Physics 221

Final Examinations

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FINAL EXAM; PHYS 221 - 003/004; 10 December 2003

1. [25 points] A small particle of mass m and charge $+Q$ is suspended to a wall via a very light string of length L. The wall is electrically non-conducting, with a uniform charge per unit area $+\sigma$ (Coul/m^2) imbedded in it.

   - Using a diagram, show all the forces that act upon the mass m.
   - Determine the magnitude and direction of each force on m.
   - Obtain an expression for the angle θ at the location(s) of stable static equilibrium of the mass.

   Answer: $\tan\theta = Q\sigma/(2mg\varepsilon_o)$.

2. [25 points] An air-filled cylindrical capacitor is constructed with inner radius a, outer radius b, and length L. Note: Assume that $L \gg b$, and ignore fringing fields. (Why?) A total charge $+Q$ is placed on the outer conducting surface, and total charge $-Q$ on the inner conducting surface.

   - Determine the magnitude and direction of the electric field everywhere between the surfaces.
   - Determine the capacitance of this system.
   - Determine the electrostatic potential energy stored in the capacitor.
   - Show that exactly half of the electrostatic potential energy is stored in the region $a \leq r \leq \sqrt{ab}$, where r is the distance to the axis of the concentric cylinders.

   Answer: $E = Q/(2\pi\varepsilon_o Lr)$, inward toward wire;

   $2\pi\varepsilon_o L/(\log b/a)$; $Q^2\log(b/a)/(4\pi\varepsilon_o L)$.

3. [25 points] The earth's lower atmosphere contains negative and positive ions that are produced by radioactive elements in the soil and cosmic rays. In a certain region, the atmospheric electric field is 100 Volts/meter, directed vertically down. This field causes singly charged positive ions ($6\times10^{+8}$ ions per cubic meter) to move downward and singly charged negative ions ($6\times10^{+8}$ ions per cubic meter) to move upward. The electrical resistivity is measured to be $\rho = 2.0\times10^{+14}\Omega m$.

- Calculate the ion drift speed v_D, which is assumed to be the same for positive and negative ions.
- Determine the magnitude and direction of the net current density J, in Amp/m^2.

Answer: $0.0025 m/s$; $5 \times 10^{-13} A/m^2$.

4. [25 points] A straight conducting wire carrying a total current $2I$ splits into identical concentric semicircular arcs, each of which carries current I. Determine the magnetic field at C, the center of the semicircular arcs.

   Answer: 0.

5. [25 points] An inductor L and a resistor R are connected in parallel. The combination is then hooked in series with a resistor R, and inductor L, an ideal DC Voltage source providing Voltage V_0, and a switch S. The switch is closed at time $t = 0$.

   - Determine the current everywhere in the circuit just after the switch is closed.
   - Determine the steady-state current everywhere in the circuit.
   - Optional: Determine the current passing through the resistor on the bottom at all times.

   Answer: $0, 0$; $V_0/R, 0$,

6. [25 points] A plane electromagnetic wave with wavelength 5.0 meters travels in a vacuum in the positive y-direction, with an electric field $\vec{E}$ of maximum amplitude 200 Volts per meter, in the positive z-direction.

   - What is the frequency of the wave, in Hertz?
   - What is the maximum amplitude, as well as the direction, of the magnetic field $\vec{B}$ corresponding to this wave.
   - What is the time-average energy flow associated with this wave, expressed in Watts per meter squared?
   - If the wave falls on a perfectly absorbing surface of area 2.0 square meters, lying in the $(x - z)$ plane, what is the net force on this surface that is produced by the wave?

$60MHz$; $6.7\times10^{-7}T$; $50W/m^2$; $3\times10^{-7}N/m^2$.

Answer:

7. [Extra Credit; 10 points] In a standing wave experiment, an electrically driven tuning fork is attached to one end of a light string 1.0 meter in length. The small tuning fork is driven transversely to the direction of the string, with very small amplitude, at a frequency of 400 Hz. The string is 1.0 meters long, and its total mass is 5 grams.

   - Explain why, for resonant transverse oscillations, there is a node on at, or very close to, the location of the tuning fork. What is the boundary condition at the other end of the string? Why?
   - What tension [in Newtons] must be applied to the other end of the string [by hanging weights over a pulley, or by pulling], if the system undergoes resonant transverse oscillations in its fundamental mode?

   Answer: $3200N$.

FINAL EXAM; PHYS 221 - 003/004; 15 December 2004

1. [25 points] A charge $+Q$ is uniformly distributed along an insulator that forms a semi-circular arc, with radius R.

   - Determine the magnitude and direction of the electric field at the center of the arc.
   - Determine the magnitude and direction of the electric field at a distance r away from the center of the arc and on its axis. Assume that $r \gg R$, and keep only the leading non-vanishing term.

   Answers: away from arc $wkQ/(\pi R^2)$; kq/r^2.

2. [25 points] In a certain cyclotron an alpha particle moves in a circle of radius 3.0 m. The magnitude of the magnetic field is 4.0 T, perpendicular to the orbit of the alpha particle.

   - What is the frequency of rotation of the alpha particle, in Hertz?
   - What is the kinetic energy of the alpha particle, in electron Volts?

Note:

- mass of alpha particle: $m_\alpha = 6.2 \times 10^{-27}$ kg
- $m_\alpha c^2 = 3.7 \times 10^9$ eV
- $q_\alpha = 3.2 \times 10^{-19}$ Coul

Answers: $32.7 MHz$; $7.4 \times 10^9 eV$.

3. [25 points] A total amount of charge $+Q$ is distributed uniformly throughout the volume of a solid insulating sphere of radius a. A very thin concentric spherical shell of radius b ($b > a$) completely surrounds the insulating sphere. That thin shell also has a total amount of charge $+Q$ spread uniformly on its surface.

   - Determine the magnitude and direction of the electric field everywhere in space.
   - With the convention that the electrostatic potential V vanishes at infinity, determine that electrostatic potential V everywhere.

   Answers: $r < a : kQr/a^3, -kQr^2/(2a^3)$; $a < r < b : kQ/r^2, kQ/r + kQ/b$; $r > b : 2kQ/r^2, 2kQ/r$.

4. [25 points] Suppose that a parallel-plate capacitor has circular plates of radius 0.2 meters, with a plate separation of 0.005 meters. Suppose also that a sinusoidal voltage $V_0 sin\omega t$ is applied to the plates. The frequency of variation of the voltage is $f = 60$ Hz and $V_0 = 150$ Volts. Neglect fringing fields.

   - Determine the (time-dependent) magnetic field everywhere inside the capacitor, expressed in terms of r, the distance to the central axis.
   - What is the maximum electrical energy stored inside the capacitor, in Joules?
   - What is the maximum magnetic energy stored inside the capacitor, in Joules?

   Answers: $\omega = 377 Hz; 6.3 \times 10^{-11} T \cos \omega t$; $2.5 \times 10^{-6} J$; $7 \times 10^{-14} J$.

5. [25 points] A resistor of resistance $R = 50\Omega$ and an inductor of inductance $L = 0.01$ Henry are placed in series with each other, and then connected to a sinusoidal Voltage source with EMF given by $V_0 sin\omega t$. The maximum voltage of the source is $V_0 = 400$ Volts, and the oscillation frequency of the voltage is $f = 1000$ Hertz. The following questions refer to "steady state" conditions, when all transient effects are gone, and the current oscillates periodically with the same frequency as the EMF.

   - Determine the maximum current passing through the resistor, in Amps.
   - Determine the average power lost through the resistor R, in Watts.
   - Determine the maximum magnetic energy stored in the inductor, in Joules.

   Answer: $4.98A$; $620W$; $0.12J$.

6. [25 points] A capacitor with capacitance $C = 2.00\mu F$ is "leaky", in the sense charge travels through the capacitor, so that the capacitor becomes neutralized over the course of time. Suppose that the capacitor is given an initial charge $Q_0 = 3.0\mu C$, and the charge is reduced to $Q_1 = 1.0\mu C$ in 50 seconds.

   - What was the initial voltage across the capacitor, in Volts?
   - Determine the "effective resistance" across the plates of the capacitor, in Ohms.
   - Determine the total time elapsed (in seconds) at which 99.99 percent of the charge has leaked off the plates of the capacitor.

   Hint: Make an RC circuit model.

   Answers: $1.5V$; $23M\Omega$; $420s$.

7. [Extra Credit; 10 points] A circus trapeze artist swings on a very light, yet very strong bar trapeze, which consists of a horizontal bar that is connected to two identical ropes that pivot at the top of the arena. The period for one (rather small amplitude) oscillation is measured to be 8.0 seconds, when the trapeze artist is lying across the bar, with her center of mass at the bar. On the other hand, when the trapeze artist stands upon the bar, the period is measured to be 7.75 seconds.

Determine the length of the ropes from pivot point to the bar, as well as the distance between the center of mass of the trapeze artist and the bottom of her feet. Express both answers in meters.

Answers: 15.88m; 0.98m.

FINAL EXAM; PHYS 221 - 007/008; 13 December 2004

1. [25 points] A point charge $+Q$ is located at the center of an insulating sphere of radius R. In addition, a total charge of $-Q$ is distributed uniformly through the volume of the insulating sphere.

   - Determine the magnitude and direction of the electric field everywhere in space.
   - Determine the electrostatic potential everywhere, with the convention that it is defined to be zero at infinity.

   Answers: inside (out) $kQ(1/r^2 - r/R^3), kQ(1/r + r^2/(2R^3) - 3/(2R)$; outside $0, 0$

2. [25 points] A small permanent magnet is pivoted about its center of mass, so that it is free to rotate in a horizontal plane. The magnetic dipole moment of the magnet is $M = 2 \times 10^{-4} Am^2$, and its moment of inertia about this vertical axis of rotation is $I = 3 \times 10^{-5} kgm^2$. A horizontal magnetic field of magnitude $B = 4 \times 10^{-3}$ T is present. Ignore gravity.

   - Determine the torque acting on the magnet when it lies at a small angle θ (counterclockwise, as seen from above) to the direction of the magnetic field.
   - Determine the frequency of small horizontal oscillations of the magnet about its equilibrium position, in Hertz.

   Answers: $-MB\theta$; 0.025Hz.

3. [25 points] A capacitor with capacitance $C_1 = 2.00\mu F$ is charged to capacity by a DC battery with of EMF of 40 Volts.

   - Determine the charge on the capacitor, in Coulombs.
   - Determine the electrical energy stored in the capacitor, in Joules.

The fully charged capacitor is then disconnected from the battery, and then connected across a second capacitor, with capacitance $C_2 = 4.00\mu F$.

- Determine the final charge on each of these capacitors, in Coulombs.
- Determine the energy stored in each capacitor, in Joules.
- Is the total electrical energy the same as before? Explain why or why not.

Answers: $8\times10^{-5}C, 1.6\times10^{-3}J; 2.7\times10^{-5}C, 5.3\times10^{-5}C; 3.6\times10^{-4}J, 1.8\times10^{-4}J$ No, resistance necessary for equililbrium.

4. [25 points] Two identical batteries, which produce direct current at an EMF of 12 Volts, each have an internal resistance $r = 1.0\Omega$. The batteries are placed in parallel, and then attached across an inductor with inductance $L = 0.1H$. The switches are closed simultaneously at $t = 0$ (sec).

   - For $t > 0$, determine the current passing through the inductor, the magnetic energy stored in the inductor, the power provided by each battery (chemical energy converted into electrical energy), and the power dissipated as heat within each battery.
   - What are the steady state values of these quantities, which are present after a long time?

   Answers:

   $$12(1-e^{-5t})A;\ 28.8(1-e^{-5t})^2J;\ 144(1-e^{-5t})^2W$$

   $12A$; $28.8J$; $144W$.

5. [25 points] A plane electromagnetic wave with wavelength 8.0 meters travels in a vacuum in the positive y-direction, with an electric field $\vec{E}$ of maximum amplitude 50 Volts per meter, in the positive z-direction.

   - What is the frequency of the wave, in Hertz?
   - What is the maximum amplitude (in Tesla), as well as the direction, of the corresponding magnetic field $\vec{B}$ for this wave?
   - What is the time-averaged energy flux associated with this wave, expressed in Watts per square meter?

- If the wave falls on a perfectly absorbing surface of area 3.0 square meters lying in the $(x - z)$ plane, what is the average force produced by the wave on this surface, in Newtons?

<u>Answers</u>: $37.5 MHz$; $1.7 \times 10^{-5} T$, $3.3 W/m^2$; $3.3 \times 10^{-8} N$.

6. [25 points] Three charges of magnitudes $+Q$, $+2Q$, and $-3Q$, respectively lie at the vertices of an equilateral triangle of side a? Take $a = 0.1$ meters, and $Q = 2 \times 10^{-6}$ Coulombs.

   - (a) How much net energy (in Joules) would be required to assemble this configuration of charges by bringing them in from infinity?
   - (b) Determine the magnitude and direction of the net electrostatic force on the negative charge, caused by the two positive charges?

   <u>Answers</u>: $-2.52 J$; $29 N, 11^o$ toward $+Q$ relative to altitude at $-3Q$.

7. [Extra Credit; 10 points] A wire coil of 200 turns is wrapped tightly around a long wire solenoid. In turn, the inner radius of the solenoid is 0.05 meters, and there are 1000 turns per meter on it. Determine the mutual inductance M of this system, in Henries.

   <u>Answer</u>: $3.95 \times 10^{-3} H$.

FINAL EXAM; PHYS 221 - 003/004; 12 May 2005

1. [25 points] Masses Hung by a Vertical Spring

   A particle of mass m is attached to a light ideal spring of spring constant k, which is hung at the ceiling. At equilibrium the spring is stretched by a distance D beyond its natural length.

   When the mass m is pulled down below its equilibrium position and released, the mass-spring system undergoes vertical oscillations with a period of 1.5 seconds. When an additional mass $M = 4.0$ kilograms is then attached to the spring, the spring stretches by an additional distance D_0 at equilibrium. The period of vertical oscillations about the new equilibrium point is 2.54 seconds.

   Determine the following quantities:

   - The original mass m, in kilograms.

- The spring constant, k, in Newtons/meter.
- The distances D and D_0, in meters.

Answers: $2.25kg$; $40N/m$; $0.55m$; $0.98m$.

2. [25 points] Spherical Dielectric Capacitor

   Two thin conducting metallic spherical shells, with radii a and b, respectively ($a < b$) are located concentrically. An insulating material of dielectric constant κ fills the region between the two spheres. A total charge $+Q$ is placed on the outer conducting shell of this capacitor, whereas a total charge $-Q$ is located on the inner conducting shell.

   Determine the following quantities (SI units):

   - The capacitance of the system, C.
   - The magnitude and direction of the net electric field everywhere inside the dielectric material that lies between the plates of the capacitor.
   - The difference in electrostatic potential ΔV between the shells. Which shell is at higher potential?
   - The total electrostatic potential energy stored in this capacitor.

   Answers: $4\pi\varepsilon_0\kappa ab/(b-a)$; $-kQ/(\kappa r^2$ inward;
   $kQ/\kappa b-a/(ab)$; $kQ^2/(2\kappa)(b-a)/(ab)$.

3. [25 points] Hall Effect

   A conducting metallic strip is 10 cm long, 1 cm wide, and 0.1 cm thick. It moves with constant velocity v in the "long" direction of the strip through a uniform magnetic field $\vec{B} = 1$ milli-Tesla, directed parallel to the thin side of the strip. A potential difference of 4 micro-Volts is measured across the width of the strip. Calculate the speed v of the strip, in meters/second.

   Answer: $0.4m/s$.

4. [25 points] Inductor with Toroidal Geometry

   A hollow torus of side s, with square cross section, and with inner radius a is formed wrapped with N turns of conducting wire – all in the same direction of rotation around this toroidal inductor. A steady current I flows in the wire.

- What is the magnetic field everywhere inside this torus, expressed in terms of the parameters (s, a, N) and the distance r to the central axis of the torus?
- What is the inductance of this toroidal inductor?
- How much (magnetic) potential energy is stored in this toroidal inductor?

Answers: $\mu_0 NI/(2\pi r)$; $\mu_0 N^2 s/(2a)\log(1+s/a)$; $1/2LI^2$.

5. [25 points] Oscillating LC Circuit

   An oscillating LC circuit consists of a $2.0nF$ capacitor and a $4.0mH$ coil. Under steady state operating conditions, there is a maximum voltage of $3.0V$ across the capacitor.

   - What is the maximum charge on the capacitor, in Coulombs?
   - What is the period of oscillation of the charge on the capacitor, in seconds?
   - What is the maximum current through the inductor, in Ampères?
   - What is the maximum energy stored in the magnetic field of the coil, in Joules?

   Answers: $1.8\times 10^{-5}s$; $2.1\times 10^{-3}A$; $9\times 10^{-9}J$.

6. [25 points] Plane Electromagnetic Wave

   A plane electromagnetic wave with wavelength 5.0 meters travels in a vacuum in the positive y-direction, with an electric field $\vec{E}$ of maximum amplitude 30 Volts per meter, in the positive z-direction.

   - What is the frequency of the wave, in Hertz?
   - What is the maximum amplitude (in Tesla), as well as the direction, of the corresponding magnetic field $\vec{B}$ for this wave?
   - What is the time-averaged energy flux associated with this wave, expressed in Watts per square meter?
   - If the wave falls on a perfectly absorbing surface of area 4.0 square meters lying in the $(x-z)$ plane, what is the time-averaged force produced by the wave on this surface, in Newtons?

Answers: $60mHz$; $10^{07}T$, x-direction; $1.2W/m^2$; $1.6\times10^{-8}N$.

7. [Extra Credit; 10 points] Lightning Precautions

   Discuss the rationale behind the following list of safety precautions during a thunderstorm or during significant lightning activity, using the basic principles of electricity:

   - Do not stand outside under a large tree.
   - Do not stand outside in an open field; i.e., on a golf course.
   - Do not get into the water, or stay in the water, at an outdoor swimming pool. (What about indoor pools?)
   - Do not remain outside on a small boat offshore in open water, either on a lake, a river, or the ocean. (What about surfing near the shore, at a beach?)
   - It is considered safe to remain inside an automobile on the open road, but do not touch metallic objects, such as the key in the ignition switch or the housing for the cigarette lighter. (Is it safe to be inside an airplane during a lightning strike?)
   - Do not get out of the car just after a nearby lightning strike.

   Answers: Along with water, our bodies are good conductors of electricity, compared with our surroundings. Lightning follows the path of least resistance to ground.

FINAL EXAM; PHYS 221 - 007/008; 12 December 2005

1. [25 points] The oscillations of a tuning fork at a frequency of 500 Hz set up a transverse standing wave on a string clamped at both ends. The wave speed for the spring is 200 meters/second. The standing wave has four loops, with a maximum displacement from equilibrium of 1.0 mm.

   - What is the length L of the string?
   - Write an equation for the displacement of the string as a function of x, the position along the string, along with the time t. Assume that the string is unstretched at time $t = 0$.

   Answers: $0.8m$; $A\sin kx\sin\omega t$; $k = 15.7m^{-1}$; $\omega = 3140r/s$.

2. [25 points] Two (positive) point charges of magnitude $+Q$ are placed symmetrically along the z-axis, at points $z = \pm d$. A thin insulating wire is placed along the x-axis, passing through the origin. A bead of mass m and (negative) charge $-q$ slides freely along the wire. Neglect gravity.

   - Determine the net force on the bead, when it is located at a distance x from the origin.
   - Determine the frequency of small oscillations of the bead about the origin.

   Answers:

$$-\frac{2kQqx}{(d^2+x^2)^3/2};\ \frac{1}{2\pi}\sqrt{\frac{2kQq}{md^3}}\ .$$

3. [25 points] The drum of a photocopying machine is a metallic cylinder of length 40 cm and diameter 20 cm. For proper functioning an electric field of 2×10^5 N/C must be present just outside lateral surface of the drum.

   - How much charge (in micro-Coulombs) must be deposited on the lateral surface of the drum?
   - For the desktop version of the machine, the drum length is 20 cm and the drum diameter is 10 cm. How much charge (in micro-Coulombs) must be deposited on its lateral surface?

   Answers: $0.44\mu C$; $0.11\mu C$.

4. [25 points] A camera flash lamp circuit has a capacitor (capacitance C) and resistor (resistance $R = 200$ kiloOhms) in series connected across a 12 Volt DC battery. The lamp fires when the potential across the capacitor reaches 8 Volts.

   - What capacitance is required if the lamp is to fire 2 seconds after the switch is closed?
   - How much energy is stored in the capacitor just before the lamp is fired?

   Answers: $9.1 \times 10^{-6}F$; $2.9 \times 10^{-4}J$.

5. [25 points] An electron with a kinetic energy of 4000 electron Volts is moving in a direction perpendicular to a uniform magnetic field of magnitude 0.3 Tesla. Find the speed of the electron, and radius of its circular orbit.

   Note:

   - Mass of electron $m_e = 9.1 \times 10^{-31}$ kg
   - $m_e c^2 = 5.1 \times 10^5$ eV
   - $q_e = -1.6 \times 10^{-19}$ Coul
   - $1eV = 1.6 \times 10^{-19} J$

   Answers: $3.76 \times 10^7 m/s$; $0.7mm$.

6. [25 points] A $24V$ battery is connected with a resistor of resistance $R = 12\Omega$, It is then connected in parallel with another $24V$ battery and a resistance $r = 6\Omega$. Calculate the magnitude and direction of the current generated by each battery, the power provided by each battery, and the power dissipated in each resistor.

   Answers: 0, $4A$; $96W$; 0, $96W$.

7. [Extra Credit; 10 points] At a certain place the earth's magnetic field is 4×10^{-5} Tesla. A flat circular coil of wire of radius 0.1 meters has 150 turns of wire, and lies in a circuit with a total resistance of 20Ω. The coil initially lies in a plane perpendicular to the direction of the magnetic field of the earth. It is then "flipped" by a half-revolution about a diameter. How much total charge flows through the coil circuit during this "flip"?

   Answer: $18.9\mu C$.

FINAL EXAM; PHYS 221 - 003/004; 14 December 2005

1. [25 points] A block of mass 2.0 kg is sitting at rest with respect to the horizontal surface of a "shake table". The shake table, in turn, is driven back and forth horizontally in simple harmonic motion (SHM) at a frequency of 3.0 Hertz. The coefficient of static friction between the block and the horizontal surface is $\mu_s = 0.4$.

   - How great is the critical amplitude of SHM just before the block begins to slip on the horizontal surface? At what point on its motion does the block begin to slip in this critical case?

- Determine the magnitude and direction of all the forces acting on the block, (including the earth's gravity) for the critical case.

Answers: $11cm$ maximum displacement; $f_s = 7.84N$, $mg = N = 19.6N$.

2. [25 points] Three charges of magnitude $+Q$, $+2Q$, and $-4Q$, respectively, lie at vertices of an equilateral triangle of side a. Take $a = 0.2$ meters, and $Q = 3 \times 10^{-7}$ Coulombs.

   - How much energy (in Joules) would be required to assemble this configuration of charges by bringing them in from infinity?
   - Determine the magnitude and direction of the electric field at the center of the line joining the two positive charges.
   -

   Answers: $-4.05 \times 10^{-2} J$; $4.5 \times 10^5 V/m$; 37^o from negative charge.

3. [25 points] A point charge $+Q$ is placed at the center of an insulating sphere of radius R. In addition, a net charge $-Q$ is imbedded uniformly throughout the volume of the insulating sphere.

   - Determine magnitude and direction of the electric field inside the sphere.
   - Determine magnitude and direction of the electric field outside the sphere.
   - With the convention that the electrostatic potential vanishes at infinity, determine the electrostatic potential everywhere — inside the sphere, as well as outside the sphere.

   Answers: $kQ(1/r^2 - r/R^3$ out; 0; $kQ(1/r + r^2/(2R^3) - 3/(2R^2)$; 0.

4. [25 points] The current in an LR circuit attached across a 10 Volt battery reaches one-third of its steady-state value ten seconds after the switch is closed. The resistance of the resistor is 10^5 Ohm.

   - Determine the time constant for this circuit.
   - Determine the inductance of the inductor (in Henries).
   - Determine the steady-state current, in Amps.

Answers: $24.7s$; $2.6 \times 10^6 H$; $10^{-4} A$.

5. [25 points] A parallel plate capacitor with circular plates of radius $R = 0.2$ meters is being discharged. A circular loop of radius $r = 0.1$ meters, centered on the central axis of the capacitor, halfway between the plates. A displacement current of 2 Amp passes through that circular loop.

   - Determine the rate of change of the electric field between the plates.
   - Determine the rate at which the capacitor is being discharged — the discharge electric current.

   Answer: $7.2 \times 10^1 2V/(ms)$; $8A$.

6. [25 points] A plane electromagnetic wave of frequency $200 MHz$ travels in a vacuum in the positive x-direction, with its electric field $\vec{E}$ of maximum amplitude of 2.0 Volts per meter, in the positive y-direction.

   - What is the wavelength of the wave, in meters?
   - What is the maximum amplitude of the magnetic field $\vec{B}$ (in Tesla) for this wave? What is its direction?
   - What is the time-averaged energy flow associated with this wave, in Watts per square meter?

   Answers: $1.5m$; $6.67 \times 10^{-7} T$ $+y$-direction; $5.3 \times 10^{-3} W/m^2$.

7. [Extra Credit; 10 points] A wire coil of 150 turns is wrapped around central part of a long wire solenoid. In turn, the inner radius of the solenoid is $0.10m$, and there are 500 turns per meter on it. Determine the mutual inductance M of this system, in Henries.

   Answer: $2.96 \times 10^{-5} H$.

FINAL EXAM; PHYS 221 - 003/004; 11 May 2006

1. [25 points] Vibrating String

   A string 80 cm long, with a mass of 3 grams, is held under a tension of 120 Newtons, with its ends held fixed. The string undergoes transverse vibrations in its fundamental mode, with the maximum amplitude of 1 cm.

- Determine the wavelength and frequency of the vibrating string, as well as the speed of these transverse vibrations.
- Determine the total energy stored in the vibrating string, in Joules.
- The speed of sound in the air surrounding the string is 350 meters/second. Determine the wavelength and frequency of the sound in the air produced by the string.

Answers: $1.6m$, $112Hz$; $179m/s$; $0.076J$; $3.13m$, $112Hz$.

2. [25 points] Cylindrical Dielectric Capacitor

   Two long hollow conducting cylinders of length L are placed concentrically. The inner cylinder has radius a, and the outer cylinder has radius b: $b > a$. A dielectric material of dielectric constant κ fills the region between the cylinders. A charge $+Q$ is present on the surface of the inner conductor, whereas a charge $-Q$ lies on the outer conductor. Neglect fringing fields.

   - Determine the electric field (magnitude and direction) everywhere between the conducting plates.
   - Determine the potential difference between the conducting plates.
   - Determine the capacitance per unit length of this system.
   - Determine the electrostatic potential energy per unit length stored in this capacitor.

   Answers: field outward

$$\frac{Q}{2\pi\varepsilon_0 L\kappa}\frac{1}{r};\ \frac{Q}{2\pi\varepsilon_0 L\kappa}\log(b/a);\ \frac{2\pi\varepsilon_0\kappa}{\log(b/a)};\ \frac{Q^2}{4\pi\varepsilon_0\kappa L^2}\log(b/a)$$

3. [25 points] Configuration of Point Charges

   Four point charges are placed at points in the $x-y$ plane as indicated in the table below:

Charge	Location
$+Q$	$(a,0)$
$+2Q$	$(0,a)$
$-3Q$	$(0,-a)$
$-4Q$	$(-a,0)$

- Determine the magnitude and direction of the electric field at the point (0,0).
- Determine the electric potential at the point (0,0).
- How much electrostatic potential energy is stored in this configuration of charges?

Answers: $5\sqrt{2}kQ/a$; 225^o from x-axis; $-4kQ/a$; $kQ^2/a(3/\sqrt{2}-8)$.

4. [25 points] Defibrillator RC Circuit

   A cardiac defibrillator consists of a capacitor bank of (equivalent) capacitance $C = 150$ microFarads, which is charged to a potential of $V = 3000$ Volts.

   Determine the electrostatic potential energy stored in the capacitor, in Joules.

   The capacitor is discharged through human tissue, with an equivalent resistance $R = 500,000$ Ohms.

   - What is the total current initially passing through the body (in Amps)?
   - How long does it take for the potential across the capacitor to come down to a "safe"level of 10 Volts?

   Answers: $675J$; $0.006A$; $428s$.

5. [25 points] Mass Spectrometer

   A singly-ionized Carbon 12 atom (mass $12AMU$) is accelerated through a potential of $10,000$ Volts, and then put into a mass spectrometer, with a uniform magnetic field of magnitude 2 Tesla perpendicular to the direction of motion of the atom.

   - Determine the radius of the circular path followed by the carbon atom.
   - How accurately must that radius be measured to distinguish Carbon 12 from Carbon 14 (mass $14AMU$) in this spectrometer?

   Note:

   - $e_0 = 1.6 \times 10^{-19}C$
   - $1AMU = 931MeV/c^2 = 1.66 \times 10^{-27}kg$

Answers: $0.025m$; 8 percent.

6. [25 points] LC Oscillations

   Consider an oscillating LC circuit with inductance $L = 40mH$ and capacitance $C = 60\mu F$. Initially, at $t = 0$, there is a current of 10 A in the inductor, with no charge on the plates of the capacitor.

   - At what time will the capacitor become fully charged for the first time?
   - How much charge is there on the plates of the capacitor when this occurs?
   - What is the Voltage across the plates of the capacitor at this time?
   - How much energy is stored in the capacitor at this time?
   - What is the maximum energy stored in the inductor? When does it occur?

   Answers: $2.4ms$; $0.015C$; $200V$; $2J$; $2J$; initially.

7. [Extra Credit; 10 points] Lucite^{TM} Light Pipe

   Light is incident at the end of a long cylindrical pipe made out of Lucite, with index of refraction $n = 1.50$, and surrounded by air $n = 1$. The incident light strikes the end at its center, at an angle ϕ to the axis of the cylinder. Determine the minimum value of ϕ that is necessary to guarantee total internal reflection of light along the lateral surface of the pipe.

   Answer: all angles are OK.