

Physics 221  
Second Hourly Examination  
Prepared August 2006  
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PHYS 221 - 003/004; TEST 2; 29 October 2003

1. [25 points] Charge  $Q$  is uniformly distributed around a thin circular ring of radius  $R$ . Find the electrostatic potential at a distance  $z$  from the center of the ring, along a line perpendicular to the plane of the ring. Use that potential to determine the magnitude and direction of the electric field  $E$  due to the ring on this line.

Answer:  $kQz/(z^2 + R^2)^{3/2}$  away from ring.

2. [25 points] An air-filled capacitor with plate area  $A$  and distance  $d$  between the plates is connected to a battery that produces an electrostatic potential difference  $V$  between the plates. With the battery still connected to the capacitor, the plates are physically separated to a distance  $2d$ , while maintaining electrical isolation. Calculate the final charge on the capacitor plates, the change in the potential energy stored in the capacitor, and the work done by the battery during this process.

Answers:  $\epsilon_0 AV/(2d)$ ;  $-\epsilon_0 AV^2/(2d)$

3. [25 points] A camera flash lamp has a capacitor in an  $RC$  circuit, connected to a  $DC$  voltage source, which charges to half of its equilibrium value in 2 seconds. If the capacitor charges through a 200 kilo-Ohm ( $k\Omega$ ) resistor, determine its capacitance.

Answer:  $1.44 \times 10^{-5} F$ .

4. [25 points] An electromagnetic rail gun can be constructed as follows: A conducting bar of mass  $m$  slides over two horizontal conducting rails separated by a distance  $L$ . A power source causes current  $I$  to flow through the rails and the bar. A uniform vertical magnetic field  $B$  is maintained perpendicular to the plane of the rails and bar. If the bar is initially at rest, how fast will it be moving after it has moved a distance  $D$ ?

Answer:  $\sqrt{2ILBD/m}$ .

5. [Extra Credit; 10 points] A spherical electrode with radius  $a$  is coated with a layer of material with electrical resistivity  $\rho$ . The thickness of the layer is  $d$ , and its outer layer is coated with a conducting electrode. Compute the resistance of the spherical coating between the inner and outer conductors.

Hint:  $E = \rho j$  in the resistive medium.

Answer:  $\rho d/(4\pi a(a + d))$ .

PHYS 221 - 003/004; TEST 2; 03 November 2004

1. [25 points] Determine the electric potential  $V$  at a point along the central axis of a uniformly charged ring of radius  $R$  that contains a total charge  $+Q$ , at a distance  $z$  from the center of the ring. Then, determine the electric field at that point. Express your answer in terms of  $Q$ ,  $R$ , and  $z$ .

Answer:  $kQz/(z^2 + R^2)^{3/2}$ , away from ring.

2. [25 points] A parallel plate capacitor filled with dielectric material has a plate area of 0.01 meters-squared, plate separation of 0.001 meters, and charges of  $\pm 2 \times 10^{-6}$  Coulombs on the plates. The potential across the capacitor is 300 Volts. Determine the capacitance of the capacitor, as well as the dielectric constant of the material inside it.

Answers:  $6.7nF$ ; 75.

3. [25 points] A 12 Volt battery (left side) and a 6 Volt battery (right side) are connected to identical resistors having a resistance of 10 Ohms. These combinations are then placed in parallel across another 10 Ohm resistor. Determine the current flowing through each resistor, the power lost in each resistor, and the power generated by each battery.

Answers: 0A, 0.6A, 0.6A; 0W; 3.6W, 3.6W; 0W, 7.2W.

4. [25 points] An alpha particle (charge  $2e_0$ , mass  $3700MeV/c^2$ ) moves in a circular orbit perpendicular to a uniform magnetic field  $B$  of magnitude 2 Tesla. The radius of that orbit is 0.1 meters. Determine the speed of the alpha particle (in meters/second) the kinetic energy (in electron Volts), and the period of revolution of the particle (in seconds).

Note:  $e_0 = 1.6 \times 10^{-19}$  Coulombs, mass of alpha particle  $m_\alpha = 6.4 \times 10^{-27}$  kilograms,  $m_\alpha c^2 = 3.7 \times 10^9$  eV.

Answers:  $10^7 m/s$ ;  $2MeV$ ;  $9.3 \times 10^{-8} s$ .

5. [Extra Credit; 10 points] A capacitor in a R-C circuit is initially charged to a potential of 10,000 Volts. How long (in seconds) must one wait after closing the switch and completing the circuit for the potential across the capacitor to be reduced to the "safe" level of 10 Volts?

Note:  $R = 2 \times 10^6$  Ohms;  $C = 2 \times 10^{-6}$  Farads.

Answer: 28s.

PHYS 221 - 007/008; TEST 2; 05 November 2004

1. [25 points] A steel rail track 10 km in length has a potential difference of 600 Volts along its length. The rail is approximately 10 cm by 20 cm in cross section. Determine the resistance of this track (in Ohms), the current flowing in the track (in Amperes) and the power lost to heat (in Watts).

Note: the resistivity of steel is  $3 \times 10^{-7} \Omega m$ .

Answers:  $0.15\Omega$ ; 4000A; 2.4MW.

2. [25 points] An insulating plastic rod of length  $L$  has the non-uniform charge per unit length  $\lambda x$ , as measured from its left end. Determine the electric potential at a distance  $y$  to the left of its left end. Use that result to determine the electric field at this point.

Answer:

$$k\lambda \left[ L - y \log \frac{L+y}{y} \right]; k\lambda \left[ \log \frac{L+y}{y} - \frac{L}{L+y} \right].$$

3. [25 points] Two 12 Volt batteries (left and right sides) are connected to identical resistors, each having a resistance of 10 Ohms. Each combination is connected in parallel across another 10 Ohm resistor. Determine the current flowing through each resistor, the power lost in each resistor, and the power generated by each battery.

Answers: 1.2A each; 14.4W first two resistors; 0 third resistor; 14.4W each battery.

4. [25 points] An electron is fired with a speed of  $10^5$  meters/sec through a region of space that contains uniform electric and magnetic fields, which are perpendicular to one another and to the direction of motion of the electron. Determine the relation between the fields, if the electron is to be undeflected when it passes through the region.

Answers:  $E(V/m) = 10^5 B(T)$ .

5. [Extra Credit; 10 points] How much energy (in Joules) is stored in a cubic meter of air in "fair weather", when there is an upward electric field of 150 Volts per meter?

Hint: Construct a parallel plate capacitor 1 meter  $\times$  1 meter  $\times$  1 meter.

Answer:  $10^{-7} J$ .

PHYS 221 - 003/004; TEST 2; 30 March 2005

1. [20 points] Calculate the electrostatic potential along the line of symmetry of a uniformly charged ring. The ring has radius  $R$  and total charge  $+Q$ . Express your answer in terms of the distance  $z$  from the point in question to the center of the ring.

(a) What is the magnitude and direction of the electric field at that point?

(b) How much work must be done to move a positive charge  $+q$  in from infinity, to the point in question?

Answers:  $kQz/(z^2 + R^2)^{3/2}$  away from ring;  $kQq/\sqrt{z^2 + R^2}$ .

2. [20 points] A parallel plate capacitor has plate area of 0.1 meters-squared and a plate separation of 0.001 meters. Charges of  $\pm 3 \times 10^{-6}$  Coulombs are placed on the plates. Determine the magnitude of the electric field inside the capacitor, the potential difference in the plates (in Volts), and the electrostatic energy stored inside the capacitor (in Joules).

Answers:  $3.5 \times 10^6 \text{V/m}$ ; 3500V;  $5.5 \times 10^{-8} \text{J}$ .

3. [20 points] Two 12 Volt batteries are hooked up in parallel with two  $4\Omega$  resistors. These configurations are then hooked in parallel (++) and (-) across another  $4\Omega$  resistor. Determine the current through each resistor, the power provided by each battery, and the power lost through each resistor.

Answers: 3A, 3A, 0A; 36W, 36W; 36W, 36W, 0W.

4. [20 points] A copper wire of cross-sectional area  $10^{-5}$  meters-squared and length 2 meters has a current of 5 A uniformly distributed across that area.

(a) What is the magnitude of the electric field along the wire?

(b) How much electrical energy is transferred into thermal energy in 30 minutes?

Hint: the resistivity of copper is  $2 \times 10^{-8}$  Ohm-meters.

Answers: .01V/m; 180J.

5. [20 points] An electron in the earth's Van Allen belt travels in a circular path of radius 10 meters about a magnetic field line. If the (uniform) magnetic field is  $10^{-5}$  Tesla, determine the kinetic energy of the electron, in electron Volts (eV).

Note:  $eV = 1.6 \times 10^{-19}$  Joules and  $m_e = 9.1 \times 10^{-31}$  kg =  $510000eV/c^2$ .  
Charge of electron =  $-1.6 \times 10^{-19}$  Coul.

Answer:  $1000eV$ .

6. [Extra Credit; 10 points] A capacitor of capacitance 2.0 mF [micro-Farads] is “leaky”, in the sense that charge gradually leaks from one plate to the other over the course of time. If the initial charge of  $5.0\mu C$  [micro-Coulombs] is reduced to  $1.0\mu C$  in 200 seconds, determine the “effective resistance” of the capacitor system, in Ohms.

Answer:  $6 \times 10^7 \Omega$ .

PHYS 221 - 003/004; TEST 2; 31 October 2005

1. [25 points] The drum of a photocopying machine is a metallic cylinder of length 50 cm and diameter 10 cm. For proper functioning an electric field of  $2 \times 10^5$  N/C must be present just outside the drum.

How much charge (in microCoulombs) must be present on the drum?

For the desktop version of the machine, the drum length is 20 cm and the drum radius is 6 cm. For the electric field to be the same as before,  $2 \times 10^5$  N/C, how much charge (in microCoulombs) must be present on the desktop drum?

Answers:  $2.6 \times 10^{-7} C$ ;  $1.3 \times 10^{-7} C$ .

2. [25 points] A charge  $Q$  is uniformly distributed throughout the interior of a solid sphere of radius  $R$ . With the convention that the electric potential vanishes far away from the sphere, determine the electric potential at these locations:

- at a point on the surface of the sphere.
- at the center of the sphere.

Answers:  $kQ/R$ ;  $3kQ/(2R)$ .

3. [25 points] A capacitor of capacitance  $C_1 = 3$  nanoFarads initially contains a charge of  $Q = 6$  microCoulombs. After a switch is closed, its plates are connected to those of a second capacitor with capacitance  $C_2 = 9$  nanoFarads. After equilibrium is achieved, determine the charge on each capacitor. In

addition, determine the energy stored in the capacitor  $C_1$  before the switch is closed, as well as the energy present in each capacitor in equilibrium. Is energy conserved? Explain why or why not.

Answers:  $1.5\mu\text{C}$ ;  $4.5\mu\text{C}$ ;  $6 \times 10^{-3}\text{J}$ ;  $0.4 \times 10^{-6}\text{J}$ ;  $1.1 \times 10^{-6}\text{J}$ ; resistive losses necessary for equilibrium to occur.

4. [25 points] A flashbulb on a camera is operated by first charging a capacitor with a battery to a potential of 9.0 Volts. The capacitor is then discharged through the bulb, which has an effective resistance of  $0.05\Omega$ . If a current of 1.0A is required to produce the flash, determine the minimum Capacitance that is required in the circuit.

Answer: It always flashes;  $I(t = 0) = 180\text{A}$ .

5. [Extra Credit; 10 points] A certain brand of hot dog (Wiener) cooker works by applying a potential difference of 120 Volts across opposite ends of a hot dog and allowing it to cook by means of the thermal energy produced. A current of 10A is present, and about 60 kiloJoules of energy are required to cook a single hot dog. How long will it take to cook 6 hot dogs in this apparatus?

Answer: 5 minutes.

PHYS 221 - 007/008; Test 2; 31 October 2005

1. [25 points] Space vehicles traveling through Earth's radiation belts can intercept a significant number of electrons, and the resulting charge buildup can damage electronic components. Suppose that a spherical metal satellite of diameter 2.0 meters accumulates 10 micro-Coulombs of charge in one orbital revolution. During this orbit it travels at 8000 meters/second for an interval of 4.0 hours.

Determine the surface charge density on the satellite.

Determine the electric field just outside the satellite.

If the satellite captures all the electrons that strike it during its travel, determine the average density of electrons (per cubic meter) along its path.

Answers:  $7.9 \times 10^{-7}\text{C}/\text{m}^2$ ;  $9 \times 10^4\text{N}/\text{C}$ ;  $1.7 \times 10^{-7}$  per cubic meter.

2. [25 points] A charge  $+Q$  is uniformly distributed over a thin ring of radius  $R$ . The electric potential  $V$  is assumed to vanish at infinity.

Determine the electric potential  $V(z)$  on the axis of symmetry of the ring, at a distance  $z$  from the plane of the ring.

Use this expression for  $V(z)$  to determine the electric field along the ring axis.

Answers:  $kQ/\sqrt{z^2 + R^2}$ ;  $kQz/(z^2 + R^2)^{3/2}$ , away from ring.

3. [25 points] An air-filled parallel plate capacitor with plate area of 2.0 square meters and plate separation 3 millimeters is charged with a 24 Volt battery.

Determine the charge on the plates, and the electrical energy stored in the capacitor.

With the battery still connected to the plates, a dielectric material ( $\kappa = 6.0$ ) is inserted between the plates, completely filling the space. Determine the (final) charge on the plates of the capacitor, as well as the (final) electrical energy stored in the capacitor.

Has the stored energy changed? Why or why not?

Answers:  $1.4 \times 10^{-7}C$ ;  $1.9 \times 10^{-6}J$ ;  $8.5 \times 10^{-7}C$ ;  $1.0 \times 10^{-5}J$ ; battery pumps charge.

4. [25 points] A Nichrome heating element dissipates 1000 Watts when the applied potential difference is 125 Volts and the wire temperature is  $800^\circ C$ . When the (electrically sealed) wire is dropped into boiling water, its temperature is reduced to  $100^\circ C$ , the potential difference remaining the same. Determine the power dissipated in the wire in the latter case.

Note: the temperature coefficient of resistivity for Nichrome is  $\alpha = 4.0 \times 10^{-4}/K$ .

Answer: 1280W.

5. [Extra Credit; 10 points] Two 24 Volt batteries are each hooked in parallel with  $12\Omega$  resistors. These configurations are then hooked in parallel with another  $12\Omega$  resistor. Determine the current passing through each resistor (in A), as well as the power provided by each battery, in Watts.

Answers:  $2/3A$ ,  $2/3A$ ,  $0A$ ;  $16W$  each.

1. [20 points] Three charges  $+Q$ ,  $-Q$ , and  $+2Q$ , are located at the vertices of an equilateral triangle of side length  $a$ .

Determine the electrostatic potential at the center of the triangle Note: the center of the triangle lies at a distance  $a/\sqrt{3}$  from each vertex.

How much electrostatic potential energy is stored when the charges are held in this configuration?

Answers:  $2\sqrt{3}kQ/a$ ;  $-kQ^2/a$ .

2. [20 points] A long, thin rod is placed along the central axis of a metallic conductor, which is a long cylinder, with inner radius  $a$  and outer radius  $b$ . The inner rod contains a charge per unit length of  $\lambda$  Coulombs per meter, whereas the conductor has zero net charge. Determine the magnitude and direction of the electric field

- everywhere between the wire and the conductor
- everywhere within the conductor
- everywhere outside the conductor

Answers: away from axis;  $\lambda/(2\pi\epsilon_0 r)$ ; 0;  $\lambda/(2\pi\epsilon_0 r)$ .

3. [20 points] The conduction electrons in a semi-conducting wire are found to be moving at a speed of 50 meters per second. There is a current of 4 Amps produced by electrons in this wire, which is 0.2 meters long, with a cross-sectional area of 0.005 meters-squared. Determine the number of conduction electrons per unit volume in the wire.

Answers:  $10^{20}$  per cubic meter.

4. [20 points] A capacitor of unknown capacitance  $C$  is charged to a potential of 100 Volts, at which point its plates have charges of magnitude  $2.0 \times 10^{-8}$  Coulombs on its plates. What is the capacitance  $C$ ? How much energy is stored in the capacitor?

The plates of the capacitor are then connected through a resistor with resistance  $R$ . It is found that the potential across its plates is reduced to 10 Volts after 10 seconds. Determine the resistance  $R$ .

Answers:  $2 \times 10^{-10} F$ ;  $10^{-6} J$ ;  $2.2 \times 10^{10} \Omega$ .

5. [20 points] Two identical batteries, with electromotive potential  $E$  and internal resistance  $r$ , are connected in parallel with one another, with positive to positive terminal and negative to negative terminal. This system is then placed across an external resistor of resistance  $R$ .
- Determine the current flowing through the external resistor, and the power dissipated in the resistor.
  - For what choice of the external resistance  $R$  is the power dissipated maximized?

Answers:  $E/(R + r/2)$ ;  $R = r/2$ .

6. [Extra Credit; 10 points] A  $50nF$  (1 nanoFarad =  $10^{-9}F$ ) capacitor is initially charged to a potential of 200 Volts, and then isolated from the charging source. How much charge is present on its plates? How much electrical energy is stored in the capacitor?

This capacitor is then connected across a  $100nF$  Capacitor. How much electrostatic energy is stored in each capacitor? Is the electrostatic potential energy the same as before? Why or why not?

Answers:  $10^{-6}C$ ;  $10^{-5}J$ ;  $1/9 \times 10^0J$ ; No. Resistive losses are required for equilibrium.