
$W = \int_i^f \vec{F} \cdot \vec{dr} = -\Delta U = -(U_f - U_i)$
U_i: potential energy at point i
Gravity: $\Delta U = mg(h_f - h_i)$
$U = mgh$ (plus a constant)
Spring: $\Delta U = \frac{1}{2}k(x_f^2 - x_i^2)$
$U = \frac{1}{2}kx^2$ (plus a constant)
Mechanical Energy Conservation:
$\mathcal{E}_m = K + U$
Otherwise, energy is converted into "heat"
External force: $F(x) = -\frac{dU}{dx}$
$W_{ext} = \Delta\mathcal{E}_{ext} = \Delta K + \Delta U$
Power: $P = \frac{\Delta\mathcal{E}_{ext}}{\Delta t} \rightarrow \frac{d\mathcal{E}_{ext}}{dt}$

Chapter 9: Systems of Particles

Center of mass: $M = \sum_i m_i$
$X_{cm} = \sum_i m_i x_i / M \rightarrow (\int x dm)/M$
$\vec{R}_{cm} = \sum_i m_i \vec{r}_i / M \rightarrow (\int \vec{r} dm)/M$
$\vec{v}_{cm} = \frac{d\vec{R}_{cm}}{dt}$; $\vec{a}_{cm} = \frac{d\vec{v}_{cm}}{dt}$
Newton's Second Law: system of particles
$\vec{F}_{ext} = \vec{F}_{ext} = M\vec{a}_{cm}$
Momentum: particle $\vec{p} = m\vec{v}$; $\vec{F} = \frac{d\vec{p}}{dt}$
System: $\vec{P} = M\vec{v}_{cm}$; $\vec{F}_{ext} = \frac{d\vec{P}}{dt}$
Conservation of momentum: $\vec{P}_i = \vec{P}_f$
(closed, isolated system)
Rockets: variable mass systems
$\Delta\vec{P}_{tot} = \vec{F}_{ext}\delta t$
No external force: $\Delta\vec{P}_{tot} = 0$
Thrust $= (-\frac{\Delta m}{\Delta t})\vec{v}_{exhaust} = m\frac{\Delta\vec{v}}{\Delta t}$
$\Delta\vec{v} = \vec{v}_f - \vec{v}_i = \vec{v}_{exhaust}\ \ln\frac{m_i}{m_f}$
Impulse: $\vec{I} = \int_1^2 \vec{F} dt$
Impulse-momentum theorem:
$\vec{I} = \Delta\vec{P} = \vec{P}_f - \vec{P}_i$
Collisions: $\Delta\vec{P} = 0$ during collision
(internal forces dominate external forces)
Two body linear collision $\vec{P}_i = \vec{P}_f$
$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$
Completely inelastic collision:
objects stick together; momentum conserved
$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{v}_f$
Mechanical energy converted to heat
Elastic Collision: kinetic energy conserved
Two dimensional collision: $\vec{P}_i = \vec{P}_f$ always
Completely inelastic: objects stick together
Completely elastic: $\mathcal{E}_{mech,i} = \mathcal{E}_{mech,f}$
(mechanical energy conserved)

Chapter 10: Rotation

s: arc length of sector r: radius of sector
$s = r\theta$ (θ in radians)
1 revolution $= 360^o = 2\pi$ radians
Angular displacement $\Delta\theta = \theta_f - \theta_i$
Angular velocity $\omega_{avg} = \frac{\Delta\theta}{\Delta t}$ $\omega = \frac{d\theta}{dt}$
Angular acceleration: $\alpha_{avg} = \frac{\Delta\omega}{\Delta t}$
$\alpha = \frac{d\omega}{dt}$
Uniform angular velocity: $\alpha = 0$
$T = 2\pi/\omega_0$; $\omega = \omega_0$; $\theta = \theta_0 + \omega_0 t$
Uniform angular acceleration $\alpha = \alpha_0$
$(\theta_0 = 0)$ $\omega = \omega_0 + \alpha t$
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$\theta = (\omega + \omega_0)t/2$
$\omega^2 = \omega_0^2 + 2\alpha\theta$
Linear and angular quantities: $s = r\theta$
$v_t = r\omega$; $a_t - r\alpha$; $a_r = \omega^2 r$
Rotational kinetic energy of rigid body
$K = \frac{1}{2}I\omega^2$; Moment of inertia I
$I = \sum_i m_i r_i^2 = \int r^2 dm$
Parallel axis theorem: $I = I_{cm} + md^2$
(d: distance from parallel axis to cm axis)

Torque: $\vec{\tau} = \vec{r} \times \vec{F}$ par
Right hand rule: $\tau = r_\perp F = r f \sin\theta$
Angular form of Newton's second law:
Fixed axis of rotation: $\tau = I\alpha$
Work done by torque: $W = \int_1^2 \tau d\theta$
Power: $P = \frac{dW}{dt} = \tau\frac{d\theta}{dt} = \tau\omega$
Work-energy theorem for rotation:
$W = \Delta K = \frac{1}{2}I(\omega_f^2 - \omega_i^2)$

Chapter 11: Angular Momentum

Kinetic energy: translation and rotation
$K_{tot} = K_{trans-cm} + K_{rot-cm} = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$
Rolling (without slipping):
$v_{cm} = \omega R$; $a_{cm} = \alpha R$
$K_{tot} = \frac{1}{2}(m + I_{cm}/R^2)v_{cm}^2$
Rolling down incline: $mg\Delta h = \Delta K_{tot}$
Inclination θ: $a_{cm} = g\sin\theta(1 + I_{cm}/(mR^2))$
Angular momentum (particle): $\vec{L} = \vec{r} \times \vec{p}$
Rigid body with fixed axis:
$\vec{L} = I\vec{\omega}$; $\frac{d\vec{L}}{dt} = \vec{\tau}$
Angular momentum conservation $\vec{L}_i = \vec{L}_f$
(when no external torques are present)

Chapter 12: Equilibrium (no elasticity)

Conditions for static equilibrium:
Net force is zero: $\sum_i \vec{F}_i = \vec{0}$
Net torque about any point is zero: $\sum_i \vec{\tau}_i = \vec{0}$

Chapter 13: Gravitation

Newton's law of universal gravitation
mutual attraction of two masses m_1; m_2
$\vec{F} = \hat{n}Gm_1m_2/d^2$ ($\hat{n}$ toward other mass)
$G = 6.67 \times 10^{11}\ Nm^2/kg^2$
Gravitational force by several masses m_i
Superposition: $\vec{F} = \sum_i Gmm_i/r_1^2\hat{n}_i$
Continuous distribution of mass:
$\vec{F} = \int d\vec{F} = \int \frac{Gmdm}{r^2}\hat{n}$
Uniform spherical shell of mass
($\vec{r} = r\hat{r}$ goes to shell center)

$$\vec{F} = \begin{bmatrix} \frac{GM_{shell}m}{r^2}\hat{r} & outside \\ \vec{0} & inside \end{bmatrix}$$

Gravitational acceleration:
$mg = F = GMm/r^2$; $g = GM/r^2$

Spherically symmetric body:
gravitational force $F = GM_{enc}m/r^2$
M_{enc}: mass inside sphere of radius r
$\vec{F}$ toward center at distance r
Gravitational potential energy:
Point mass as $\vec{r}$: $U = -GMm/r$
Escape speed: $\mathcal{E} = \frac{1}{2}mv^2 - GMm/r \geq 0$
$v^2 \geq 2GM/r$
Kepler's laws of planetary motion
I: Orbits are ellipses with sun at a focus
Semi-major axis (sma) a: half of major diameter
II: Equal areas in equal times (radius)
Angular momentum conservation
III: Period-squared proportional to sma-cubed
$T^2 = \frac{4\pi^2a^3}{GM}$
Angular momentum: $L = mr^2\dot{\theta}$
Energy: $\mathcal{E} = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - \frac{GMm}{r}$
$\mathcal{E} = \frac{1}{2}m\dot{r}^2 + \frac{L^2}{2mr^2} - \frac{GMm}{r} = -\frac{GMm}{2a}$