Physics 123

Quizes and Examinations

Spring 2002

Porter Johnson

"Physics can only be learned by thinking, writing, and worrying."
-David Atkinson and Porter Johnson (2002)

"There is no royal road to geometry."
-Euclid

Quizes: Spring 2002

1. PHYS 103 - 001 QUIZ 1 05 February 2002

   An engineer standing on a bridge throws a penny straight up with a speed of $v_0 = 20$ meters/second, from a height $h = 100$ meters above the water. [Neglect air resistance.]

   - What is the speed of the penny when it hits the water below?
   - How long does it take the penny to hit the water?

   Solution:

   The final speed can be calculated from the relation

$$\begin{aligned} v^2 &= v_0^2 + 2gh \\ &= 20^2 + 2(10))100 = 2400 \\ v &= 49\, m/s \end{aligned}$$

   The time of travel can be calculated from the relation

$$\begin{aligned} h &= v_0 t + \frac{1}{2} g t^2 \\ 5t^2 - 20t &= 100 \\ t^2 - 4t - 20 &= 0 \\ t &= 2 \pm \sqrt{24} = 6.9\, sec \end{aligned}$$

   We have chosen the positive square root in the last relation, since the time of travel must be positive.

   Alternatively, we may calculate the travel time t from the relation

$$\begin{aligned} h &= \frac{v + v_0}{2} t \\ 100 &= 14.5t \\ t &= 6.9\, sec \end{aligned}$$

2. PHYS 123 - 002 QUIZ 1 05 February 2002

An engineer standing on a bridge throws a penny straight down with a speed of 20 meters/second, from a height of 100 meters above the water. [Neglect air resistance.]

- What is the speed of the penny when it his the water below?
- How long does it take the penny to hit the water?

Solution:

The final speed can be calculated from the relation

$$\begin{aligned} v^2 &= v_0^2 + 2gh \\ &= 20^2 + 2(10))100 = 2400 \\ v &= 49\ m/s \end{aligned}$$

The time of travel can be calculated from the relation

$$\begin{aligned} h &= v_0 t + \frac{1}{2} g t^2 \\ 5t^2 + 20t &= -100 \\ t^2 + 4t + 20 &= 0 \\ t &= -2 \pm \sqrt{24} = 2.9\ sec \end{aligned}$$

We have chosen the positive square root in the last relation, since the time of travel must be positive.

Alternatively, we may calculate the travel time t from the relation

$$\begin{aligned} h &= \frac{v + v_0}{2} t \\ 100 &= 39.5t \\ t &= 2.9\ sec \end{aligned}$$

3. PHYS 123 - 001 QUIZ 2 19 February 2002

A rock, thrown out of the open window of a building that is 100 meters above the (level) ground, is given an initial speed of 30 meters per second, traveling horizontally. [You make take $g = 10m/sec^2$, if you wish.]

- Draw a diagram showing the trajectory of the rock from its launch point to the ground.
- What is the (vector) velocity of the rock when it hits the ground?
- How long (in seconds) does it take the rock to hit the ground?

Solution:

The trajectory or the rock is parabolic, starting out horizontally at the window and becoming concave downward.

The coördinates of the rock at time t are

$$
\begin{aligned}
y &= 0 - \frac{1}{2} g t^2 \\
x &= v_0 t
\end{aligned}
$$

At the time of flight, we obtain $-100 = -5t^2$, or $t = \sqrt{20} = 4.4\ sec$. The horizontal position at which the rock strikes the ground is $x = v_0 t = (30)4.4 = 130\ m$.

The velocity components of the rock when it strikes the ground are

$$
\begin{aligned}
v_y &= -gt = -44\ m/s \\
v_x &= v + 0 = 30\ m/s
\end{aligned}
$$

The rock thus strikes the ground at a speed of 54 m/s, at an angle of below the horizontal.

4. PHYS 123 - 002 QUIZ 2 21 February 2002

   A rock, thrown out of the open window of a building that is high above the (level) ground, is given an initial speed of 30 meters per second, traveling horizontally. It hits the ground 3.0 seconds after the launch. [You make take $g = 10\ m/sec^2$, if you wish.]

   - Draw a diagram showing the trajectory of the rock from its launch point to the ground.
   - What is the (vector) velocity of the rock when it hits the ground?

- From what height (in meters) was the rock launched?

Solution:

The trajectory or the rock is parabolic, starting out horizontally at the window and becoming concave downward.

In a time $t = 3\ sec$, the rock travels a horizontal distance $x = v_x t = 30 \cdot 3 = 90\ m$, and a vertical distance $h = 1/2gt^2 = 5(3)^2 = 45\ m$.

The velocity components when it strikes the ground are $v_x = 30\ m/s$ and $v_y = 10 \cdot 3 = 30\ m/s$. It thus moves with a speed of 42 m/s, at an angle 45^o below the horizontal.

5. PHYS 123 - 001 QUIZ 3 05 March 2002

A smaller box of mass m_1 is placed on top of a larger box of mass m_2, which, in turn, is placed on a frictionless table, as shown. The coefficient of static friction of the smaller box on the rough surface between the two boxes is given by μ_s. A horizontal force F is applied to the top box.

- Draw a diagram showing all the forces acting on each box.
- Determine the acceleration of the bottom box, assuming that the top box does not slip on the surface of the bottom box.
- In terms of the parameters m_1, m_2, μ_s, and g, determine the maximum force F that be applied before the top box slips.

Solution:

The forces on the top block are its weight $m_1 g$ (downward), the normal force from the second block $N_1 = mg$ (upward), the applied force F (to the right), and static friction f_s (to the left).

The vertical forces on the bottom block are its weight $m_2 g$ (downward), the normal force from the top block $N_1 = m_1 g$ (downward), and the normal force from the table $N_2 = (m_1 + m_2)g$ (upward). In addition, there is static friction from the upper block f_s to the right.

We apply Newton's second law to the horizontal motions to obtain

$$\begin{aligned} F - f_s &= m_1 a \\ f_s &= m_2 a \end{aligned}$$

Thus, $F = (m_1 + m_2)a$, or $a = F/(m_1 + m_2)$.

Consequently, for the critical case, $f_s = \mu_s m_1 g = m_2 a$, so that $a = \mu_s g m_1/m_2$.

The maximum applied force is $F = \mu_s g(m_1 + m_2)m_1/m_2$.

6. PHYS 123 - 002 QUIZ 3 05 March 2002

   A smaller box of mass m_1 is placed on top of a larger box of mass m_2, which, in turn, is placed on a frictionless table. The coefficient of static friction of the smaller box on the rough surface between the two boxes is given by μ_s. A horizontal force F to the right is applied to the bottom box.

   - Draw a diagram showing all the forces acting on each box.
   - Determine the acceleration of the bottom box, assuming that the top box does not slip on the surface of the bottom box.
   - In terms of the parameters m_1, m_2, μ_s, and g, determine the maximum force F that be applied before the top box slips.

   Solution:

   The forces on the top block are its weight $m_1 g$ (downward), the normal force from the second block $N_1 = mg$ (upward), and static friction f_s (to right).

   The vertical forces on the bottom block are its weight $m_2 g$ (downward), the normal force from the top block $N_1 = m_1 g$ (downward), and the normal force from the table $N_2 = (m_1 + m_2)g$ (upward). In addition, there is static friction from the upper block f_s to the left, and the applied force F to the right.

   We apply Newton's second law to the horizontal motions to obtain

$$\begin{aligned} f_s &= m_1 a \\ F - f_s &= m_2 a \end{aligned}$$

   Thus, $F = (m_1 + m_2)a$, or $a = F/(m_1 + m_2)$.

   Consequently, for the critical case, $f_s = \mu_s m_1 g = m_1 a$, so that $a = \mu_s g$.

   The maximum applied force is $F = \mu_s (m_1 + m_2)g$.

7. PHYS 123 - 001 QUIZ 4 28 March 2002

   An $m = 10\ kg$ block is released from rest on a $\theta = 30^o$ frictionless incline. Below the block on the incline there is a spring, which can be compressed by 2.0 cm by a force of 200 Newtons. The block stops momentarily after it compresses the sprng by $d = 6.0\ cm$.

   - How far does the block move up the incline from its initial rest position to this stopping point?
   - What is the speed of the block just as it touches the spring?

   Solution:

   The spring constant is $k = 200/(0.05) = 10^4\ N/m$. If the ball slides a distance ℓ before coming to the spring, then it follows from conservation of mechanical energy that

$$\begin{aligned} 0 &= \frac{1}{2}kd^2 - mg(\ell + d)\sin\theta \\ 10(10)(\ell + 0.06)\frac{1}{2} &= \frac{1}{2}(10^4)(0.06)^2 \\ 50(\ell + 0.06) &= 18 \\ \ell &= 0.30\ m \end{aligned}$$

   According to energy conservation, the speed of the block when it touches the spring is

$$\begin{aligned} \frac{1}{2}mv^2 &= mg\ell\sin\theta \\ v^2 &= 2(10)(0.3)\frac{1}{2} = 3.0 \\ v &= 1.7\ m/s \end{aligned}$$

8. PHYS 123 - 002 QUIZ 4 28 March 2002

   An $m = 10\ kg$ block is launched up a $\theta = 30^o$ frictionless incline, using a spring that is compressed a distance of $x = 10\ cm$. The spring can be compressed by 1.0 cm by a force of 200 Newtons. The block stops momentarily after it travels some distance up the incline.

- How far does the block move up the incline from its initial rest position to this stopping point?
- What is the speed of the block just after it leaves the spring?

Solution:

The spring constant is $k = 200/(0.01) = 2\times 10^4\ N/m$. We use conservation of mechanical energy to determine the distance ℓ tht the block goes beyond the string:

$$
\begin{aligned}
\frac{1}{2}kx^2 + 0 + 0 &= 0 + 0 + mg\ell\,\sin\theta \\
\frac{1}{2}(2\times 10^4)(0.1)^2 &= (10)(10)\ell\frac{1}{2} \\
100 &= 50\ell \\
\ell &= 2\ m
\end{aligned}
$$

The speed of the block when it leaves the spring is given by

$$
\begin{aligned}
\frac{1}{2}mv^2 &= \frac{1}{2}kx^2 \\
v^2 &= (2\times 10^4)(0.1)^2/10 = 20 \\
v &= 4.5\ m/s
\end{aligned}
$$

9. PHYS 123 - 001 QUIZ 5 09 April 2002

A particle of mass $m = 1.0\ kg$, which is initially moving freely in the horizontal direction with a speed of $v_0 = 10$ meters/second, collides elastically with a target particle of equal mass, and then travels at an angle of $\theta = 37^o$ to the initial direction.

- Determine the speed of the incident particle, after the collision.
- Determine the speed and direction of motion of the target particle.

Solution:

Let v be the final speed of the projectile, V the final target speed, and ϕ the angle of travel of the target relative to the $+x$-axis. The laws of conservation of momentum and energy are

$$\begin{aligned} mv_0 &= mv\cos\theta + mV\cos\phi \\ 0 &= mv\sin\theta - mV\sin\phi \\ \frac{1}{2}mv_0^2 &= \frac{1}{2}mv^2 + \frac{1}{2}mV^2 \end{aligned}$$

The momentum conservation relations may be written

$$\begin{aligned} V_x &= v_0 - v\cos\theta \\ V_y &= v\sin\theta \end{aligned}$$

Substitute these into the energy conservation relation $v^2 + V^2 = v_0^2$ to obtain

$$\begin{aligned} V^2 = v_0^2 - v^2 &= (v_0 - v\cos\theta)^2 + (v_0\sin\theta)^2 \\ -2v^2 &= -2vv_0\cos\theta \\ v &= v_0\cos\theta = (10)\cos(37^o) = 8\ m/s \end{aligned}$$

The target particle thus travels at speed $V = \sqrt{v_0^2 - v^2} = 6\ m/s$. Its direction is given by the relation $\sin\phi = v\sin\theta/V = 4/5$, or $\phi = 53^o$.

10. PHYS 123 - 002 QUIZ 5 11 April 2002

A particle of mass $m = 1.0\ kg$, which is initially moving freely in the horizontal direction with a speed of $v_0 = 10$ meters/second, collides elastically with a target particle of equal mass, and then travels at a speed of $v = 6$ meters/second.

- Determine the direction of motion of the incident particle after the collision, relative to its direction of motion before the collision.
- Determine the speed and direction of motion of the target particle.

Solution:

Let v be the final speed of the projectile, V the final target speed, and ϕ the angle of travel of the target relative to the $+x$-axis. The laws of conservation of momentum and energy are

$$\begin{aligned} mv_0 &= mv\cos\theta + mV\cos\phi \\ 0 &= mv\sin\theta - mV\sin\phi \\ \frac{1}{2}mv_0^2 &= \frac{1}{2}mv^2 + \frac{1}{2}mV^2 \end{aligned}$$

From the energy consevation relation we obtain $V = \sqrt{v_0^2 - v^2} = \sqrt{10^2 - 6^2} = 8\ m/s$. From the momentum consevation we obtain

$$\begin{aligned} v_x^2 + v_y^2 &= (v_0 - V\cos\phi)^2 + (V\sin\phi)^2 \\ 36 &= 100 + 64 - 160\cos\phi \\ \cos\phi &= 128/160 = 0.8 \\ \phi &= 37^o \end{aligned}$$

11. Phys 123-001 QUIZ 6 25 April 2002

A particle of mass m is hung by a massless string, which is wrapped over a frictionless massive pulley. The string is attached to a mass M that is initially at rest on a frictionless table. The pulley has radius R and moment of inertia I about its central axis. It rotates freely about its central axis. Assume that the string does not slip on the pulley.

- Show all forces acting on the pulley and the two masses.
- Determine the downward acceleration of the mass m.
- Determine the tensions in the string, both above and below the pulley.

Solution:

The net force on the mass M on the table is the string tension T_1, so that $T_1 = Ma$. The net (clockwise) torque on the pulley is $(T_2 - T_1)R$, so that $(T_2 - T_1)R = I\alpha = Ia/rR$. The net downward force on the mass m is $mg - T_2$, so that $mg - T_2 = ma$. In summary:

$$\begin{aligned} T_1 &= Ma \\ (T_2 - T_1) &= \frac{I}{r^2}a \\ mg - T_2 &= ma \end{aligned}$$

We add these equations to obtain

$$a = \frac{mg}{m + M + I/R^2}$$

Substituting this result, we obtain the tensions:

$$\begin{aligned} T_1 &= \frac{M}{m + M + I/R^2} mg \\ T_2 &= \frac{M + I/R^2}{m + M + I/R^2} mg \end{aligned}$$

12. PHYS 123 - 002 QUIZ 6 25 April 2002

    A particle of mass m is hung by a massless string, which is wrapped over a frictionless massive pulley. The string is attached to an equal mass m that is initially at rest on a frictionless table, as shown. The pulley has radius R, and it rotates freely about its central axis. Assume that the string does not slip on the pulley. The downward acceleration of the particle is measured to be $g/4$.

    - Show all forces acting on the pulley and the two masses.
    - Determine the moment of inertia I of the pulley, in terms of m, g, and R.
    - Determine the tensions in the string, both above and below the pulley.

    Solution:

    The force on the mass on the table is $T_1 = ma = mg/4$. The force on the suspended mass is $mg - T_2 = mg/4$, so that $T_2 = 3g/4$. Finally, the net torque on the pulley is $(T_2 - T_1)R = mgR/2 = I\alpha = Ia/R = Ig/(4R)$. Consequently, the moment of inertia of the pulley is $I = 2mr^2$.

13. PHYS 123 - 001 QUIZ 7 30 April 2002

    A round ball of mass $m = 0.1$ kg and radius $R = 0.05$ meters rolls without slipping down a track after being released from rest at a height $h = 0.5$ meters above the level portion of the track.

    Hint: The ball is a uniform sphere, with moment of inertia about its center of $2/5mR^2$.

- Draw a diagram showing all the forces acting on the ball as it rolls down the curved portion of the track.
- Determine the (translational) speed v of the ball just as it reaches the bottom of the curved portion of the track.

Solution:

The forces are the weight of the ball mg downward, the normal force N perpendicular to the track, and the (static) friction force f_s up the track.

Applying energy conservation, we obtain

$$\begin{aligned} mgh &= K_f + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \\ &= \frac{1}{2}mv^2 + \frac{1}{2}\frac{2}{5}mR^2\frac{v^2}{R^2} \\ &= \left(\frac{1}{2} + \frac{1}{5}\right)mv^2 = \frac{7}{10}mv^2 \\ v^2 &= \frac{10}{7}gh = \frac{10}{7}(9.8)(0.5) = 7.2 \\ v &= 2.7\ m/s \end{aligned}$$

14. PHYS 123 - 002 QUIZ 7 02 May 2002

A round ball of mass m, radius R, and moment of inertia I (with respect to its center of mass) rolls without slipping down the track through a height h.

Note : The center of mass of the ball lies at its geometric center.

- Draw a diagram showing all the forces acting on the ball as it rolls down the curved portion of the track.
- Express the height h in terms of the (translational) speed v of the ball just as it reaches the bottom of the curved portion of the track, as well as the other parameters.

Solution:

The forces are the weight of the ball mg downward, the normal force N perpendicular to the track, and the (static) friction force f_s up the track.

Applying energy conservation, we obtain

$$
\begin{aligned}
mgh &= K_f = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \\
&= \frac{1}{2}\left(m + \frac{I}{R^2}\right)v^2 \\
h &= \frac{v^2}{2g}\left(1 + \frac{I}{mR^2}\right)
\end{aligned}
$$

15. PHYS 123 - 001 QUIZ 8 07 May 2002

    Two small objects of equal mass m, which lie far away from everything else in the universe, travel along the same circular orbit with the same speed v, and always lie at precisely opposite locations, as shown.

    - Draw a diagram showing all the forces on this system.
    - What is the radius R of the circle, expressed in terms of v, m, and G?
    - Determine the rotational period T of this system, expressed in terms of G, m, and R.

    Solution:

    The force of gravitational attraction between the bodies is $F = Gm^2/(2R)^2$. Thus,

$$
\begin{aligned}
F &= \frac{mv^2}{R} = \frac{Gm^2}{(2R)^2} \\
v^2 &= \frac{Gm}{4R} \\
R &= \frac{Gm}{4v^2} \\
T &= \frac{2\pi R}{v} = \frac{4\pi R^{3/2}}{\sqrt{Gm}}
\end{aligned}
$$

16. PHYS 123 - 002 QUIZ 8 09 May 2002

    Two small objects, each of mass 100 kilograms, are located far from everything else in the universe. Under mutual gravitational attraction, they travel along the same stationary circular orbit of radius 1000 meters, always lying at precisely opposite locations, as shown.

- Draw a diagram showing all the forces on this system.
- What are their (tangential) speeds in this orbit, in meters/second?
- Determine the rotational period T, in seconds.

Hint: $G = 6.67 \times 10^{-11}\ [Ntm^2]/kg^2$.

<u>Solution</u>:

The force of gravitational attraction between the bodies is $F = Gm^2/(2R)^2$. Thus,

$$
\begin{aligned}
F &= \frac{mv^2}{R} = \frac{Gm^2}{(2R)^2} \\
v^2 &= \frac{Gm}{4R} = \frac{6.67 \times 10^{-11} \cdot 100}{4 \times 10^3} = 1.67 \times 10^{-12} \\
v &= 1.3 \times 10^{-6}\ m/s
\end{aligned}
$$

The period is

$$T = \frac{2\pi r}{v} = \frac{2\pi 10^3}{1.3 \times 10^{-6}} = 4.8 \times 10^9\ sec = 170\ years$$

Spring 2002 Examinations:

PHYS 123 - 001/002 TEST 1 25 February 2002

1. [25 points] An automobile of mass 1000 kg on the Autobahn is traveling at 200 kilometers/hour on level ground. Seeing a stopped car in his/her lane a distance of 500 meters ahead, the driver immediately locks the brakes. Assume that the car slides to rest on the road just before striking the stopped car. Determine the coefficient of kinetic friction for the car, μ_k, under these circumstances.

   <u>Solution</u>:

$$v_0 = 200 \frac{km}{hr} \cdot \frac{1\ m/s}{3.6\ km/hr} = \ 55 m/s$$

Since $v_x = v_0 - at$, the stopping time is $t = v_0/a$. Furthermore, since $x = v_0 t - at^2/2$, we obtain $L = v_0 \cdot v_0/a - a(v_0/a)^/ 2 = v_0^2/2a$. As a consequence, the deceleration is

$$a = \frac{v_0^2}{2L} = \frac{(55)^2}{2 \cdot 500} = 3.1 m/s^2$$

The coefficient of kinetic friction is

$$\mu_k = \frac{f_k}{N} = \frac{ma}{mg} = \frac{a}{g} = 0.32$$

2. [25 points] Two masses, A and B are connected across a massless, frictionless pulley by a massless cord. Mass A sits on a frictionless surface, tilted $\theta = 30^o$ to the horizontal. The cord remains taut, and Mass B [10 kilograms] falls, with a downward acceleration of $a = 3$ meters/second-squared.

   - Indicate all forces acting on each mass.
   - Determine the tension in the cord [in Newtons].
   - Determine the mass of A, [in kilograms].

   Solution:

   The forces acting on mass A are its weight $m_A g$ (down), the normal force $N = mg\cos\theta$ perpendicular to the table, and the tension T in the cord (up the table). It experiences an acceleration a up the table, where $T - m_A \sin\theta = m_a a$.

   The forces acting on mass B are its weight (down) and the tension in the string (up). It experiences a downward acceleration a, where $m_B g - T = m_B a$. Thus, $T = m_b(g-a) = 10(9.8-3.0) = 68\,N$.

   Finally, we have $T = m_A(a + g\sin\theta)$ or $68 = m_A(3.0 + 9.8\ 2) = 7.9 m_A$. Therefore, $m_A = 68/(7.9) = 8.6\,kg$.

3. [25 points] On level ground, a steel spherical ball (used in shot-put) of mass 1 kilogram is thrown at an angle of 30^o above the horizontal with an initial speed of 15 meters per second (Neglect air resistance).

   - What is the maximum altitude of the ball [in meters], relative to the launch point?

- How long does the ball stay in the air above the launch point [in seconds]?

Solution:

The basic equation for trajectory motion are:

$$\begin{aligned} v_x &= v_0 \cos\theta \\ x &= v_0 \cos\theta t \\ v_y &= v_0 \sin\theta - gt \\ y &= v_0 \sin\theta t - \frac{1}{2} g t^2 \end{aligned}$$

The maximum altitude occurs when $v_y = 0$, at the time $t = v_0 \sin\theta / g = 15 \sin 30^o / 9.8 = 0.76\ sec$. The corresponding altitude is

$$y_{max} = v_0 \sin\theta (\frac{v_0 \sin\theta}{g}) - \frac{1}{2} g (\frac{v_0 \sin\theta}{g})^2 = \frac{v_0^2 \sin^2\theta}{2g} = \frac{15^2 0.5^2}{2 \cdot 9.8} = 2.87\ m$$

The landing $y = 0$ occurs at a time $t = 2v_0 \sin\theta / g = 1.53\ sec$, so that the range is

$$R = v_0 \cos\theta (\frac{2v_0 \sin\theta}{g}) = 15 \cos 30^o (1.53) = 19.9\ m$$

4. [25 points] An $m = 5$ kilogram mass is attached to a massless cord. It slides on a frictionless horizontal plane along a circular path of radius $R = 1$ meter, the cord being tied to a point at the middle of the plane. The cord can safely withstand a tension of $F = 1000$ Newtons without breaking. What is the maximum number of revolutions per second permitted for safe operation?

   Solution:

   The tension in the cord is

$$T = m a_{cent} = \frac{m v^2}{R}$$

$$
\begin{aligned}
1000 &= \frac{5 \cdot v^2}{1} \\
v^2 &= 200 \\
v &= 14.1\, m/s
\end{aligned}
$$

The corresponding frequency of revolution is

$$
f = \frac{1}{T} = \frac{v}{2\pi R} = \frac{14.1}{2\pi \cdot 1} = 2.25\, Hz
$$

5. [Extra Credit; 10 points] Calculate the angle between the [three-dimensional] vectors $\vec{A}$ and $\vec{B}$, where

$$
\begin{aligned}
\vec{A} &= 2\hat{i} + 3\hat{j} + 4\hat{k} \\
\vec{B} &= 3\hat{i} + 4\hat{j} + 2\hat{k}
\end{aligned}
$$

Note that $\hat{i}$, $\hat{j}$, and $\hat{k}$ are unit vectors in the x, y, and z directions, respectively.

Solution:

Let us compute the scalar product:

$$
\vec{A} \cdot \vec{B} = a_x b_x + a_y b_y + a_z b_z = 2 \times 3 + 3 \times 4 + 4 \times 2 = 26
$$

The respective magnitudes of these vectors are

$$
\begin{aligned}
|A| &= \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29} \\
|B| &= \sqrt{b_x^2 + b_y^2 + b_z^2} = \sqrt{3^2 + 4^2 + 2^2} = \sqrt{29}
\end{aligned}
$$

Since $\vec{A} \cdot \vec{B} = |A||B| \cos\theta$, if follows that $26 = (\sqrt{29})^2 \cos\theta$, or $\cos\theta = 26/29$, so that $\theta = 26^o$.

Alternatively, one may use the relation $|\vec{A} \times \vec{B}| = |A||B| \sin\theta$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 2 \end{vmatrix} = 10\hat{i} - 8\hat{j} - \hat{k}$$

Thus

$$|\vec{A}\times\vec{B}| = \sqrt{10^2 + (-8)^2 + (-1)^2} = \sqrt{165}$$

and $\sin\theta = \sqrt{165}/29$, or $\theta = 26^o$.

PHYS 123 - 001; 002 TEST 2 03 April 2002

1. [25 points] A bullet of mass $m = 10$ grams and initial speed $v_0 = 1000$ meters/second passes straight through a block of mass $M = 1000$ grams, leaving at the other end with speed of $v = 300$ meters/second. The bullet travels a distance of $\ell = 30\ cm$ through the block. Calculate the following:

   - the recoil velocity V of the block.
   - the energy $\Delta\mathcal{E}_b$ lost by the bullet.
   - the amount of energy $\mathcal{E}_{lost}$ converted into heat by this process.
   - the average force F_b acting on the block while the bullet was passing through it.

   Solution:

   Momentum is conserved in the collision, so that

$$\begin{aligned} mv_0 &= mv + MV \\ (0.01)(1000) &= (0.01)(300) + 1V \\ V &= 10 - 3 = 7\ m/s \end{aligned}$$

   The energy lost by the bullet is

$$
\begin{aligned}
\Delta\mathcal{E}_b &= \frac{1}{2}mv_0^2 - \frac{1}{2}mv^2 = \frac{1}{2}(0.01)(1000)^2 - \frac{1}{2}(0.01)(300)^2 \\
&= 5000 - 450 = 4550\ J
\end{aligned}
$$

The energy concerted into heat is

$$\Delta\mathcal{E}_{lost} = \Delta\mathcal{E}_b - \Delta\mathcal{E}_{recoil} = \Delta\mathcal{E}_b - \frac{1}{2}MV^2 = 4550 - \frac{1}{2}1(7)^2 = 4525\ J$$

The average force on the bullet is given by

$$F_b = \Delta\mathcal{E}_b/\ell = 4550/0.3 = 1.5 \times 10^4\ N$$

2. [25 points] A car (mass m) is traveling around a horizontal circular track at a speed of $v = 30$ meters per second. The radius of curvature of the track is $R = 200$ meters.

   - Draw a diagram showing all forces acting on the car.
   - Calculate the acceleration of the car in the curved track.
   - What minimum coefficient of kinetic friction μ_k is required to keep the car from sliding off the track?

   <u>Solution</u>:

   The forces acting on the car are its weight $W = mg$ (downward), the normal force $N = mg$ from the track (upward), and kinetic friction f (toward center of track). The net force acts toward the center of the track:

$$f = \frac{mv^2}{R} = ma_c$$

   Thus

$$a_c = \frac{v^2}{R} = \frac{(30)^2}{200} = 4.5\ m/s^2$$

   The coefficient of kinetic friction is

$$\mu_k = \frac{f}{N} = \frac{ma}{mg} = \frac{a}{g} = \frac{4.5}{9.0} = 0.46$$

3. [25 points] A block of mass $m = 2\ kg$ is released from rest on a frictionless inclined plane tilted at an angle of $\theta = 30^o$ to the horizontal. It slides down the track for $\ell = 1$ meter before beginning to compress a spring, and comes to rest after compressing the spring by $x = 20\ cm$.

   - What is the spring constant k of the spring, in Newtons per meter?
   - How fast was the block going when it first touched the spring?

   Solution:

   The block slides down the inclined plane a total distance $\ell + x = 1.2\ m$ before stopping. The initial gravitational potential energy is converted entirely into energy stored in the spring. Thus

$$\begin{aligned} mg(\ell + x)\sin\theta &= \frac{1}{2}kx^2 \\ 2 \times 9.0 \times 1.2 \times \frac{1}{2} &= \frac{1}{2}k(-/2)^2 \\ k &= \frac{2 \times 9.8 \times 1.2}{0.04} = 588\ N/m \end{aligned}$$

   The speed of the block when it just touches the spring is also determined from energy conservation:

$$\begin{aligned} \frac{1}{2}mv^2 &= mg\ell\sin\theta \\ v^2 &= 2g\ell\sin\theta = 2 \times 9.8 \times \frac{1}{2} = 9.8 \\ v &= 3.13\ m/s \end{aligned}$$

4. [25 points] A rocket of initial mass $m = 500$ kilograms, momentarily at rest in free space, is expelling spent fuel at the rate of $-\Delta m/\Delta t = 10\ kg/sec$ out of its back, at a speed of $v_e = 500$ meters/second. Calculate the magnitude of the initial forward acceleration a of the rocket, in meters/second-squared.

   Solution:

   The total momentum of the rocket plus exhauseted gas is conserved. Thus, over a short time Δt, the mass of expelled gas is $-\Delta m$, and we obtain

$$
\begin{aligned}
(m+\Delta m)\cdot(v+\delta v) &+ (-\Delta m)(v-v_e) = mv \\
mv+\Delta mv+m\Delta v+(drop)\Delta m\Delta v &- \Delta mv+\Delta mv_e = mv \\
m\Delta v &= +\Delta mv_e \\
m\frac{\Delta v}{\Delta t} &= -\frac{\Delta m}{\Delta t}v_e \\
a=\frac{\Delta v}{\Delta t} &= (-\frac{\Delta m}{\Delta t})\frac{v_e}{m} \\
a &= 10\frac{500}{500} = 10\ m/s^2
\end{aligned}
$$

5. [Extra Credit; 10 points] A block of mass m starts from rest on a smooth (frictionless) curved track, with its center of mass a height h above where it would be when the block slides onto the rough horizontal track, which has coefficient of kinetic friction μ_k. What distance ℓ does the block slide on the rough horizontal track?

   Solution:

   We use the work energy theorem, setting the initial potential energy mgh equal to the work done against friction, to obtain

$$
\begin{aligned}
mgh = \frac{1}{2}mv^2 &= f\ell = \mu_k mg\ell \\
\ell = \frac{h}{\mu_k}
\end{aligned}
$$

PHYS 123 - 001; 002 FINAL EXAM 15 May 2002

1. [25 points] A small projectile is launched at level ground at speed v_0, at an angle $\theta = 30^o$ above the horizontal. It remains in the air for 6 seconds before returning to ground. Neglecting air resistance, and taking $g = 9.8\ m/sec^2$, determine the initial speed v_0, its maximum height h, and the horizontal distance covered, expressed in SI units.

   Solution:

   The intiial components of the velocity are $v_{x0} = v_0\cos\theta = v_0\sqrt{3}/2$ and $v_{y0} = v_0\sin\theta = v_0/2$. The horizontal component of velocity remains unchanged.

The rocket stays in the air for 6 seconds – three seconds going up and three seconds going down. Its vertical speed starts out at $v_0/2$, and ends up at $-v_0/2$, being 0 at the top of the trajectory. Thus $0 = v_0/2 - gt$, or $v_0 = 2 \times 9.8 \times 3 = 59 m/s$.

The horizontal distance covered (Range) is $R = v_x t = v_0\sqrt{3}/2t = 305\ m$. From the relation $v_y^2 = v_{y0}^2 - 2gy$, we obtain the height h by setting the vertical speed to zero: $v_y = 0$:

$$h = \frac{v_{y0}^2}{2g} = \frac{29.4^2}{19.6} = 44\ m$$

2. [25 points] A disk of mass M, radius R, and moment of inertia I has a light string wrapped around its rim, which is attached to the ceiling. The string remains taut while it unwinds, and disk falls.

   - Draw a diagram showing all the forces acting on the disk.
   - Calculate the tension T in the string.
   - Calculate the downward acceleration a of the center of mass of the disk.

   Solution:

   The forces acting on the disk are its weight $W = mg$ (downward) and the tension T in the string (upward). The downward acceleration of the disk is given through Newton's Second Law: $ma = mg - T$.

   In addition, the torque about the center of mass of the disk is equal to the rate of change of its angular momentum:

$$I\alpha = I\frac{a}{R} = TR$$

   Thus,

$$\begin{aligned} a &= g - \frac{T}{m} \\ a &= g - a\frac{I}{mR^2} \\ a &= g\frac{mR^2}{I + mR^2} \end{aligned}$$

The tension in the string is thus

$$T = \frac{Ia}{R^2} = mg\frac{I}{I + mR^2}$$

3. [25 points] A banked track with radius of curvature R and inclination $\theta = 30^o$ is designed so that a vehicle traveling at $v = 40$ meters/second will move around the track without the need for frictional forces. Determine the radius R of the track, in meters. Also, if the coefficient of static friction is 0.1, determine the maximum speed the vehicle can have without slipping off the track.

   Solution:

   The forces acting on vehicle are its weight $W = mg$ (downward) and the normal force of the track N perpendicular to the track. The resultant of these two forces, the net force on the vehicle is horizontal, and of magnitude $F = mv^2/R$. The vertical component of the normal force must thus cancel the weight; $N\cos\theta = mg$, and the horizontal component of the normal force is the resultant force: $N\sin\theta = F$. Consequently $F = mg\tan\theta$ and

$$\begin{aligned} \frac{mv^2}{R} &= mg\tan\theta \\ R &= \frac{v^2}{g\tan\theta} \\ R &= \frac{40^2}{9.8 \times 0.577} = 280\ m \end{aligned}$$

4. [25 points] A bullet of mass 30 grams and initial speed $v_0 = 1000$ meters/second passes straight through a block of mass 500 grams, leaving at the other end with a speed of $v = 500$ meters/second. The bullet travels a distance of $\ell = 10\ cm$ through the block. Calculate the following:

   - the recoil velocity V of the block.
   - the energy $\Delta\mathcal{E}_b$ lost by the bullet.
   - the amount of energy converted into heat by this process.
   - the average force F acting on the block while the bullet was passing through it.

Solution:

The energy lost by the bullet is

$$\begin{aligned}\Delta\mathcal{E}_b &= \frac{1}{2}mv_0^2 - \frac{1}{2}mv^2 \\ &= \frac{1}{2}(0.03)\times(1000^2 - 500^2) = 11250\ J\end{aligned}$$

The total momentum of the system (bullet plus block) is conserved, so that

$$\begin{aligned}mv_0 &= mv + MV \\ V &= \frac{m(v_0 - v)}{M} = \frac{0.03\times(1000-500)}{0.5} = 30\ m/s\end{aligned}$$

The energy converted into heat is

$$\Delta(\mathcal{E}_b - \mathcal{E}_{block}) = 11250 - \frac{1}{2}(0.5)\cdot(30)^2 = 11250 - 225 = 11025\ J$$

The average frictional force F on the bullet, multiplied by the distance ℓ traveled in the block, equals the energy converted to heat, so that $F = 11025/0.3 = 3.7\times 10^4\ N$.

5. [25 points] A mass m is attached to a massless string of length L and swung in a vertical plane. The string remains taut at all times, and at the top of the swing the mass has the minimum speed necessary for the string to remain taut at that point. In terms of m, g, and L, calculate the speed of the mass and the tension in the string at points T [bottom] B [string horizontal] and H [top].

   Solution:

   At the top of the path the tension in the string is zero, and the weight of the mass must be responsible for its centripetal acceration: $mg = mv_T^2/L$, or $v_T = \sqrt{gL}$. There is gravitional potential energy $2mgL$ as well.

   Since the mechanical energy is conserved, at the bottom of the path, with zero gravitational potential energy, we have

$$\begin{aligned}\frac{1}{2}mv_T^2 + 2mgL &= \frac{1}{2}mv_B^2 \\ \frac{5}{2}mgL &= \frac{1}{2}mv_B^2 \\ v_B &= \sqrt{5gL}\end{aligned}$$

When the string is horizontal, conservation of mechanical energy leads to the relation

$$\begin{aligned}\frac{5}{2}mgL &= \frac{1}{2}mv_H^2 + mgL \\ v_H &= \sqrt{3gL}\end{aligned}$$

6. [25 points] A lump of clay of mass $m = 50$ grams is dropped from rest at a height of $h = 1.0$ meters above a turntable. The turntable is initially rotating freely about its center at $\omega_0 = 45$ revolutions per minute. The turntable is a uniform disk of mass $M = 1000$ grams, radius $R = 50\ cm$, and moment of inertia of $I_0 = 1.25\ kg\ m^2$. The lump of clay sticks to the rotating turntable at a point $r = 25\ cm$ from its center. Determine the speed of the clay just as it hits the turntable [in m/s], the final angular velocity of the turntable [in rpm], and the energy lost in the collision [in Joules].

   Solution:

   The speed of the class as it hits the turntable is determined from energy conservation:

$$\begin{aligned}\frac{1}{2}mv^2 &= mgh \\ v^2 &= 2gh = 2 \times 9.8 \times 1.0 = 19.6 \\ v &= 4.42\ m/s\end{aligned}$$

   The angular momentum of the turntable is unchanged in the collision with the clay. Thus

$$
\begin{aligned}
I_0\omega_0 &= I\omega = (I_0 + mr^2)\omega \\
\omega &= \frac{\omega_0}{1 + (mr^2)/I_0} \\
\omega &= \frac{45}{1 + (0.05 \times (0.25)^2)/1.25} \\
\omega &= 44.89\, rpm
\end{aligned}
$$

The angular velocities are $\omega_0 = 4.71\, rad/sec$ and $\omega = 4.70\, rad/sec$, respectively.

The energy lost in the collision is

$$\Delta\mathcal{E} = \frac{1}{2}mv^2 + \frac{1}{2}I_0\omega_0^2 - \frac{1}{2}I\omega^2 = 0.49 + 13.88 - 13.84 = 0.53\; J$$

7. [Extra Credit; 10 points] The Jovian satellites Io and Callisto travel around the planet Jupiter in circular orbits, with orbit radii R and periods P as given:

Satellite	Orbit Radius (km)	Orbit Period (days)
Io	4.22×10^5	1.77
Callisto	18.8×10^5	–

Determine the period of Callisto about the planet Jupiter, in days, as well as the mass of planet Jupiter, in kilograms.

Hint: $G = 6.67 \times 10^{-11}\; Nt\; m^2/kg^2$.

Solution:

It follows from Newton's second law and Newton's Law of Universal gravitation that, for a body of mass m moving in a circular orbit of radius R at speed v, we have

$$
\begin{aligned}
\frac{mv^2}{R} &= \frac{GmM}{R^2} \\
\omega^2 = \left[\frac{v}{R}\right]^2 &= \frac{GM}{R^3} \\
P = \frac{2\pi}{\omega} &= 2\pi\sqrt{\frac{R^3}{GM}}
\end{aligned}
$$

Thus

$$\frac{P_{Callisto}}{P_{Io}} = \left[\frac{R_{Callisto}}{R_{Io}}\right]^{3/2}$$

$$P_{Callisto} = 1.77 \times \left(\frac{18.8}{4.22}\right)^{3/2} = 16.6\ days$$