

Physics 221
First Hourly Examination
Prepared August 2006
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PHYS 221 - 003/004; TEST 1; 22 September 2003

1. [25 points] The G string of a guitar has a length of 0.6 meters, with a fundamental frequency of 200 Hz. We may effectively shorten the length of the string by pressing down on a fret (a small ridge on the neck of the guitar). How far from the end of the string should the fret be, if you wish to produce a frequency of 250 Hz?

[Assume that the string is uniform, and that the tension in it does not change as you press down on the fret.]

Answer: 12cm.

2. [25 points] A thin ring of mass M and radius R is pivoted about an axis on its edge, perpendicular to the plane of the ring. Determine the period of small angle oscillations about this axis.

Hints:

- The moment of inertia of a thin ring of mass M and radius R about its center is MR^2 .
- Parallel axis theorem $I' = I_{cm} + Md^2$, where I_{cm} is the moment of inertia about an axis passing through the center of mass, and I' is the moment of inertia about a parallel axis a distance d from the center of mass.

Answer: $2\pi\sqrt{2R/g}$.

3. [25 points] Consider a “semi-infinite” wire, which begins at $x = 0$, and lies at positive x , with a uniform charge per unit length of λ Coulombs per meter. Compute the electric field at a point in the plane perpendicular to the wire, at a distance d below the end of the wire. Show that that electric field makes an angle of 45° with the wire.

Answer: $E_x = E_y = -k\lambda/d$.

4. [25 points] A point charge $+Q$ is placed at the center of a hollow, conducting sphere of inner radius a , and outer radius b . Determine the electric field everywhere and the charge on the inner and outer surfaces of the conducting sphere, which is electrically neutral and isolated. Sketch the direction of the field lines in every region.

Answers: outward $r < a : E = kQ/r^2$; $a < r < b : E = 0$; $r > b : E = kQ/r^2$.

5. [Extra Credit; 10 points] Two infinite, parallel, non-conducting sheets carry surface charge densities $+\sigma$ and $-\sigma$, respectively. Determine the electric field between the sheets, as well as outside the sheets on either side. Draw a picture indicating which sheet is charged positively and which is charged negatively, showing the direction of the field lines.

Answers: left, right, left: $\sigma/(2\epsilon_0)$ everywhere.

PHYS 221 - 007/008; TEST 1; 27 September 2004

1. [25 points] A certain guitar string has a density of 5 grams per meter and is held under a tension of 100 Newtons. The fixed supports are 1 meter apart. The string is oscillating in a standing wave pattern with two internal nodes - its third harmonic. Determine the wavelength, frequency, and speed of the traveling waves that produce this standing wave.

Answers: $0.67m$; $210Hz$; $140m/s$.

2. [25 points] Two small particles of mass m are connected by a short light rod of length D , which is pivoted and free to rotate about its center in a horizontal plane. The two particles have net charges of equal magnitude Q , and opposite signs, forming an electric dipole. A uniform electric field E is applied in a particular horizontal direction. Determine the period of small oscillations of this dipole about its stable equilibrium position, in seconds. Ignore gravity.

Take $D = 10^{-3}$ meters, $Q = 10^{-6}$ Coul, $m = 10^{-3}$ kg, and $E = 10^5$ Nt/Coul.

Answer: 0.014 sec.

3. [25 points] A particle of charge $q_1 = 10^{-6}$ Coul is placed on a horizontal surface. A second charge $q_2 = 2 \times 10^{-6}$ Coul is placed on the same horizontal surface two meters East of the first charge.

- Roughly sketch the lines of force of the electric field in the horizontal surface.
- Determine magnitude and direction of the electric field two meters North of the first charge.
- If a charge $q_3 = 3 \times 10^{-6}$ Coul is located on the horizontal surface 2 meters North of the first charge, determine the net force on it (in Newtons) produced by the other two charges.

Answers: 4560N/C ; 23° W of N; 0.012N .

4. [25 points] Charge is uniformly imbedded throughout a long (infinite) solid circular cylinder of radius R meters, with the uniform charge density ρ Coul/m^3 throughout the cylinder.
- Determine the charge per unit length of the cylinder, λ , in terms of ρ and R .
 - Determine the magnitude and direction of the electric field everywhere inside the cylinder.
 - Determine the magnitude and direction of the electric field everywhere outside the cylinder.

Answers: $\rho\pi R^2$; $\rho r/(2\epsilon_0)$; $\rho R^2/(2\epsilon_0 r)$.

5. [Extra Credit; 10 points] A total charge Q is distributed uniformly on a semi-circular arc of radius R . Determine the magnitude and direction of the electric field at the center of that semi-circular arc.

Answer: $2kQ/(\pi R^2)$.

PHYS 221 - 003/004; TEST 1; 21 February 2005

1. [20 points] A particular string on a musical instrument is tightened to a tension of 100 Newtons (about 25 pounds). The fundamental mode of the string then corresponds to a frequency of vibration of 250 Hertz. The length of the string between the points of attachment is 0.6 meters.
- Determine the wavelength of the fundamental mode of vibrations of the string.
 - Determine the mass per unit length of the string.
 - Determine the speed of transverse vibrations of the string.

The string is then “shortened” to an effective length of 0.5 meters by placing a finger gently on it (at a fret). Determine the new frequency of the fundamental mode of the string.

Answers: 1.2m ; $1.1\text{g}/\text{m}$; $300\text{m}/\text{s}$; 300Hz .

2. [20 points] Two sound sources, A and B , produce isotropic sound waves at a frequency of 400 Hz with the same acoustic power, $P = 0.01$ Watts. An observer is located 5 meters from source A , as well as 6 meters from source B .

- Determine the intensity of sound (in Watts/meter-squared) detected by the observer when (a) only source A is turned on I_A and (b) only source B is turned on, I_B .
- Determine the relative intensity in these two cases (using I_A and I_B) in decibels (dB).
- Determine the intensity at the observer when sources A and B are both turned on and driven in phase.

Answers: $3.1 \times 10^{-5} \text{ w/m}^2$; $2.2 \times 10^{-5} \text{ w/m}^2$; 1.6 dB ; $8.3 \times 10^{-5} \text{ w/m}^2$.

3. [20 points] A total charge $+Q$ is distributed uniformly along an insulating wire of length $2L$. The point P is located directly above the center of the wire, C , a distance L from the center.

What is the direction of the electric field at the point P ? Why?

Determine the magnitude of the electric field at the point P .

Answers: away from wire; $kQ/(\sqrt{2}L^2)$.

4. [20 points] An insulating bead of mass m , which contains a total (negative) charge $-q$, slides freely along a horizontal wire. Positive charges $+Q$ are symmetrically placed, a distance D above and below the wire, respectively.

Note: Neglect gravity and friction in the wire.

- When the bead is a distance x to the right of the line joining the two positive charges, determine the net electrostatic force on the bead.
- Determine the frequency of small oscillations of the bead about the center line.

Answers: $-2kQqx/(x^2 + D^2)^{3/2}$; $1/(2\pi)\sqrt{2kQq/(mD^2)}$

5. [20 points] An isolated, hollow, conducting sphere with inner radius a and outer radius b contains a TOTAL electrical charge $+Q$. IN ADDITION, a point charge $+Q$ is placed at the CENTER of that sphere.

- Determine the charge lying on the inner surface of the hollow conducting sphere, as well as on its outer surface.
- Determine the magnitude and direction of the electric field in these three regions: $r < a \dots a < r < b \dots r > b$.

Answers: outward; kQ/r^2 ; 0; $2kQ/r^2$.

6. [Extra Credit; 10 points] The shock wave from a supersonic jet aircraft arrives at a detector on the ground. The airplane is flying at an altitude of 12 kilometers above the ground, with a speed of 500 meters per second, and it passed directly over the detector on the ground. What is the angle between the jet aircraft and the zenith line at the location of the detector when the shock wave arrives?

Note: The average speed of sound over the relevant region of the atmosphere is about 300 meters per second.

Answer: 53° .

PHYS 221 - 003/004; TEST 1; 29 September 2004

1. [25 points] A string of length 1 meter and mass 2 grams with fixed ends, which is held under a tension of 50 Newtons, undergoes standing wave oscillations in its fundamental mode. If the maximum total kinetic energy in the string is 0.01 Joules, determine the maximum amplitude of vibration of the string, in meters.

Answer: 9mm.

2. [25 points] A large (infinite) insulating sheet has a uniform surface charge $\sigma = +2 \times 10^{-6} \text{ Coul}/\text{m}^2$ imbedded into it. A particle of mass $m = 10$ grams and charge $q = +3 \times 10^{-4} \text{ Coul}$ is attached one end to a light rod of length $L = 10$ cm, the other end of which is attached to the sheet. The rod pivots freely about an axis parallel to the sheet. Neglect gravity.

- Determine the magnitude and direction of the electric field at the location of the charge
- Determine the frequency of small oscillations of the charge about its equilibrium position, in Hertz.

Answers: $1.1 \times 10^5 \text{ N}/\text{C}$ to right; 29 Hz .

3. [25 points] Three charges, $+Q$, $+Q$, and $-Q$, are placed at vertices of an equilateral triangle of side a . Determine the magnitude and direction of the electric field at the center of that triangle.

Answer: toward negative charge: $6kQ/a^2$.

4. [25 points] A long, thin rod is placed along the central axis of a conductor, which is a long cylinder of inner radius a and outer radius b . The conductor has zero net charge, and the rod contains a charge per unit length of λ Coulombs per meter. Determine the magnitude and direction of the electric field

- everywhere between the wire and the conductor.
- everywhere within the conductor.
- everywhere outside the conductor.

Answers: $r < a : E = \lambda/(2\pi\epsilon_0 r)$; $a < r < b : E = 0$; $r > b : E = \lambda/(2\pi\epsilon_0 r)$.

5. [Extra Credit; 10 points] A certain positive charge $+Q$ is separated into two parts: $Q = q_1 + q_2$, where q_1 and q_2 are positive. The two parts are then separated by a distance d . For what value(s) of q_1 and q_2 is the force between these two parts as large in magnitude as it can be? Express the magnitude and direction of the force for this case in terms of Q and d .

Answers: $q_1 = q_2 = Q/2$; $kQ^2/(4d^2)$, repulsive.

PHYS 221 - 007/008; TEST 1; 26 September 2005

1. [25 points] One end of a strong, light ideal spring of spring constant k is attached to a sled of mass m that lies on a frictionless horizontal surface. The other end of the string is attached to a stationary wall. The period of small oscillations of the system about equilibrium is 2.0 seconds. When a person of mass 60 kg rides in the sled, the period of oscillations is measured to be to 4.0 seconds. Determine the mass m of the sled in kg, as well as the spring constant k , in Nt/m.

Answers: $20kg$; $200N/m$.

2. [25 points] Two identical sound sources, A and B , are driven in phase to produce sound at a wavelength of 3 meters (the speed of sound is 350 meters/second). The power produced by each source is 10^{-6} Watts. A listener

sits 5 meters from source A and 10 meters from source B . Determine the amplitude of the signal heard by the listener in Watts/m^2 . How many decibels does this signal correspond to?

Answers: $2.4 \times 10^{-9} \text{W}/\text{m}^2$; 34dB .

3. [25 points] Three charges of magnitudes $-Q$, Q , and $2Q$ are placed at the vertices of an equilateral triangle of side a . Determine the magnitude and direction of the electrostatic force on a charge q placed at the center of that triangle.

Note: the distance from the center of the triangle to a vertex is $a/\sqrt{3}$.

Answers: $7.9kQq/a^2$; 19° from $-Q$ on side of $+Q$.

4. [25 points] An infinite line is uniformly charged, with 2.0×10^{-6} Coulombs per meter. Determine the magnitude and direction of the electric field at a distance of 3.0 meters from the line, and directly above it.

Answers: upward; $1.2 \times 10^4 \text{N}/\text{C}$.

5. [Extra Credit; 10 points] One end of a string of mass m and length L is attached to an end of a string of mass $4m$ and length L . The other ends of the strings are attached to fixed locations, and then stretched to a tension T . Determine the lowest frequency of vibration of the system, f , for which the point at which the strings are attached is a node.

Note: express f in terms of m , L , and T .

Answer: $1/(2\pi)\sqrt{T/(mL)}$.

PHYS 221 - 003/004; TEST 1; 28 September 2005

1. [25 points] An electric dipole of dipole moment p , pivoted about its center, is free to rotate about a vertical axis. The dipole makes a small angle θ with an external field of fixed magnitude E , which is also horizontal. The moment of inertia of the dipole about this axis is I .

- Compute the magnitude and direction of the torque on the dipole.
- Determine the frequency of small oscillations of the dipole about this axis.

Answers: $-pE\sin\theta$; $1/(2\pi)\sqrt{pE/I}$.

2. [25 points] An organ pipe is $L_0 = 2.0$ meters long, with open ends. In a room in which the velocity of sound is 350 meters per second, determine the frequency f_0 of the fundamental mode – the lowest frequency of vibration of the pipe. In addition, determine the next lowest frequency.

How long (L) must a pipe be to have the same fundamental frequency f_0 , if one end is open and the other end is closed? Determine the next lowest frequency for that case, as well.

Do the two organ pipes sound the same? Explain.

Answers: 88Hz ; 175Hz ; 1m ; 263Hz ; no.

3. [25 points] Four charges, with values $+Q$, $+Q$, $-Q$, and $-Q$, respectively, are held in place at the vertices of a square of side a . A charge q_0 is placed at the center of the square. Determine the magnitude and direction of the net electrostatic force on the charge q_0 . Note: the distance from the center of the square to any vertex is $a/\sqrt{2}$.

Answers: between negative charges: $4\sqrt{2}kQq_0/d^2$.

4. [25 points] Two identical rings of radius R are placed in parallel planes, along a common horizontal central axis, a distance $2R$ from one another, center to center. Each ring is uniformly charged, the left ring containing a total charge $+Q$, whereas the right ring contains a total charge $-Q$. The point P lies on the ring axis, halfway between the rings.

- Determine the magnitude and direction of the electric field produced at P by the ring on its left.
- Determine the magnitude and direction of the electric field produced at P by the ring on its right.
- Determine the total electric field at the point P produced by the rings.

Answers: $kq/(2\sqrt{2}R^2)$ to right; $kq/(2\sqrt{2}R^2)$ to right; $kq/(\sqrt{2}R^2)$ to right.

5. [Extra Credit; 10 points] There are 88 strings inside a standard piano, with the notes that cover a range of more than seven octaves — or more than a factor of 100 in frequency. Using the physics of transverse vibrations of a string between fixed ends, explain the following:

- Why are the strings shorter for high frequency (treble) notes than for low frequency (bass) notes?

- Why are the strings thicker for low frequency (notes) than for high frequency (treble) notes? (Assume that they are all made out of the same material - piano wire.)
- Why is the tension in the strings greater for high frequency (treble) notes than for low frequency (bass) notes?

Answers: $f = 1/(2L)\sqrt{T/\mu}$; large t ; small μ ; small L .

PHYS 221 - 003/004; TEST 1; 20 February 2006

1. [20 points] The ends of a string 0.5 meter in length with a mass of 20 grams are held fixed, with a tension of 120 Newtons in the string.

- Determine the wavelength and frequency of the lowest vibrational frequency (fundamental mode) of the string.
- Determine the wavelength and frequency of the sound produced in air by the vibration of the spring, if the speed of sound is 350 meters/second.

Answers: $1m$; $55Hz$; $6.4m$; $55Hz$.

2. [20 points] Two sound sources, A and B , produce sound waves of the same frequency, as detected by a fixed receiver the same distance from each source. However, the sound from source B is one produced quarter-period later than that of source A . If the receiver gets 50 decibels of intensity from source A alone, and it receives 55 decibels from source B alone, what is the intensity at the receiver in decibels when both sources are turned on?

Answer: $55dB$.

3. [20 points] Four positive charges are placed at the locations in the (x,y) plane, that are given below:

Amount of charge	(x,y) coordinates
Q	$(d,0)$
$2Q$	$(0,d)$
$3Q$	$(-d,0)$
$4Q$	$(0,-d)$

Determine the magnitude and direction of the net electric field at the origin of the coordinate system, $(0,0)$.

Answers: $\sqrt{20}kQ/d^2$, 45° between Q and $2Q$.

4. [20 points] A semicircular arc of radius R contains a total charge Q , which is imbedded uniformly along it.
- Determine the magnitude and direction of the electric field at the center of the arc.
 - Determine the magnitude and direction of the electric field at a great distance z from the center of the arc, and lying along its symmetry axis. Assume that z is much larger than the arc radius R .

Answers: $2kq/(\pi R^2)$ away from arc; kQ/r^2 away from arc.

5. [20 points] Two positive charges of magnitude $+Q$ are placed symmetrically at a distance d above and below an horizontal insulating wire. A particle of mass m and negative charge $-q$ slides without friction on the wire.
- Determine the net horizontal electric force on the particle, when it is placed a distance x from the line joining the two charges $+Q$.
 - Show that, if the particle is placed at rest a small distance from the center of the wire (ie, $x = 0$), it executes simple harmonic motion about $x = 0$. Determine the frequency of that motion.

Answers: $2kQqx/(x^2 + d^2)^{3/2}$; $1/(2\pi\sqrt{2kQq/(md^3)})$.

6. [Extra Credit; 10 points] Two strings on a guitar each produce sound. The strings on the guitar have the same length, and are made of the same material. The first string is stretched to a tension of 100 Newtons, and it produces sound of frequency 300 Hz in the fundamental mode. What possible tensions can be present in the second string, in order that 4 beats per second are produced when the fundamental mode of each string is excited at the same time?

Answer: 103N or 97N.