

Physics 221: Old Quizzes

Prepared August 2006

Porter Johnson

Chapter 15: Oscillations

SHM: $x = A \cos(\omega t + \phi)$; ϕ (phase)

ω (angular frequency); A (amplitude)

Period: $T = 2\pi/\omega = 1/f$.

Frequency $f = 1/T = \omega/(2\pi)$

Velocity: $v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$

Acceleration: $a = \frac{dv}{dt}$

$a = -\omega^2 A \cos(\omega t + \phi) = -\omega^2 x$

Ideal Spring: $m\ddot{x} = F = -kx$

Energy $E = 1/2mv^2 + 1/2kx^2$

Frequency $\omega^2 = k/m$

Simple Pendulum: $I\ddot{\theta} = m\ell^2\ddot{\theta} = \tau = -mg\ell \sin\theta \approx -mg\ell\theta$

$\ddot{\theta} + (g/\ell)\theta = 0$; $\omega^2 = g/\ell$

Damped Harmonic Motion:

$m\ddot{x} + b\dot{x} + kx = 0$

$x = A \exp[-bt/(2m)] \sin(\omega't + \phi)$

$\omega'^2 = k/m - b^2/(4m^2)$

Underdamped $\omega'^2 > 0$

Overdamped $\omega'^2 < 0$

Critically damped $\omega'^2 = 0$

Resonant frequency: $\omega^2 = k/m$

Previous Quizes

- Although California is known for earthquakes, there are large regions dotted with precariously balanced rocks that would easily be toppled by even a mild earthquake. Evidence shows that the rocks have stood this way for thousands of years, suggesting that major earthquakes have not occurred there during that time. If an earthquake were to put such a rock in a sinusoidal oscillation (parallel to the ground) at 3.0 Hertz, an amplitude of oscillation of 2.0 cm would cause a particular rock to topple. Under these conditions, compute the maximum speed of the rock and its maximum acceleration.

Answers: $0.36m/s$; $7.1m/s^2$.

- A 5.0 kilogram block hangs from a spring, extending it 10 cm from its unstretched position. What is the spring constant?

The block is removed, and a 2 kilogram block is hung from the same spring. If the spring is then stretched and released, what is its period of oscillation?

Answers: $490N/m$, $0.4s$.

- A block rests on a piston that is moving vertically in simple harmonic motion, with an amplitude of 0.1 meters. What is the maximum frequency of motion for which the block and the piston remain in constant contact?

Answer: $1.6Hz$.

- A block is on a horizontal surface (a shake table) that is moving back and forth horizontally with simple harmonic motion (SHM) of frequency 4.0 Hertz. The coefficient of static friction between the block and surface is 0.60. How great can the amplitude of the SHM be if the block is not to slip along the surface?

Answer: $9.3mm$.

- A performer seated on a trapeze is swinging back and forth with a period of 6.0 seconds. When she stands up, the center of mass of the system (trapeze plus performer) rises by 0.4 meters. What is the new period of the system?

Hint: Treat trapeze and performer as a simple pendulum.

Answer: $5.86s$.

- A block of mass M , at rest on a horizontal frictionless table, is attached to a rigid support by a spring of constant k . A bullet of mass m and speed v strikes the block, and becomes embedded in the block. Determine the following quantities:
 - The speed of the block immediately after the collision
 - The amplitude of the resulting simple harmonic motion
 - The period of motion of the system.

Answers:

$$\frac{mv}{M+m}, \frac{mv}{2\sqrt{k(M+m)}}, 2\pi\sqrt{\frac{M+m}{k}}.$$

- A particle of mass m is attached to a light ideal spring of spring constant k , which is attached to the ceiling. When the mass m is pulled down below its equilibrium position, it undergoes vertical oscillations with a period of 1.0 seconds.

When an additional mass of 1.25 kilograms is attached to the spring, the period of vertical oscillations increases to 2.0 seconds. Determine the original mass, m , as well as the spring constant k .

Answers: 0.42kg, 16N/m.

- A performer seated on a trapeze is swinging back and forth with a period of 9.0 seconds. When she stands up, the center of mass of the system of trapeze and performer rises by 0.5 meters.
 - Draw a diagram showing all the forces acting on the trapeze and performer
 - What is the period of the new system?

You may treat the system of “trapeze plus performer” as a simple pendulum, and the rope on the trapeze as fixed in length.

Answer: 8.88s.

Chapter 16: Waves I

transverse or longitudinal

$$y(x, t) = A \sin(kx - \omega t + \phi)$$

A : (amplitude); k (wave number);

ω (angular frequency); ϕ (phase).

$$k = 2\pi/\lambda; \omega = 2\pi f; \omega/k = \lambda f = v$$

(v = wave velocity).

Stretched string: $v = \sqrt{T/\mu}$.

T : (tension); μ (mass per unit length).

Superposition of waves:

$$y(x, t) = y_1(x, t) + y_2(x, t)$$

Interference of waves:

$$A \sin(kx - \omega t) + A \sin(kx - \omega t + \phi)$$

$$= 2A \cos(\phi/2) \cos(kx - \omega t + \phi/2)$$

$\phi = 0 \text{ mod } 2\pi$: (constructive int)

$\phi = \pi \text{ mod } 2\pi$: (destructive int)

Standing waves:

$$A \sin(kx - \omega t) + A \sin(kx + \omega t) = 2A \sin kx \cos \omega t$$

$kL = n\pi$ (L is string length).

Number of internal nodes: $n - 1$.

$$y(x, t) = B \sin(n\pi x/L) \sin(\omega t + \phi)$$

$$k = n\pi/L; \lambda = 2\pi/k = 2L/n$$

$$f = v/\lambda = nv/(2L)$$

Chapter 17: Waves II

Speed of Sound: (longitudinal) $v = \sqrt{B/\rho}$

B : (bulk modulus); ρ : (density)

343 meters/sec at 20°C;

Speed increases with temperature.

Air displacement and pressure differential 90° out of phase.

Interference phase $\phi = 2\pi(\Delta L)/\lambda$

ΔL : (path difference); λ (wave length).

Constructive Int: $(\Delta L)/\lambda = 0, 1, 2, \dots$

Destructive Int: $(\Delta L)/\lambda = 1/2, 3/2, 5/2, \dots$

I : Sound intensity:

Power per unit area.

Threshold $I_{th} = 10^{-12}$ Watts/meter².

decibels (dB): $\beta = 10 \log_{10} I/I_{th}$

Standing waves in a Pipe of length L

Both ends open: $n\lambda = 2L$.

$$f = v/\lambda = nv/(2L)$$

One end open: $(n + 1/2)\lambda = 2L$.

$$f = v/\lambda = (2n + 1)v/(4L)$$

Beats: $f_{beat} = |f_1 - f_2|$

Doppler Effect:

$$f' = f \times (v \pm v_D)/(v \mp v_S)$$

top sign: motion toward

bottom sign: motion away

v_D : velocity of detector

v_S : velocity of source

Shock waves, Mach cone:

$$\sin \theta = v_{sound}/v_{projectile}$$

Previous Quizzes

- A string of length 1.5 meters has a mass of 15 grams. It is stretched with a tension of 12 Newtons between fixed supports.
 - What is the speed of transverse waves for this string?
 - What is the lowest resonant frequency of the string?

Answers: 35m/s ; 12Hz .

- In an experiment on standing waves, a string of length 80 cm is attached to the prong of an electrically driven tuning fork that oscillates perpendicular to the length of the string at a frequency of 60 Hz. A standing wave is set up, with an amplitude of 2.0 cm and four loops. The mass of the string is 20 grams.
 - What is the wavelength of this mode?
 - What is the tension in the string under these circumstances?

Answers: 40cm ; 14.4N .

- Oscillation of a 500 Hz tuning fork sets up standing waves in a string clamped at both ends. The wave speed for the string is 200 meters/second. The standing wave has four loops, with a maximum transverse displacement of 2.0 mm.
 - What is the length of the string?
 - Write an equation for the displacement of the string as a function of position along the string and time.

Answers: 0.2m ; $.002\text{m}\cos(10\pi x - 1000\pi t)$.

- A string of length 10 meters with both ends fixed has a mass of 100 grams. That string is under a tension of 100 Newtons. Calculate the velocity of transverse waves on the string. In addition, determine the wavelength and frequency of the fundamental mode, the smallest frequency of transverse vibrations.

Answers: 100m/s ; 20m , 5Hz .

- A uniform string of a particular fixed length is tied at the ends under a tension T . The frequency of the fundamental mode in that case is f_1 , and its wavelength is λ_1 . The tension is increased to the value $4T$, everything else being kept the same. Determine the new wavelength and frequency of the fundamental mode.

Answers: $\lambda_2 = \lambda_1$; $f_2 = 2f_1$.

Chapter 21: Electric Charge

$\leftarrow \cdot Q \quad Q \cdot \rightarrow$ (Like charges Repel)

$Q \cdot \rightarrow \quad \leftarrow \cdot -Q$ (Unlike charges attract)

Coulomb's Law: $F = (kq_1q_2)/r_{12}^2$

$k = 9 \times 10^9 (N \cdot m^2 / C^2) = 1 / (4\pi\epsilon_0)$

$\epsilon_0 = 8.85 \times 10^{-12} C^2 / (N \cdot m^2)$

Charge: CONSERVED – QUANTIZED

Electron charge: $e_0 = 1.6 \times 10^{-19} \text{ C}$

Conductor: No free charges inside.

Uniformly charged spherical shell

(radius R ; charge Q)

test charge q at distance \vec{r} from center:

outside: $\vec{F} = (kqQ\hat{r})/r^2$.

inside: $\vec{F} = \vec{0}$

Previous Quizes

- Two positive charges of magnitude $Q = 2.0$ microCoulombs are held at fixed locations along the x -axis, at a distance of 1.0 meters to the left and right of the origin, respectively. A particle of positive charge $q = 5.0$ microCoulombs and mass $m = 0.3$ kilograms is placed on the y -axis one meter above the origin. Determine the magnitude and direction of the acceleration of the particle.

Answers: $0.21m/s^2$ up.

- Two very small water droplets of mass 10^{-12} grams are created with charges that are equal in magnitude and opposite in sign. When separated by a distance of 1.0×10^{-6} m in the absence of gravity, they are seen to accelerate toward one another with accelerations $a = 2.3$ cm/sec². Determine the amount of charge on them, expressed in terms of the fundamental charge, 1.6×10^{-19} Coulombs.

Answers: $0.53 \times 10^{-19}C = e_0/3$.

Chapter 22: Electric Field

test charge q_0 ; $\vec{E} = \vec{F}/q_0 = kQ\hat{r}/r^2$

Electric field lines:

away from (+) charges: $\leftarrow (+) \rightarrow$

and toward (-) charges: $\rightarrow (-) \leftarrow$

Two opposite charges: lines from - to +

Two like charges: lines away from each

Dipole: $(-q) \rightarrow (+q)$: \vec{p} from - to +

$p = qd$

along dipole axis (z): $E_z = (2kp)/z^3$

Continuous distribution of charge:

$$\vec{E} = \int d\vec{E} = k \int dq\hat{r}/r^2$$

Point charge in uniform electric field:

$$\vec{F} = m\vec{a} = q\vec{E}$$

Dipole in uniform electric field:

$$\text{Torque: } \tau = \vec{p} \times \vec{E}$$

Potential Energy: $U = -\vec{p} \cdot \vec{E}$

Previous Quizes

- A total charge Q is uniformly distributed along a thin rod of length L . Determine the magnitude and direction of the electric field a distance D to the right of the rod.

NOTE: $dE = kdq/r^2$

Answers: to right;

$$\frac{kQ}{D(D+L)}$$

- A total charge Q is uniformly distributed along a thin rod of length L . Determine the magnitude and direction of the electric field a distance D to the left of the rod.

Answers: to left;

$$\frac{kQ}{D(D+L)}$$

- Two equal positive charges, $+Q$, are placed in left half of the $x - y$ plane at points (d, d) and $(d, -d)$, respectively. Two charges of the same magnitude and opposite sign, $-Q$, are placed in the left half of the $x - y$ plane at points $(-d, d)$ and $(-d, -d)$, respectively. Determine the magnitude and direction of the electric field at the origin, $(0, 0)$.

Note: Magnitude of electric field at distance x from a point charge q : $E = kq/x^2$.

Answers: $-x$ direction;

$$\sqrt{2} \frac{kQ}{d^2}$$

- Four equal charges are placed in the x - y plane ($z=0$) at these locations (x, y, z) :

$$(R, 0, 0); (0, R, 0); (-R, 0, 0); (0, -R, 0)$$

Determine the magnitude and direction of the electric field at the point $(x, y, z) = (0, 0, Z)$.

Note: Magnitude of electric field at distance D from a point charge q : $E = kq/D^2$.

Answers: upward;

$$\frac{kQZ}{(R^2 + Z^2)^{3/2}}$$

- A small bead of mass m and (negative) charge $-q$ is constrained to move along an insulating wire. The wire lies along the symmetry axis perpendicular to a thin ring of radius R , which contains total (positive) charge $+Q$ distributed uniformly over the ring. You may neglect gravity and sliding friction of the bead on the wire.
 - If the bead lies a distance x away from the center of the ring, show that the electric field has magnitude

$$E = kQx/[R^2 + x^2]^{3/2}$$

- What is the direction of that electric field?
- Determine the angular frequency of small oscillations of the bead about the center of the ring.

Answers: to right;

$$\frac{kQq}{mR^3}$$

- Two equal positive charges, Q are placed in the right half of the $x - y$ plane at points (d, d) and $(d, -d)$, respectively. Two charges of the same magnitude and opposite sign, $-Q$, are placed in the left half of the $x - y$ plane at points $(-d, d)$ and $(-d, -d)$, respectively. Determine the magnitude and direction of the electric field at the origin, $(0, 0)$.

Note: The magnitude of the electric field at a distance D from a point charge q is $E = kq/D^2$.

Answers: away from wire;

$$\frac{kQ}{z\sqrt{z^2 + L^2}}$$

- A charge Q is uniformly distributed along a insulating wire of length $2L$. At a point P , which is located a distance z from the center of the wire along its central axis, determine the magnitude and direction of the electric field \vec{E} .

- Four equal positive charges $+Q$ are placed in the $x - y$ plane at these locations:

$$(x, y, z) = (R, 0, 0) \quad (-R, 0, 0) \quad (0, R, 0) \quad (0, -R, 0)$$

Determine the magnitude and direction of the electric field at the point $(x, y, z) = (0, 0, z)$.

Note: The magnitude of the electric field at a distance D from a point charge q is $E = kq/D^2$.

Answers: vertical;

$$\frac{4kQzD}{(z^2 + R^2)^{3/2}}$$

- A thin insulating rod of length L contains a total charge $+Q$, which is uniformly distributed along it. Determine the magnitude and direction of the electric field at the point P , which is a distance b beyond the end of the rod.

Answers: away from wire;

$$\frac{kQ}{b(L+b)}$$

Chapter 23: Gauss's Law

$$\epsilon_0 \Phi_E = \epsilon_0 \oint \vec{E} \cdot d\vec{S} = Q_{enc}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / (\text{N m}^2)$$

Any excess charge sits on outer surface of conductor; $\vec{E} = \vec{0}$ inside.

At surface, $E = \sigma / \epsilon_0$ (outward), σ : charge per unit area on surface.

Infinite line charge: λ : charge per unit length.

$$E = \lambda / (2\pi\epsilon_0 r) \text{ away from line.}$$

Non-conducting sheet on either side:

$$E = \sigma / (2\epsilon_0)$$

Spherical shell. electric field radially out

$$E = 0 \text{ inside; } E = Q / (4\pi\epsilon_0 r^2) \text{ outside.}$$

Spherical charge distribution:

$$E = Q_{enc} / (4\pi\epsilon_0 r^2) \text{ radially outward.}$$

Q_{enc} : charge inside sphere of radius r .

Uniformly charged sphere:

$$E = \rho r / (3\epsilon_0) \text{ inside.}$$

$$E = kQ_{tot} / r^2 \text{ outside: } Q_{tot} = 4\pi\rho R^3 / 3.$$

Gauss's Law + symmetry:

Planar, cylindrical, or spherical

Previous Quizes

- A long wire of length L and negligible radius contains a total charge $+Q$, uniformly distributed along its length. It is surrounded by a concentric, open cylindrical shell of radius R (of negligible thickness), which is also of length L . (That is, the wire lies along the symmetry axis of the cylinder.) The cylinder contains total charge $-Q$, which is uniformly distributed over its lateral surface. The length L is very large in comparison of the radius of the shell, so that “fringing fields” may be neglected. Determine the magnitude and direction of the electric field everywhere inside the (hollow) shell.

If the shell radius R is 0.1 meters, and the charge per unit length on the wire is $+10^{-8}$ Coulombs/meter, determine the magnitude the electric field just inside that shell, in Newtons/Coulomb.

Potentially useful information Coulomb's Law $F = kQ_1Q_2/d^2 \dots F/q_0 = E$
 $k = 9 \times 10^9 \text{m}^2/\text{C}^2 \dots \epsilon_0 = 8.85 \times 10^{-12} \text{C}^2/(\text{Nm}^2)$ Gauss's Law: $\epsilon_0 \oint \vec{E} \cdot \vec{ds} = Q_{\text{enclosed}}$.

Answers: radial electric field

$$E_r = \frac{Q}{2\pi\epsilon_0 L r}$$

1800N/C.

- An infinite line of charge, with charge per unit length λ , produces an electric field of 50 Newtons per Coulomb at a distance of 2 meters. Determine λ , in Coulombs per meter.

Answers: $5.5 \times 10^{-9} \text{C}/\text{m}$.

- A total charge of +2 picoCoulombs is spread uniformly throughout an insulating sphere of radius 3 meters. Determine the magnitude and direction of the electric field \vec{E} at the following distances from the center of that sphere
 - 2 meters
 - 4 meters

Note: 1 picoCoulomb = 10^{-12} Coulomb

Answers: outward; $1.3 \times 10^{-3} \text{N}/\text{C}$; $1.1 \times 10^{-3} \text{N}/\text{C}$.

- A very large non-conducting slab of thickness 20 millimeters contains a uniform charge throughout its volume of 50 femtoCoulombs per cubic meter. Use Gauss's law to determine the electric field at these points:
 - At the left outside surface of the slab.
 - At the central plane of the slab.
 - At a point 5 mm to the right of the central plane of the slab
 - At the right outside surface of the slab.

Answers: $5.7 \times 10^{-5} N/c$ left; 0; $2.97 \times 10^{-5} N/c$ right; $5.7 \times 10^{-5} N/c$ right.

- Two thin, long, concentric conducting cylindrical shells contain electric charge. The inner cylindrical shell, with radius a , contains λ Coulombs per meter, whereas the outer shell, with radius b , contains $-\lambda$ Coulombs per meter. Determine the electric field, expressed in terms of the distance r from the central axis of the (concentric) shells. Consider these regions:
 - $r < a$
 - $a < r < b$
 - $r > b$

Answers: $Q/(4\pi\epsilon_0 r^2)$ outward, 0, $Q/(4\pi\epsilon_0 r^2)$ outward.

- A point charge Q sits at the center of an isolated, hollow, metallic spherical shell with inner radius a , and with outer radius b . The TOTAL charge on the shell is zero. Determine the electric field everywhere, expressed in terms of the distance r to the center of the shell, dividing it into the following regions:
 - $r < a$
 - $a < r < b$
 - $r > b$

Answers: 0; $\lambda/(2\pi\epsilon_0 r)$ outward; 0.

- Charge is uniformly distributed inside an infinitely long cylinder of radius R . Let ρ be the volume charge density in the cylinder.

- Show that the magnitude of the electric field inside the cylinder at a distance $r < R$ from its center is $E = rR/(2\epsilon_0)$.
- What is the direction of \vec{E} ?
- Determine the magnitude and direction the electric field everywhere outside the cylinder

Answers: outward; $E = \rho R^2/(2\epsilon_0 r)$ outward.

- Charge of uniform volume density $\rho = 2\mu\text{C}/\text{m}^3$ fills a non-conducting sphere of radius 5 cm.
 - What is the magnitude of the electric field 2 cm from the center of the sphere?
 - What is the magnitude of the electric field 10 cm from the center of the sphere?

Answers: $1500\text{N}/\text{C}$; $930\text{N}/\text{C}$.

Chapter 24: Electric Potential

Work done by electric field $dW_E = -\Delta U$.

$$\Delta V = V_f - V_i = -W_E/Q.$$

Electric potential: 1 Volt = 1 Joule/Coulomb.

Equipotential surface (constant V):

Perpendicular to electric field lines.

$$dV = -\vec{E} \cdot d\vec{\ell}$$

$$\Delta V = -\int_i^f \vec{E} \cdot d\vec{\ell}$$

Potential of point charge: $V = kQ/r$.

Combination of charges (superposition) $V = k\sum_i Q_i/r_i$.

Continuous distribution: $V = k\int dQ/r$.

$V(x, y, z) \rightarrow (E_x, E_y, E_z)$:

$$E_x = -\partial V/\partial x; E_y = -\partial V/\partial y; E_z = -\partial V/\partial z.$$

Electric potential energy of pair of charges: $\mathcal{E} = kq_1q_2/r_{12}$

(superposition of pairs to get total)

Charged conductor: Potential constant inside.

Previous Quizes

- Determine the potential at a point along the central axis of a thin ring of radius R , with charge $+Q$ uniformly distributed along it, and at a distance z from the center of the ring. Express you answer in terms of Q , R , and z .

Hint: $V = kq/r$

Answer:

$$\frac{kQ}{\sqrt{R^2 + z^2}}$$

- Four charges, $Q, Q, -Q, -Q$, are placed each vertex of a square with side of length a . The two positive charges are diagonally opposite each other, as are the negative charges. Initially, the charges are infinitely far from one other. Determine the net amount of work that must be done to bring the charges into this final configuration

Hint: $V = kq/r$.

Answer: $-2.6kQ^2/a$.

- A spherical drop of water carries 20 pico-Coulombs of charge and is held at a electrostatic potential of 1000 Volts at its surface. (with $V = 0$ at infinity). Determine the radius of the drop.

If two drops of water of that size and charge merge (with no loss of charge) to form a single drop, determine the electrostatic potential of that new drop.

Hint: $V = k q / r$

Answers: $0.18mm; 3100V$.

- An insulating plastic rod having (negative) charge - 5 pico-Coulombs spread uniformly along its length is bent to form a semi-circle of radius 10 cm. Determine the electric potential V at the center of the rod, in Volts, with the convention $V = 0$ at infinity.

Hint: $V = kq/r$.

Answer: $-0.45V$.

- Charges of respective sizes $+2Q, -3Q$, and $-4Q$ are placed at the vertices of an equilateral triangle with side length b . Determine the following quantities.

- Determine the electrostatic potential V at the center of the equilateral triangle.
- Determine the net energy required to separate the three charges to infinity (in 3 different directions!).

Note: the distance to from any vertex to the center of the equilateral triangle is given by $b/\sqrt{3} \approx 0.577b$.

Answers: $5\sqrt{3}kQ/b$; $2kQ^2/b$.

- A total charge $+Q$ is uniformly distributed along a straight wire of length L . Determine the electrostatic potential at a point P , which is a distance L to the left of the closest portion of the wire.

Answer: $0.70kQ/L$.

- Positive charged objects Q , $2Q$, and $3Q$, are located at vertices of an equilateral triangle of side a . How much electrostatic potential energy is stored in this case?

Answer: $11kQ^2/A$.

Chapter 25: Capacitance

Two conductors; charges $+Q$ and $-Q$, respectively.

Potential between conductors: ΔV : $Q = C\Delta V$.

C : capacitance (1 Farad = 1 Volt/Coul).

Parallel plate capacitor: $C = \epsilon_0 A/d$.

plate area A ; plate separation d .

Cylindrical capacitor (length ℓ ; radii a, b): $C = 2\pi\epsilon_0\ell/\ln(b/a)$.

Spherical capacitor (radii a, b):

$$C = 4\pi\epsilon_0 ab/(b - a).$$

Capacitors in parallel:

$$C_{tot} = C_1 + C_2 + C_3 + \dots$$

Capacitors in series:

$$1/C_{tot} = 1/C_1 + 1/C_2 + 1/C_3 + \dots$$

Potential energy stored in capacitor:

$$U = 1/2CV^2 = Q^2/(2C)$$

Energy density stored in electric field: $u = 1/2\epsilon_0 E^2$.

Dielectric-filled capacitor:

$$C \rightarrow \kappa C_0 = \kappa\epsilon_0 A/d,$$

κ : dielectric constant.

Gauss's law for dielectrics:

$$\epsilon_0 \oint \kappa \vec{E} \cdot d\vec{S} = Q_{enc}$$

Previous Quizes

- A capacitor of Capacitance $C_1 = 1.00\mu F$ (micro-Farad; $10^{-6}F$) is charged to an electric potential of 100 Volts.
 - Determine the charge on the capacitor, in Coulombs.
 - Determine the electrical energy stored in the Capacitor, in Joules.

This fully charged capacitor is then connected to a capacitor with a capacitance $C_2 = 2.00\mu F$, while maintaining electrical isolation.

- Determine the final charge on each of these two capacitors in equilibrium.
- Determine the equilibrium energy stored in each of these capacitors, in Joules.
- Is the total electrical energy the same as before? Explain.

Answers: $10^{-4}C$; 5×10^3J , $33\mu C$ ($0.6 \times 10^{-3}J$); $67\mu C$ ($1.2 \times 10^{-3}J$).

- A parallel plate capacitor with plate area $A = 0.1m^2$ and plate separation $d = 0.001$ meters has its plates connected across a 10 Volt battery.

Determine the charge on the capacitor (in Coulombs) and the electrical energy stored in the capacitor (in Joules).

With the battery still connected to it, the capacitor is then filled with a dielectric material with dielectric constant $\kappa = 8.0$.

Determine the charge on the capacitor (in Coulombs) and the electrical energy stored in the capacitor (in Joules).

Where has the energy come from?

Answers: $9 \times 10^{-9}C$; $4 \times 10^{-8}j$, $7 \times 10^{-8}C$; $3 \times 10^{-7}j$; battery.

- A parallel plate capacitor has a total plate area of $A = 4 \text{ cm}^2$ and plate separation of $d = 2$ millimeters. The left half of the gap is filled with material of dielectric constant $\kappa_1 = 30$, whereas the right half of the gap contains material with dielectric constant $\kappa_2 = 90$, as shown.
 - Determine the capacitance of this configuration, in Farads.

- The plates are then charged to a potential difference of 50 Volts.
 - * How much electrical energy (in Joules) is stored in the capacitor?
 - * How much energy lies in each of the two halves?

Answers: 110pF ; 14nJ ; 3.5nJ on left; 10.5nJ on right.

- A capacitor of unknown capacitance C is charged to a potential of 100 Volts. Then it is connected across an (initially uncharged) $50\mu\text{F}$ (microfarad) capacitor. The potential difference is then measured to be 20 Volts. Determine the capacitance C , as well as the energy stored in each capacitor before and after the connection is made.

Answers: $13\mu\text{F}$; $6.3 \times 10^{-2}\text{J}$; $1.3 \times 10^{-3}\text{J}$.

Chapter 26: Current and Resistance

Charge ΔQ passes by in time Δt :

$$I = \Delta Q / \Delta t. \quad 1 \text{ Amp} = 1 \text{ Coul/sec.}$$

Current density \vec{J} : current per unit area

$$I = \int \vec{J} \cdot d\vec{A}$$

$$\vec{J} = ne_0\vec{v}_d; \text{ charge: } e_0; \text{ drift speed: } v_d.$$

n : num charge carriers per unit vol.

Electrical resistance:

Current I flowing through circuit;

Voltage drop ΔV along circuit.

$$R = \Delta V / I. \quad 1(\Omega) \text{ Ohm} = 1 \text{ Volt/ Amp.}$$

$$\text{Resistivity } \rho: \vec{E} = \rho\vec{J}.$$

Wire cross-section area A , length ℓ :

$$R = \rho\ell/A.$$

Metals: resistivity increases with temperature

$$\rho = \rho_0(1 + \alpha\Delta T).$$

$$\alpha = (\Delta\rho/\Delta T)/\rho_0$$

$$\text{Ohm's Law: } V = RI.$$

Resistivity of metal (electron gas):

$$\rho = m/(ne_0^2\tau): \text{ Collision time: } \tau.$$

Power dissipated inside resistor:

$$P = I\Delta V = I^2R = \Delta V^2/R.$$

Semiconductor ρ decreases with T .

Superconductor (low temp) $\rho = 0$.

Previous Quizes

- Two identical batteries of electromotive force \mathcal{E} and internal resistance r are connected in parallel, + to + and - to -. A resistance R is then connected across them. Calculate the magnitude and direction of the current passing through each battery, the power generated by each battery, and the power dissipated in each resistor.

Answer:

$$\frac{\mathcal{E}}{R+2r}; \frac{2\mathcal{E}}{R+2r}; \frac{\mathcal{E}^2 r}{(R+2r)^2}; \frac{4\mathcal{E}^2 R}{(R+2r)^2}$$

- Two identical resistors, which have the same resistance of 120 Ohms, are connected across a battery that has an EMF of 20 Volts. Determine the current flowing in each resistor (Amps), the Voltage across each resistor (Volts), and the power dissipated in each resistor (Watts) for these two cases.
 - The resistors are hooked in series with each other, and then across the battery.
 - The resistors are hooked in parallel with each other, and then across the battery.

Answer: 0.08A; 10V; 0.3W each; 0.17A; 20V; 3.3W each.

- A 1200 Watt radiant heater is connected to operate at 120 Volts.
 - What will the current be in the heater?
 - What is the resistance of the heating coil?
 - How much thermal energy is produced in 1 hour by the heater?

Answer: 10A; 12Ω; 4.23MJ.

- Two wires of cylindrical cross-section and length L are connected in series. A voltage V is placed across this series combination. The wires are each made of the same of resistivity ρ , and the radius of the left wire ($2R$) is twice the radius of the right wire (R). Determine the following:
 - The current flowing through each wire.
 - The power dissipated in each wire.

Answers:

$$\frac{4\pi R^2 V}{5 \rho L}; \frac{4\pi R^2 V^2}{25 \rho L}; \frac{16\pi R^2 V^2}{5 \rho L}$$

- A uniform cylindrical resistor of radius 2.0 cm and length 50 cm is made out of material that has resistivity $\rho = 4.0 \times 10^{-5} \Omega m$. What is the current density ($J = I/A$) passing through the resistor, as well as the potential difference V along the resistor, when the energy dissipation rate inside the resistor is 3.0 Watts?

Answers: $1.9 \times 10^{-4} A/m^2$; 0.22V.

- A resistor R of resistance 0.1Ω is connected across a 1.5 Volt battery with unknown internal resistance r . Thermal energy is generated inside the resistor at a rate of 10 Watts. Determine the potential difference across the resistor, as well as the internal resistance of the battery.

Answers: 0.5V; 0.05Ω .

Chapter 27: Circuits

Work done by EMF source (battery)

Chemical energy \rightarrow Electrical energy

$$dW = \mathcal{E}dQ$$

Kirchhoff's Laws

Loop rule: sum of voltage changes around closed loop = 0.

Junction rule: net current into junction = 0.

Single loop circuit: Load resistor R .

Battery \mathcal{E} with internal resistance r ;

$$I = \mathcal{E}/(R + r)$$

Power provided by battery: $P = I\mathcal{E}$.

Thermal power in resistor $P = I^2R$.

Resistors in series:

$$R_{tot} = R_1 + R_2 + R_3 + \dots$$

Resistors in parallel:

$$1/R_{tot} = 1/R_1 + 1/R_2 + 1/R_3 + \dots$$

RC Circuit

Charging \mathcal{E}, R, C in series:

$$\mathcal{E} = R dQ/dt + Q/C$$

$$Q(t) = \mathcal{E}C(1 - e^{-t/RC})$$

$$I = dQ/dt = \mathcal{E}/R e^{-t/RC}$$

Discharging: R, C in series

$$0 = R dQ/dt + Q/C$$

$$Q(t) = Q_0 e^{-t/RC}$$

$$I(t) = dQ/dt = -Q_0/RC e^{-t/RC}$$

Previous Quizes

- A capacitor of capacitance $10\mu F$ [microfarads] is “leaky”, in the sense that charge leaks from one plate to the other over the course of time. If the charge of $2\mu C$ [micro-coulombs] is reduced to $1.0\mu C$ over a time interval of 10 seconds, determine the “effective resistance” across the capacitor plates.

Answer: $140k\Omega$.

- A linear accelerator produces a pulsed beam of electrons over a time interval of $0.1\mu s$ (microseconds), corresponding to a current of 1.0 Amps over that interval. There are 500 pulses produced per second in the accelerator.
 - How many electrons are produced per pulse?
 - What is the time-averaged current, over a period of several seconds?
 - If the electrons are accelerated to an energy of $50MeV$ (Million electron Volts), what are the average and peak powers of the accelerator, in Watts?

Potentially useful information: $1eV = 1.6 \times 10^{-19}$ Joules; electron charge: $e_0 = -1.6 \times 10^{-19}$ Coulombs

Answers: 6×10^{11} ; $5 \times 10^{-5}A$; $2500W$; $50MW$.

- A capacitor of capacitance 5 nanoFarads is “leaky”, in that charge leaks from one plate to another over the course of time. Suppose that the capacitor is initially charged to a potential of 20 Volts. After one hour, the potential across the plates has dropped to 5 Volts. Determine the “equivalent resistance” between the plates.

Answer: $5 \times 10^{11}\Omega$.

- An ideal battery of Voltage 24 Volts is connected in series with a switch, a resistor of resistance $R = 2 \times 10^5\Omega$, and a capacitor (initially uncharged) of capacitance $C = 5$ nanoFarads. The switch is closed, and the capacitor eventually becomes fully charged. Determine the following:
 - The final charge on the capacitor.
 - The total electrostatic potential energy stored inside the capacitor.
 - The total energy converted to heat in the resistor.

- The total energy provided by the battery.

Answers: $1.2 \times 10^{-7} C$; $1.44 \mu J$; $1.44 \mu J$; $2.88 \mu J$.

- A fluorescent lamp is placed across a capacitor of capacitance $C = 0.15 \mu F$. The combination is connected in series with a resistor of resistance $R = 10^5 \Omega$ and a battery of Voltage 120 Volts. The switch is closed at time $t = 0$. How much time elapses before Voltage across the lamp reaches 70 Volts - at which point the lamp flashes briefly?

Answer: 0.013 sec.

- Two identical resistors, which have the same resistance of 60Ω , are connected across an ideal battery that produces an EMF of 20 Volts. Determine the current flowing in each resistor (Amps), the Voltage across each resistor (Volts), and the power dissipated in each resistor (Watts) for these two cases.

- The resistors are hooked in series with each other, and then across the battery.

- The resistors are hooked in parallel with each other, and then across the battery.

Answers: .17A, 10V, 1.7W; .33A, 20V; 6.6W.

- A flashing circuit consists of a battery of EMF 24 Volts, which is placed in series with a resistance R and capacitance $C = 1.0 \times 10^{-6}$ Farads. A flashing bulb, placed across the capacitor, fires when the Voltage across it reaches 16 Volts, quickly discharging the capacitor. If the flash is to occur every one second, what value of the resistance R must be chosen? How much electrostatic energy is stored in the capacitor at the instant of firing?

Answers: $9 \times 10^5 \Omega$; $1.3 \times 10^{-4} J$.

Chapter 28: Magnetic Fields

Lorentz Force: $\vec{F} = q\vec{v} \times \vec{B}$

(1 Tesla = 1 N / (A m))

Hall effect: (determine sign of charge carriers) wire of width w and thickness t

→ $I \cdots \rightarrow \cdots \vec{B}$ (out of paper) $\cdots \vec{F}$ (down)

$$J = I/A = ne_0v_d.$$

$$E = v_d B \Delta V = wE.$$

Charged particle - uniform magnetic field:

planar motion. $qvB = mv^2/R$;

$$R = mv/(qB).$$

$$\omega = qB/m, f = \omega/(2\pi) = 1/T$$

Three dimensional motion: helix

Cyclotron: Dees: voltage change produces acceleration in uniform magnetic field.

Current-carrying wire:

$$\text{straight: } \vec{F} = I \vec{\ell} \times \vec{B}$$

$$\text{curved: } d\vec{F} = I d\vec{\ell} \times \vec{B}$$

Magnetic moment of wire:

(Current I , area A , N turns)

$$\vec{M} = N I A \text{ (right hand rule)}$$

$$\text{Torque: } \vec{\tau} = \vec{M} \times \vec{B}$$

$$\text{Potential Energy: } U = -\vec{M} \cdot \vec{B}$$

Previous Quizes

- Four long parallel wires, each carrying current I in the same direction, are arranged to form a square of side length a , as seen in the plane perpendicular to the direction of current flow. Compute the magnitude and direction of the force per unit length on any one wire.

Answer: toward center line;

$$\frac{3}{\sqrt{2}} \frac{\mu_0 I^2}{2\pi A}$$

- A metal strip 10 cm long, 1 cm wide, and 0.1 cm thick moves with constant velocity v in the direction of the strip width through a uniform magnetic field $B = 1$ milli-Tesla, directed perpendicular to the strip. A potential difference of 4 micro-Volts is measured across the width of the strip, as shown. Calculate the speed v of the strip.

Answer: 40 cm/sec.

- A circular coil of radius 5 cm, which contains 100 turns of wire, has a current of 5A flowing in it. Determine the magnetic moment of the current loop. This current loop makes an angle of 45° with a uniform external magnetic field of $0.3T$. Determine the magnitude of the torque on this current loop, as well as the potential energy stored in the loop.

Answers: $0.83N - m$; 0.83 Joule.

- A Copper strip of square cross section and side 100 microns (10^{-4} meters) is placed in a uniform magnetic field of magnitude $2.0T$, with the field perpendicular to the strip. A current of 20A is passed through the strip, such that a potential difference appears across the width of the strip. (Hall effect) The number of charge carriers for Copper is 9×10^{28} electrons per cubic meter. Determine the potential difference across the strip.

Note: charge of electron is -1.6×10^{-19} Coulombs, and the mass of the electron is 9.1×10^{-31} kilograms.

Answer: $26\mu V$.

- Find the frequency of revolution of an electron with a kinetic energy of 1000eV in a uniform magnetic field of magnitude 10^{-3} Tesla. Find the radius of the circular orbit, if the velocity of the electron is perpendicular to the magnetic field.

Note: mass of electron $m_e = 9.1 \times 10^{-31}$ kg

$$m_e c^2 = 5.1 \times 10^5 \text{ eV}$$

$$q_e = -1.6 \times 10^{-19} \text{ Coul.}$$

Answers: 26MHz; 0.1m.

- In a certain cyclotron a proton moves in a circle of radius 0.5m. The magnitude of the magnetic field is 2.0T, perpendicular to the proton orbit.
 - What is the oscillator frequency?
 - What is the kinetic energy of the proton, in electron Volts?

Note: mass of proton: $m_p = 1.7 \times 10^{-27}$ kg

$$m_p c^2 = 9.38 \times 10^8 \text{ eV}$$

$$q_p = 1.6 \times 10^{-19} \text{ Coul.}$$

Answers: 30 MHz; 47 MeV.

- A 200 turn solenoid having a length of 50 cm and a diameter of 10 cm carries a current of 3A.
 - Calculate the magnitude of the magnetic field inside the solenoid.
 - Determine the magnetic dipole moment of the solenoid.

Answers: $1.5 \times 10^{-3} T$; $4.7 Am^2$.

- A long solenoid has 100 turns per centimeter, and carries current I . An electron moves within the solenoid in a circular orbit of radius 4 cm, which is perpendicular to the solenoid axis. The speed of the electron is $0.15c$, or 4.5×10^7 meters/second. Determine the current I in the solenoid, in Ampères.

Answer: 0.5A.

- An electron is moving with kinetic energy of 1000 electron Volts in a plane perpendicular to a magnetic field of magnitude 5×10^{-6} Tesla. Determine the radius of the circular path of the electron (in meters), and its frequency of revolution (in Hertz).

Answers: 21m; 140 kHz.

- A copper strip of width 100 microns (10^{-4} meters) is placed in a uniform magnetic field of magnitude $1.0T$, perpendicular to the strip. A current of $20A$ is then sent through the strip, such that a Hall potential of magnitude V appears across the width of the strip. Calculate the Hall potential V .

Note: the number of charge carriers per unit volume for copper is about 8×10^{29} per cubic meter.

Answer: $1.5 \times 10^{-6}V$.

- A strip of copper $200 \mu m$ thick and 1.0 cm wide is placed perpendicular to a uniform magnetic field of magnitude $2.0T$. A current of $30A$ passes through the strip, so that a Hall potential difference V appears across the strip.

- Determine the drift speed of conduction electrons in the strip
- Determine the Hall Voltage V .

Note: The number of conduction electrons per unit volume in Copper is 8.5×10^{28} per cubic meter, and the electron charge is $-1.6 \times 10^{-19}C$.

Answers: $1.1 \times 10^{-3}m/s$; $22\mu V$.

- The Dees of a cyclotron of radius 50 cm are operated at an oscillator frequency of 10 MHz to accelerate protons.
 - What is the (uniform) magnetic field in the cyclotron?
 - What is the kinetic energy of the protons that leave the cyclotron, in electron Volts?

Note: $e_0 = 1.6 \times 10^{-19}C$

$$m_p = 1.67 \times 10^{-27}kg$$

$$m_p c^2 = 938MeV.$$

Answers: $0.66T$; 54 MeV.

- An electron with a kinetic energy of $8 keV$ is projected into a uniform magnetic field of magnitude $0.2 T$, with its velocity vector making an angle of 80° with respect to the magnetic field. The electron thus travels along a helical path around a magnetic field line. Determine the following quantities:
 - The orbital period in the plane perpendicular to the magnetic field.

- The orbital radius.
- The pitch of the helical path.

Answers: 1.8×10^{-10} sec; $1.48 \times 10^{-3}m$; $1.65 \times 10^{-3}m$.

Chapter 29: Magnetic Fields Due to Currents

Biot-Savart Law:

$$d\vec{B} = \mu_0 I / (4\pi) d\vec{\ell} \times \vec{r} / r^3$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ Tm/A}$$

Long Straight wire - current I :

$$B_t = \mu_0 I / (2\pi r) \text{ right hand rule.}$$

$$\text{Circular arc center: } B = \mu_0 I / (2r) \cdot \theta / (2\pi)$$

Force between wires: Like currents attract; unlike currents repel:

$$F = \mu_0 I_1 I_2 / (2\pi d)$$

$$\text{Ampère's Law: } \oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$$

$$\text{Ideal Solenoid: } B = \mu_0 NI / L$$

N turns; current I ; length L .

Previous Quizes

- A circular loop of wire of radius 0.01 meters carries a current of 100A.

Use the Biot-Savart Law to determine the magnitude of the magnetic field at the center of the loop, in Tesla. Determine the energy density caused by that magnetic field at the center of the loop, in Joules per cubic meter.

Answers: 6, .3mT; 16J/m³.

- The current density inside a long, solid, cylindrical wire of radius R lies in the direction of the central axis. Its magnitude varies linearly with the distance r from that central axis, $J = J_0 r/R$. Find the magnitude and direction of the magnetic field everywhere inside the wire, and the total current flowing in the wire.

Answers: tangential

$$\frac{\mu_0 J r^2}{3R} \quad \frac{2\Pi}{3} J_0 R^2$$

- A bare copper wire of radius 3.0 mm has an electric current of 50 Ampères flowing in it. Determine the magnitude and direction of the magnetic field at the surface of the wire.

Answer: 3.3mT; tangential.

- A 10 gauge bare copper wire, with a diameter of 2.6 mm, can carry a current of 50 A without overheating. For this current, what is the magnetic field at the surface of the wire?

Answer: 7.8mT.

Chapter 30: Induction and Inductance

Magnetic Flux: $\Phi_B = \int \vec{B} \cdot d\vec{S}$

Faraday's Law:

(closed loop)-(open surface)

$$\mathcal{E} = -d\Phi_B/dt = -d/dt(\int \vec{B} \cdot d\vec{S})$$

Self-Inductance (N turns, current I):

$$N\Phi_B = LI; (1 \text{ Henry} = 1 \text{ T m}^2/\text{A})$$

Long Solenoid (length ℓ , area A , N turns):

$$L = \mu_0 N^2 A / \ell$$

LR Circuit + battery: $\mathcal{E} = LdI/dt + RI$

$$I(t) = \mathcal{E}/R(1 - e^{-Rt/L})$$

No Battery: $LdI/dt + RI = 0$

$$I(t) = I_0 e^{-Rt/L}$$

Magnetic energy in inductor: $U_B = 1/2 LI^2$.

Magnetic energy density: $u = B^2/(2\mu_0)$

Mutual Inductance of two circuits: M

$$\mathcal{E}_2 = -MdI_1/dt; \mathcal{E}_1 = -MdI_2/dt$$

Previous Quizes

- A coil of 20 turns is wrapped tightly around a long solenoid of radius 2 cm and 200 turns per meter. What is the mutual inductance M of this system?

Answers:

- The current in an $L - R$ circuit attached to a battery reaches one third of its steady-state value a time of 10 seconds after the switch is closed. Determine the time constant for this circuit. If the resistance is $R = 1000\Omega$ (Ohms), and the battery produces an EMF of 10 Volts, determine the value of the inductance [in Henries] and the steady-state current.

Answer: $6.4\mu H$.

- At a certain place the earth's magnetic field is $B = 0.6G = 6 \times 10^{-5}$ Tesla. A flat circular coil of wire with a radius of 0.2 meters has 200 turns of wire, and lies in a circuit of total resistance 40Ω . The coil initially lies in a plane perpendicular to the direction of the magnetic field. It is then "flipped" by a half-revolution about a diameter. How much total charge flows through the coil during the flip?

Answer: $7.3 \times 10^{-4}C$.

- An air conditioning unit connected to a 120V AC line (60Hz) is equivalent to a 10W resistance in series with a 0.05H inductance.

Calculate the impedance of the air conditioner, in Ohms. Determine the average rate at which energy is supplied to the circuit, in watts.

Answers: 22Ω ; $320W$.

- A typical "light dimmer" consists of a variable inductor L (whose inductance is adjustable between zero and L_{max}) in series with the light bulb. The electrical supply is 120V, at 60Hz. The light bulb is rated at 1000 W, and the resistance of the light bulb is independent of temperature

– What maximum inductance L_{max} is required if the rate of energy dissipation in the bulb is required to vary from 200W to 1000W.

– We replace the variable inductor by a variable resistor, whose resistance is adjustable between zero and R_{max} . What value of R_{max} must be used, if the rate of energy dissipation in the bulb is required to vary from 200W to 1000W.

- Why are variable inductors used, instead of variable resistors?

Answers: $0.08H$; 14Ω ; because of power loss in resistor.

- A coil of unknown inductance L is connected in series with a resistor of electrical resistance $R = 200 \text{ Ohms}$. A 50 Volt battery is connected across the two devices. At 0.005 seconds after the connection is made, the current reaches a value of 3 milli-Ampères.
 - How much energy is stored in the coil at this same moment?
 - Find the inductance of the coil, in Henries.

Answers: $3.6 \times 10^{-4}J$; $480H$.

- A coil is connected in series with a 1000Ω resistor. An ideal 20 Volt battery is applied across the two devices. The current reaches a value of 0.01 A at a time of 2×10^{-3} seconds after the circuit is closed.
 - Determine the inductance of the coil.
 - How much energy is stored in the coil at this same moment?

Answers: $288H$; $1.4 \times 10^{-4}J$.

Chapter 31: Electromagnetic Oscillations

LC Circuit: $LdI/dt + Q/C = 0$;

$$I = dQ/dt.$$

$$Q(t) = Q_0 \cos(\omega t + \phi).$$

$$\omega^2 = 1/(LC)$$

LRC Circuit: (Damped Oscillations)

$$Ld^2Q/dt^2 + RdQ/dt + Q/C = 0$$

$$Q(t) = Ae^{-Rt/(2L)} \cos(\omega't + \phi)$$

$$\omega'^2 = 1/(LC) - R^2/(4L^2)$$

Forced Oscillations (steady state):

$$Ld^2Q/dt^2 + RdQ/dt + Q/C = \mathcal{E}_0 \sin(\omega t)$$

$$I = \mathcal{E}_0/|Z| \sin(\omega t + \phi)$$

$$Z = R + i(\omega L - 1/\omega C)$$

$$\omega^2 < 1/(LC); \phi < 0; I \text{ lags } V$$

$$\omega^2 > 1/(LC); \phi < 0; I \text{ leads } V$$

$$\omega^2 = 1/(LC); \phi = 0; \text{ resonance}$$

Previous Quizes

- In an oscillating LC circuit consisting of a $1.0 \mu F$ (microFarad) capacitor and a 1.0 mH (milliHenry) inductor, the maximum Voltage across the system is 10 Volts.
 - What is the maximum charge on the capacitor?
 - What is the maximum current through the circuit?
 - What is the maximum energy stored in the magnetic field of the coil?

Answers: $10\mu C$; $0.32A$; $50\mu J$.

- An oscillating LC circuit, which consists of a 2.0 nF capacitor and a 4.0 mH coil, has a maximum voltage of 3.0 V across the capacitor.
 - What is the maximum charge on the capacitor?
 - What is the maximum current through the inductor?
 - What is the maximum energy stored in the magnetic field of the coil?

Answers: $6nC$; $2mA$; $8nJ$.

- In an oscillating LC circuit with $L = 20mH$ and $C = 50 \mu F$, the maximum voltage across the inductor is 3.0 Volts .
 - What is the maximum charge on the capacitor?
 - What is the maximum current passing through the inductor?
 - What is the maximum energy stored in the inductor?
 - What is the period of oscillation of the Voltage across the capacitor?

Answers: $150\mu C$; $0.15A$; $230\mu J$; $6.3ms$

- A solenoid having an inductance of $4 \mu H$ is connected in series with a $3k\Omega$ resistor and a 20 Volt (DC) battery. The switch is closed at time $t = 0$.
 - What is the final current flowing in the circuit a long time after the switch is closed? How much energy is stored in the inductor at that point?

- At what time after the switch is closed is the current equal to 90 percent of its final value?

Answers: $6.7mA$; $90pJ$; $3.9ns$.

Chapter 32: Magnetism;

Maxwell's Equations

Gauss's Law for Magnetism:

$$\oint \vec{B} \cdot d\vec{S} = 0.$$

Magnetic Dipole: \vec{M} in field \vec{B} :

$$U = -\vec{M} \cdot \vec{B}; \vec{\tau} = \vec{M} \times \vec{B}.$$

Orbital Magnetic Moment \vec{M} ;

angular momentum \vec{L}

$$\vec{M}_L = q/(2m)\vec{L}$$

Spin magnetic moment: $\vec{M}_S = q/(2m)\vec{S}$

Diamagnetism: $\vec{B}_{induced}$ opposite \vec{B}_{ext} .

Unpaired atomic spins are possible.

Paramagnetism: $\vec{B}_{induced} || \vec{B}_{ext}$.

Curie-Weiss Law: $M = CB_{ext}/T$

Ferromagnetism:

(Permanent magnetic moment in absence of field).

Displacement Current I_d

(extension of Ampère's Law)

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0(I + I_d)$$

$$I_d = \epsilon_0 d\Phi_E/dt = \epsilon_0 d/dt(\int \vec{E} \cdot d\vec{S})$$

Maxwell's Equations

Gauss: $\epsilon_0 \oint \vec{E} \cdot d\vec{S} = Q_{enc}$

Magnetic Gauss: $\oint \vec{B} \cdot d\vec{S} = 0$

Faraday: $\oint \vec{E} \cdot d\vec{\ell} = -d/dt(\int \vec{B} \cdot d\vec{S})$

Ampère + Maxwell:

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0[I_{enc} + \epsilon_0 d/dt(\int \vec{E} \cdot d\vec{S})]$$

Chapter 33: Electromagnetic Waves

$$\vec{E} = \hat{i}E_0 \sin(kz - \omega t)$$

$$\vec{B} = \hat{j}B_0 \sin(kz - \omega t)$$

$$c = E_0/B_0 = \omega/k$$

$$c = 1/\sqrt{(\mu_0\epsilon_0)} = 3 \times 10^8 \text{ m/sec.}$$

Poynting Vector: (Watts/m²)

energy flux per unit area per unit time.

$$\vec{S} = \vec{E} \times \vec{B}/\mu_0$$

Radiation pressure $P = F/A = |\vec{S}|/c$.

Polarization defined as direction of \vec{E}

Polarizer: light initially unpolarized.

$$I_f = 1/2I_0$$

Malus Law: $I = I_0 \cos^2 \theta$

\vec{E} makes angle θ with polarizer.

Reflection: $\theta_i = \theta_r$

angle of incidence = angle of reflection.

Refraction:

Snell's Law medium 1 - medium 2.

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

n : index of refraction.

θ angle with surface normal.

Total internal reflection: $n_2 < n_1$:

$$n_1 \sin \theta_1 > n_2.$$

Polarization by reflection:

Brewster's angle: reflected ray polarized.

(\vec{E} perpendicular to plane of reflection.)

$$\theta_1 + \theta_2 = 90^\circ.$$

Previous Quizes

- Suppose that a parallel-plate capacitor has circular plates with radius 10 cm, and a plate separation of 0.5 mm. Suppose that a sinusoidal voltage is applied to the plates, with frequency of 60 Hz and maximum voltage of 150 Volts. Determine the induced magnetic field inside the capacitor, and along the rim edge.

Answer: $6.3 \times 10^{-11} T$.

- A parallel plate capacitor with circular plates 20 cm in diameter is being charged. The current density of the displacement current between the plates is uniform, having a magnitude of $20 A/m^2$.
 - Calculate the magnetic field B at the edge of the capacitor (10 cm from its center).
 - Determine the rate of change of the electric field with respect to time, dE/dt , inside the capacitor.

Answers: $1.3 \times 10^{-6} T$; $2.5 \times 10^{12} V/(ms)$.

- A parallel plate capacitor has circular plates of radius $R = 0.2$ meters, which are separated by a distance $d = 2 \times 10^{-4}$ meters. The plates of the capacitor are being charged with a steady current of $I = 5.0$ Amps. Determine the total displacement current (in Amps) inside the capacitor, as well as the magnetic field B at the outer edge of the capacitor (in Tesla), while it is being charged.

Answers: $5A$; $5 \times 16^{-6} T$.

- A parallel plate capacitor with circular plates of radius $R = 0.3$ meters is being discharged. A circular loop of radius $r = 0.1$ meters is concentric with the capacitor, and halfway between the plates. The displacement current in that loop is measured to be 1 Amp. Determine the rate of change of the electric field between the plates.

Answers: $3 \times 10^{12} V/(ms)$.