Physics 123

Quizes and Examinations

Spring 2007

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"Physics can only be learned by thinking, writing, and worrying."
-David Atkinson and Porter Johnson (2002)

"There is no royal road to geometry."
-Euclid

Quizes: Spring 2007

1. PHYS 123 - 051 QUIZ 1 30 January 2007

   The spool of dental floss is cylindrical, with a core of inner radius $r_1 = 1\ cm$ and an outer radius of $r_2 = 5\ cm$, which is $H = 10\ cm$ in length. The dental floss is $t = 1\ mm$ by $w = 3\ mm$ in cross-section, and it is tightly wound around the spool from the inner radius to the outer radius.

   Determine the length D of dental floss that is required, in meters.

   <u>Solution</u>:

   The volume of dental floss may be calculated as the volume of the outer cylinder of radius r_2 and length H, minus the volume of the inner cylinder of radius r_1 and height H:

   $$V_{floss} = \pi r_2^2 H - \pi r_1^2 H = \pi(r_2^2 - r_1^2)H = \pi(5^2 - 1^2)10\ cm^3 = 240\pi\ cm^3$$

   It may also be calculated as the cross-sectional area of the floss multiplied by its length:

   $$V_{floss} = t \times w \times D = (0.1\ cm) \times (0.3\ cm) \times D$$

   These two volumes must be the same, so that

   $$\begin{aligned} 0.03D &= 240\pi \\ D &= 8000\pi\ cm \approx 25000\ cm = 250\ m \end{aligned}$$

   How long will this dental floss last?

2. PHYS 123 - 052 QUIZ 1 01 February 2007

   A startled armadillo leaps upward, rising $H_1 = 0.6$ meters in the first $t_1 = 0.2$ seconds.

   - What is its initial speed v_0 as it leaves the ground?
   - What is its speed v_1 at the height of 0.6 meters?
   - How much higher (y) does it go?

- How long (T) does it stay in the air?

Solution:

The height H_1 at time t_1 is expressed in terms of v_0 as

$$H_1 = v_0 t_1 - \frac{1}{2} g t_1^2$$

So that v_0 can be determined:

$$\begin{aligned} 0.6 &= 0.2 v_0 - 5 \times (0.2)^2 \\ 0.6 &= 0.2 v_0 - 0.2 \\ v_0 &= 4\ m/s \end{aligned}$$

The velocity v_1 at height H is given by

$$\begin{aligned} v_1 &= v_0 - g t_1 \\ &= 4 - 10 \times 0.2 \\ &= 2\ m/s \end{aligned}$$

The additional height y to which it climbs can be determined from the relation

$$\begin{aligned} v_{top}^2 &= v_1^2 - 2gy \\ 0 &= 2^2 - 2 \times 10 \times y \\ y &= 0.2\ m \end{aligned}$$

The total time T that the armadillo is in the air is the (non-trivial) solution of the equation

$$0 = v_0 T - \frac{1}{2} g T^2$$

That is,

$$T = \frac{2v_0}{g} = \frac{2\times 4}{10} = 0.8\,sec$$

How long can a person stay in the air? Why is it that many non-flying creatures can remain aloft for about the same amount of time?

3. PHYS 123 - 051 QUIZ 2 13 February 2007

The following two vectors are given:

$$\begin{aligned}\vec{A} &= 2\hat{i}+2\hat{j}+\hat{k}\\ \vec{B} &= \hat{i}-2\hat{j}+2\hat{k}\end{aligned}$$

- Determine the scalar product $\vec{A}\cdot\vec{B}$.
- Determine the vector product $\vec{A}\times\vec{B}$.
- Determine the angle between the vectors $\vec{A}$ and $\vec{B}$.

<u>Solution</u>:

The scalar product is expressed in terms of components as

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z = (2)(1)+(2)(-2)+(1)(2) = 0$$

Since the scalar product vanishes, the angle between the vectors is 90^o.

The vector product is

$$\vec{A}\times\vec{B} = \left| \begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{array} \right] = 6\hat{i}-3\hat{j}-6\hat{k}$$

As a consistency check, note that $|\vec{A}| = 3$, $|\vec{B}| = 3$, and $|\vec{A}\times\vec{B}| = 9$, so that

$$\begin{aligned}|\vec{A}\times\vec{B}| &= |A||B|\sin\theta\\ 9 &= (3)(3)\sin\theta\\ \sin\theta &= 1\\ \theta &= 90^o\end{aligned}$$

4. PHYS 123 - 052 QUIZ 2 15 February 2007

A long jumper leaves (level) ground at a speed of $v_0 = 10$ meters per second.

- What is the maximum distance D that the jumper will travel before returning to the ground?
- Under those conditions, determine the time of flight T and the maximum height H reached by the jumper.

Note: you may treat the jumper as a "point mass", and take $g = 10m/s^2$.

Solution:

Let the launch angle by θ. The horizontal and vertical components of velocity are, respectively,

$$\begin{aligned} v_{0x} &= v_0 \cos\theta \\ v_{0y} &= v_0 \sin\theta \end{aligned}$$

If the jumper takes off at time $t = 0$ from positions $x = 0$ and $y = 0$, the positions at time t are

$$\begin{aligned} x &= v_0 \cos\theta t \\ y &= v_0 \sin\theta t - \frac{1}{2} g t^2 \end{aligned}$$

The time T at which the jumper returns to the ground is obtained by setting $y(T) = 0$, so that

$$\begin{aligned} v_0 \sin\theta T &= \frac{1}{2} g T^2 \\ \frac{2 v_0 \sin\theta}{g} &= T \end{aligned}$$

The distance traveled, D, is given by

$$D = v_{0x} T = \frac{v_0^2}{g} (2 \sin\theta \cos\theta) = \frac{v_0^2}{g} \sin 2\theta$$

The distance D depends upon the launch angle θ. The greatest distance is obtained by setting

$$\begin{aligned} sin 2\theta &= 1 \\ 2\theta &= 90^o \\ \theta &= 45^o \end{aligned}$$

The maximum distance is

$$D = \frac{v_0^2}{g} = \frac{10^2}{10} = 10\ m$$

The maximum height reached is

$$H = \frac{1}{2} g (T/2)^2 = \frac{1}{2} g (\frac{v_0 \sin\theta}{g})^2 = \frac{1}{4}\frac{v_0^2}{g} = 2.5\ m$$

Both these numbers exceed the world record for high jump ($H = 2.45\ m$) as well as long jump ($D = 8.95\ m$).

5. PHYS 123 - 051 QUIZ 3 27 February 2007

A lamp hangs vertically down from a light cord attached to the ceiling of an elevator.

- When the elevator is descending, and decelerating at 3 m/s2, there is a tension of 80 N in the cord. What is the mass of the lamp?
- While still descending, the elevator begins to accelerate at 3 m/s2. What is the tension in the cord in that case?

Solution:

Take the positive direction to be downward. The acceleration of the elevator is $a = -3m/sec^2$. If m is the mass of the lamp, the net force acting on it is $F = ma$. The two forces on the lamp are its weight $W = mg$ (downward) and the tension $T = -80\ N$ (upward). Thus

$$\begin{aligned}
mg - T &= ma \\
m(g-a) &= 80 \\
m(10-(-3)) &= 80 \\
m &= \frac{80}{13} = 6.2\ kg
\end{aligned}$$

For the second part of the problem, $a' = +3\ m/s^2$, so that

$$\begin{aligned}
mg - T' &= ma' \\
m(g-a') &= T' \\
T' &= m(10-3) = 6.2(7) = 43\ N
\end{aligned}$$

6. PHYS 123 - 052 Quiz 3 01 March 2007

   An old streetcar rounds a flat corner of radius 10 meters at a speed of 8 meters/second. What angle with the vertical will be made by the hand straps, which hang loosely?

   Solution:

   For simplicity we assume that the straps consist of a massless string, with a mass m at its bottom. The forces acting on the mass are its weight $W = mg$ (downward) and the tension in the string T, acting along the string at an angle θ to the vertical. The net vertical component of force on the mass must vanish, whereas the net horizontal force on it must be equal to its mass multiplied by the centripetal acceleration. Thus

$$\begin{aligned}
mg &= T\cos\theta \\
\frac{mv^2}{R} &= T\sin\theta
\end{aligned}$$

   Taking the ratio, we obtain

$$\tan\theta = \frac{v^2}{Rg} = \frac{8^2}{10\cdot 10} = 0.64$$

   The angle is $\theta = 32^o$. This is a rather sharp turn!

7. PHYS 123 - 051 Quiz 4 20 March 2007

   A fully loaded, slow-moving, freight elevator cab has a total mass of $m_1 = 1500\ kg$. It is must move upward by $h = 60$ meters in $\Delta t = 2.0$ minutes, starting and ending at rest. The mass of the elevator counterweight is only $m_2 = 1000\ kg$, so that the elevator motor must help in the ascent.

   What average power P (in Watts) is required of the force that the motor exerts on the cab by means of the cable?

   <u>Solution</u>:

   The change (an increase) in the gravitational potential of the energy of the elevator cab is

$$\Delta\mathcal{E}_1 = m_1\, g\, h = 1500 \cdot 10 \cdot 60 = 9 \times 10^5\ J$$

   The change (a decrease) in gravitational potential energy of the counterweight is

$$\Delta\mathcal{E}_2 = -m_2\, g\, h = -1000 \cdot 10 \cdot 60 = -6 \times 10^5\ J$$

   The net change in gravitational potential energy is

$$\Delta\mathcal{E} = \Delta\mathcal{E}_1 - \Delta\mathcal{E}_2 = 3 \times 10^5\ J$$

   The energy must be provided by the elevator motor, over a period $\Delta t = 120$ seconds. Thus, the power provided by the motor is

$$P = \frac{\Delta\mathcal{E}}{\Delta t} = \frac{3 \times 10^5\ J}{120\ sec} = 2.5 \times 10^3\ W = 2.5\ kW$$

   Since one horsepower is approximately 750 Watts, the motor must provide energy at the rate of about $3.3\ HP$.

   Elevators are rated according to the "maximum load" that they can carry, according to the maximum power that can be provided by the driving motor at a certain speed. Roughly speaking, the mass of the counterweight should match the mass of the elevator plus its "average" load.

8. PHYS 123 - 052 Quiz 4 22 March 2007

   You push an $m = 5.0\ kg$ block against a light horizontal spring, compressing that spring against a wall by $x = 20\ cm$. Then you release the block, and the spring sends it across a tabletop. It stops at $d = 80\ cm$ from where you released it. The spring constant is $300\ N/m$.

   What is the coefficient of kinetic friction μ_k between the block and the table?

   <u>Solution</u>:

   All of the potential energy stored in the spring is given to the block during the release:

$$\mathcal{E} = \frac{1}{2}\,k\,x^2 = \frac{300 \times (0.2)^2}{2} = 6\ J$$

   This energy is all dissipated as the block slides across the floor. When a kinetic friction force f_k acts over a distance d, the dissipated energy is $f_k\,d$, so that

$$\begin{aligned} f_k\,d &= \mathcal{E} \\ f_k(0.8) &= 6 \\ f_k &= 7.5\,N \end{aligned}$$

   The normal force of the block on the bable is equal in magnitude to its weight; $N = mg = 5 \cdot 10 = 50\,N$. Thus, the coefficient of kinetic friction is

$$\mu_k = \frac{f_k}{N} = \frac{7.5}{50} = 0.15$$

9. PHYS 123 - 051 QUIZ 5 03 April 2007

   An object of mass $m = 3\ kg$ is initially moving with a speed of $v_0 = 8$ meters per second in the $+x$-direction. It has an elastic collision with an object of mass M, which is initially at rest. After the collision the object of mass M has a speed of $V = 6$ meters per second, also in the $+x$-direction.

   - Determine the mass M.

Solution:

The momentum (horizontal) and energy are both conserved, so that

$$\begin{aligned} mv_0 &= mv + MV \\ \frac{1}{2}mv_0^2 &= \frac{1}{2}mv^2 + \frac{1}{2}MV^2 \end{aligned}$$

We solve the first equation for v:

$$v = v_0 - \frac{M}{m}V$$

We substitute this into the second equation to obtain

$$\begin{aligned} v_0^2 &= \left(v_0 - \frac{M}{m}V\right)^2 + \frac{M}{m}V^2 \\ \frac{M}{m}(2V\,v_0 - V^2) &= \left(\frac{M}{m}\right)^2 V^2 \\ \frac{M}{m} &= \frac{2v_0 - V}{V} = \frac{2 \cdot 8 - 6}{6} = \frac{5}{3} \end{aligned}$$

Thus, $M = 5\ kg$.

10. PHYS 123 - 052 QUIZ 5 05 April 2007

An $M = 4000\ kg$ block falls vertically through $h = 12$ meters, and then collides with a a $m = 500\ kg$ pile, driving it $x = 20\ cm$ into solid ground, before the block comes to rest on top of the pile.

Determine the average force F on the pile by the solid ground. during its collision with the block.

Note: consider the collision between the block and the pile to be completely inelastic.

Solution:

The speed of the block before the collision with the pile is given by equating its initial gravitational potential energy to its kinetic energy just before the collision:

$$\begin{aligned}\frac{1}{2}MV^2 &= Mgh \\ V &= \sqrt{2gh} = \sqrt{2\cdot 10\cdot 40} = \sqrt{240} = 15.5\ m/sec\end{aligned}$$

During the completely inelastic collision between the block and the pile, the momentum is conserved:

$$\begin{aligned}(M+m)V' &= MV \\ V' &= \frac{M}{M+m}V = \frac{400}{450}15.5 = 13.8\ m/sec\end{aligned}$$

The kinetic energy after the collision equals the work done against friction

$$\begin{aligned}F\,x &= \frac{1}{2}(M+m)\,V'^2 \\ F(0.2) &= \frac{1}{2}(450)(13.8)^2 = 4.0\times 10^5\ J \\ F &= 2.0\times 10^6\ N\end{aligned}$$

11. PHYS 123 - 051 QUIZ 6 17 April 2007

A thin uniform rod of length $\ell = 2$ meters and mass $m = 0.1\ kg$ is pivoted about a horizontal, frictionless pin put through one end. It is released at an angle $\theta = 60^o$ above the horizontal. Use energy conservation to determine the speed v of the moving end of the rod as it passes through the horizontal position.

Solution:

The initial potential energy of the rod is $m\,g\,h$, where $h = \ell\sin\theta/2$ is the height of its center of mass above the pivoted end. As it swings through the horizontal position, its kinetic energy is $K = 1/2I\,\omega^2$, where ω is its angular velocity at that instant, and L is the moment of inertia of the rod about its pivot point. We divide the rod into infinitesimal slices of thickness dx and mass $dm = m\,dx/\ell$. The moment of inertia of the rod about its end is

$$I = \int x^2 dm = \int_0^\ell x^2 \frac{m\, dx}{\ell} = \frac{m}{\ell} \int_0^\ell x^2 dx = \frac{m}{\ell} \frac{\ell^3}{3} = \frac{m\ell^2}{3}$$

Because of conservation of energy we obtain

$$\frac{1}{2} mg\ell \sin 60^o = \frac{1}{2} I\, \omega^2 = \frac{1}{2}\, \frac{1}{3}\, m\, \ell^2\, \omega^2$$

Thus

$$\begin{aligned} \omega^2 &= \frac{3\sqrt{3}}{2}\, \frac{g}{\ell} = 2.6 \times 5 = 13 \\ \omega &= 3.6\, rad/sec \end{aligned}$$

The speed at the moving end is $v = \omega\, \ell = 7.2\ m/s$.

12. PHYS 123 - 052 QUIZ 6 19 April 2007

A bowler throws a bowling ball of radius $R = 10\ cm$ along a horizontal lane at an initial speed $v_0 = 8\ m/s$. The ball slides along the lane with initial angular velocity $\omega_0 = 0$. The coefficient of kinetic friction between the ball and the lane is $\mu_k = 0.2$ This kinetic friction force produces a torque on the ball, resulting in an angular acceleration of the ball. When the linear speed v and angular velocity ω satisfy $v = \omega\, R$, the ball begins rolling without slipping.

Note: The moment of inertia of the ball about its center is $I = 2m\, R^2/5$.

What is the linear speed v of the ball when it begins to roll without slipping?

How far (D) does the ball travel before it begins to roll without slipping?

Solution:

There are three forces: the weight mg downward, the normal force $N = mg$ upward, the friction force $f_k = \mu_k\, mg$ (left), opposite the direction of motion. The net force is equal to the friction force. Thus, the component of acceleration in the direction of motion is $a = -\mu_k\, g$. The friction force also produces a (clockwise) torque about the center of the ball: $\tau = f_k R$. Thus, there is an angular acceleration $\alpha = \tau/I = 5\ f_k/2\ mR^2$. The velocity v and angular velocity ω at time t are

$$\begin{aligned} v &= v_0 - \mu_k g t \\ \omega &= \alpha t = \frac{5\mu_k\ mg\ t\ R}{2\ mR^2} \end{aligned}$$

We set $v = \omega R$ to determine the time at which slippage stops:

$$\begin{aligned} v_0 - \mu_k\ g\ t &= \frac{5\mu_k\ g\ t}{2R} R \\ v_0 &= \left[\mu_k g - \frac{5\mu_k g}{2}\right] t \\ 8 &= \left[0.2 \cdot 10 - \frac{5 \cdot 0.2 \cdot 10}{2}\right] t = 7t \\ t &= 1.14\, sec \end{aligned}$$

The velocity at this time is

$$v = v_0 - 2 \cdot 1.14 = 5.7\ m/s$$

The distance traveled is

$$d = \frac{v_0 + v}{2} t = \frac{13.7}{2} 1.14 = 7.8\ m$$

After this point, the ball begins to roll without slipping. Typically, a bowling ball "hooks" at this point, because of the initial spin given to it by the bowler. For the best shots a right-handed player should hit the $1-3$ pocket, to compensate for this spin.

13. PHYS 123 - 051 QUIZ 7 24 April 2007

A uniform ladder of length $L = 10$ meters has a mass $m = 40$ kilograms. It is leaned against a frictionless vertical wall at an angle θ to the vertical, while being supported at its base on a horizontal surface. The coefficient of static friction between the ladder and the horizontal surface is $\mu_s = 0.15$.

Determine the maximum tilt angle θ at which the ladder can be in static equilibrium.

Solution:

The force F produced by the wall on the ladder is out of the wall, whereas the static friction f_s at the base is toward the wall. The weight W is downward at the center of the ladder, whereas the normal force N at the base of the ladder is upward. For equilibrium, $F = f_s$ and $N = W = m\,g$.

We also require that the net torque about the base of the ladder is zero. The normal force N and static friction f_s produce no torque at this point. The force F produces a torque that would cause the ladder to rotate away from the wall, whereas the weight produces a torque to cause rotation toward the wall. We obtain

$$\begin{aligned} F\,L\,\cos\,\theta &= m\,g\,\frac{L}{2}\,sin\,\theta \\ F &= \frac{m\,g}{2}\,\tan\,\theta \end{aligned}$$

For the critical case $f_s = \mu_s N$, so that

$$\begin{aligned} \frac{m\,g}{2}\,\tan\,\theta &= m\,g \\ \tan\,\theta &= 2\mu_s = 0.3 \\ \theta &= 17^o \end{aligned}$$

14. PHYS 123 - 052 QUIZ 7 26 April 2007

One end of a uniform beam of mass $m = 20$ kilograms and length $L = 1$ meter is attached to a wall with a hinge. The beam hangs at an angle $\theta = 30^o$ below the horizontal direction. The other end of the beam is supported by a light wire, which is attached to the wall above the beam. The wire makes an angle $\phi = 30^o$ with the vertical direction.

- Find the tension T in the wire.
- What are the horizontal H and vertical V components of the force at the hinge?

Solution:

PHYS 123 - 051/052 TEST 1 19 February 2007

1. [20 points] A villain in a car is traveling at 30 meters per second on a straight, horizontal road. He fires a projectile at a launch speed of 50 meters per second relative to the car, in the plane determined by the vertical direction and the direction of the path of the car. A hero standing nearby observes the projectile to travel straight up.

   - What was the launch angle (relative to the direction of the car) as viewed by the villain?
   - What height does the projectile attain?
   - How far ahead of the projectile is the villain's car when the projectile hits the ground?

   <u>Solution</u>:

   The initial velocity of the projectile with respect to the (heroic) observer $\vec{v}_{po}$ is the sum of the velocity of the projectile relative to the car $\vec{v}_{pc}$ and the velocity of the car relative to the observer $\vec{v}_{co}$. That is,

   $$\vec{v}_{po} = \vec{v}_{pc} + \vec{v}_{co} = 50(\cos\theta\hat{i} + \sin\theta\hat{j}) + 30\hat{i} = (50\cos\theta + 30)\hat{i} + 50\sin\theta\hat{j}$$

   Since the horizontal component of $\vec{v}_{po}$ is zero, we have

   $$\begin{aligned} 50\cos\theta + 30 &= 0 \\ \cos\theta &= -0.6 \\ \theta &= 123^o \end{aligned}$$

   The initial vertical speed of the projectile relative to the observer is thus $v_{yo} = 50\cos\theta = 40\ m/s$.

   The projectile travels upward to a height $h = v_{yo}^2/(2g) = 40^2/20 = 80\ m$, The projectile stays in the air for a time $T = 2v_{0y}/g = 8\ sec$. The villain's car travels a distance of $30\ m/sec \times 8\ sec = 240m$ during this time.

2. [20 points] A rifle is aimed horizontally at a target 50 meters away. The bullet hits the target 4 cm below the aiming point.

- What is the time of flight of the bullet?
- What is the speed of the bullet as it emerges from the rifle?

Solution:

The vertical distance fallen by the bullet at time t is given by

$$y = \frac{g}{2}t^2$$

Thus $0.04\, m = 5\, m/sec^2 t^2$, or $t^2 = 0.008\, sec^2$, or $t = .089\, sec$.

The bullet travels a horizontal distance $L = 50\, m$ in this time, so that its horizontal speed is

$$v_x = \frac{L}{t} = \frac{50}{0.089} = 560\, m/s$$

3. [20 points] A boy whirls a stone attached to a string above his head in a circle of radius 1 meter lying in a horizontal plane, at a height of 2 meters above ground level. The string breaks, and the stone flies off in a horizontal direction. The stone strikes the ground after traveling a horizontal distance of $30m$.

   - What is the initial speed of the stone?
   - What is the centripetal acceleration of the stone in the circular path?
   - What is the period of motion of the stone in the circular path?

   Solution:

   The stone falls a distance $y = 2\, m$ in a time t, so that

$$y = \frac{g}{2}t^2$$
$$2 = \frac{10}{2}t^2$$
$$\begin{aligned} t^2 &= 0.4 \\ t &= 0.63\, sec \end{aligned}$$

It travels a horizontal distance $L = e0\ m$ during this time, so that its horizontal speed is

$$v_0 = \frac{L}{t} = \frac{30}{0.63} = 47\ m/s$$

The period of motion around a circular path of radius $R = 1\ m$ at this speed is

$$T = \frac{2\pi R}{v_0} = \frac{6.28}{47} = 0.13\ sec$$

The centripetal acceleration is

$$a_c = \frac{v_0^2}{R} = \frac{47^2}{1} = 2250\ m/s^2$$

4. [20 points] Two rocks are dropped from the top of a tall building at the same point, one second apart.

   - Determine the time after the first rock is dropped, at which the rocks are 10 meters apart.
   - Determine the distances fallen by each rock at this time.

   <u>Solution</u>:

   The first rock falls for a time t, traveling downward by a distance

   $$D_1 = 1/2gt^2$$

   The second rock falls for a time $t-1$, travelling downward by a distance

   $$D_2 = 1/2g(t-1)^2$$

   Since $D_1 = D_2 + 10$, we have

$$\begin{aligned}
\frac{g}{2}t^2 &= \frac{g}{2}(t-1)^2+10 \\
5t^2 &= 5(t-1)^2+10 \\
0 &= -10t+5+10 \\
10t &= 15 \\
t &= 1.5\,sec
\end{aligned}$$

The distances traveled are $D_1 = 5(1.5)^2 = 11.25\ m$ and $D_2 = 5(.5)^2 = 1.25\,m$.

5. [20 points] An explorer is caught in a blizzard in which she cannot distinguish the ground from the sky, while returning to camp. She had intended to travel due North for 6 *km*, but instead has traveled 8 *km* at 30^o East of North. How far, and in what direction, must she now travel to reach base camp?

   Solution:

   The vector from the original location to the camp is

   $$\vec{R} = 0\hat{i} + 6\hat{j}$$

   Whereas the vector corresponding to the original travel is

   $$\vec{R}_1 = (8\sin 30^o)\hat{i} + (8\cos 30^o)\hat{j} = 4\hat{i} + 6.92\hat{j}$$

   She must travel along $\vec{R}_2$, where $\vec{R} = \vec{R}_1 + \vec{R}_2$, so that

   $$\vec{R}_2 = \vec{R} - \vec{R}_1 = 6\hat{j} - (4\hat{i} + 6.92\hat{j}) = -4\hat{i} - 0.92\hat{j}$$

   The path length is 4.1 *km*, and the direction is 13^o South of West.

6. [Extra Credit; 10 points] A human heart pumps blood at the rate of about 100 cubic centimeters per second. The total blood volume is 6 liters.

   - How long does it take for the blood to complete one trip through the circulatory system?

- If the average path length for circulation is about 3 meters, what is the average speed of blood in the body?

Solution:

The rate of blood flow is 100 cm^3 sec, and the total volume of blood is 6000 cm^3. Thus, the average time of circulation is

$$T = \frac{6000\; cm^3}{100\; cm^3/sec} = 60\; sec$$

The average speed of blood flow is

$$v_{avg} = \frac{3\; m}{60\; sec} = 0.05\; m/s$$

PHYS 123 - 051/052 TEST 2 02 April 2007

1. [20 points] A 0.2 kg hockey puck has a velocity of 3 meters per second toward the East as it slides across the frictionless horizontal surface of an ice rink. A player strikes the puck with his hockey stick, causing it to change its velocity to 4 meters per second, to the South. The time interval for contact of the stick with the puck is 0.4 seconds. Assume that the applied force is constant in magnitude and direction over this time interval.

   - What is the magnitude and direction of the force on the puck applied by the hockey stick?

   Solution:

   The initial momentum of the hockey puck is $\vec{p}_i = (0.2)3\hat{i} = 0.6\hat{i}$, and its final momentum is $\vec{p}_f = -(0.2)4\hat{j} = -0.8\hat{j}$. Its change in momentum is

   $$\vec{\Delta p} = \vec{p}_f - \vec{p}_i = -0.6\hat{i} - 0.8\hat{j}$$

   The average force acting on the puck is

   $$\vec{F}_{avg} = \frac{\vec{\Delta p}}{\Delta t} = \frac{-0.6\hat{i} - 0.8\hat{j}}{0.4} = -1.5\hat{i} - 2.0\hat{j}$$

   The average force is of magnitude 2.5 N, acting at 233^o to the positive x-axis. In other words, it is 53^o South of West.

2. [20 points] An $m = 70$ kilogram crate is dragged across a floor by pulling on a rope attached to the crate, which is inclined at an angle of $\theta = 20^o$ above the horizontal.

- If the coefficient of static friction is $\mu_s = 0.5$, what is the minimum magnitude on the force T on the rope required to set the crate into motion?
- If the coefficient of kinetic friction is $\mu_k = 0.3$, what is the magnitude of the initial acceleration a of the block, with the minimum force T applied?

<u>Solution</u>:

The forces acting on the block are its weight mg(downward), the normal force N (upward), the static friction force f_s to the left, and the applied force T to the right and upward. We set the net force equal to zero:

$$\begin{aligned} N &= mg - T\sin\theta \\ f_s &= T\cos\theta \end{aligned}$$

In the critical case $f_s = \mu_s N$, so that

$$\begin{aligned} T\cos\theta &= \mu_s\left[mg - T\sin\theta\right] \\ T\left[\cos 20^o - 0.5\sin 20^o\right] &= 0.5 \cdot 70 \cdot 10 = 350 \\ T &= \frac{350}{1.11} = 315\,N \end{aligned}$$

When the block moves, we have kinetic friction, so that

$$\begin{aligned} ma &= F_x = T\cos\theta - 0.3(mg - T\sin\theta) \\ 70a &= (315)(0.92) - 0.3(700 - 315 \cdot 0.34) = 115 \\ a &= 1.65\,m/s^2 \end{aligned}$$

3. [20 points] An $m = 200$ gram block is dropped onto a relaxed light vertical pan balance spring of spring constant $k = 300\ N/m$. The block becomes attached to the spring, and compresses the spring by $x = 20\ cm$ before stopping momentarily.

   - From what height h above the spring was the block initially dropped?
   - What was the speed v of the block when it made impact with the spring?

   Solution:

   The block falls a distance $h + x$, and loses gravitational potential energy $mg(h+x)$, compressing the spring a distance x, and storing potential energy $kx^2/2$ in the spring. Thus

$$\begin{aligned} mg(h+x) &= \frac{1}{2}kx^2 \\ h &= \frac{kx^2}{2mg} - x = \frac{300 \cdot (0.2)^2}{2 \cdot 0.2 \cdot 10} - 0.2 = 3.0 - 0.2 = 2.8\ m \end{aligned}$$

   The speed of the block when it hits the spring is given by

$$\begin{aligned} \frac{1}{2}mv^2 &= mgh \\ v^2 &= 2gh = 2 \cdot 10 \cdot 2.8 = 56 \\ v &= 7.5\ m/s \end{aligned}$$

4. [20 points] A factory worker accidentally releases an $m = 200\ kg$ crate that was being held at rest at the top of a ramp that is $d = 3$ meters long, and inclined at $\theta = 30^o$ to the horizontal. The coefficient of kinetic friction between the crate and the ramp is $\mu_k = 0.2$. The crate continues to move on a horizontal floor, with the same coefficient of kinetic friction.

   - How fast v is the crate moving when it reaches the bottom of the ramp?
   - After reaching the bottom of the ramp, how far ℓ does the crate slide across the horizontal floor?

Solution:

The gravitational potential energy decreases by an amount $mgh = mgd\sin\theta$. This equals the sum of the kinetic energy of the object $mv^2/2$, and the energy lost to friction, $f_k d = \mu_k N d = \mu_k mgd\cos\theta$:

$$\begin{aligned}
\frac{1}{2}mv^2 + \mu_k mgd\cos\theta &= mgd\sin\theta \\
v^2 &= 2gd(\sin 30^o - \mu_k\cos 30^o) \\
v^2 &= 2\cdot 10\cdot 3(0.5 - 0.2\cdot 0.87) = 20 \\
v &= 4.5\, m/s
\end{aligned}$$

The distance traveled across the floor is given by equating the kinetic energy of the crate at the bottom with the energy lost to friction on the horizontal floor:

$$\begin{aligned}
\mu_k mg\ell &= \frac{1}{2}mv^2 \\
\ell &= \frac{v^2}{2\mu_k g} = \frac{20}{2\cdot 0.2\cdot 10} = 5\, m
\end{aligned}$$

5. [20 points] A projectile of mass $m = 500$ kilograms is initially moving horizontally at a speed of 300 meters per second, at a height of 2 kilometers above the earth's surface. A sudden explosion separates it into two components, of mass $m_1 = 400$ kilograms and $m_2 = 100$ kilograms, respectively. The 400 kg fragment falls straight downward, starting from rest just after the collision, as seen by an observer at rest on the ground.

   - What is the initial velocity v_2 of the 100 kg fragment?
   - How long t does it take the 100 kg fragment to strike the ground?
   - What is the velocity $\vec{v}$ of the 100 kg fragment when it hits the ground?
   - How far ℓ from the 400 kg fragment does the 100 kg fragment land?

   Solution:

   Momentum is conserved during the explosion, so that

$$\begin{aligned}(500)(300) &= 100\,v \\ v &= 1500\,m/s\end{aligned}$$

The time it takes for each fragment to hit the ground is given by

$$\begin{aligned}h = &= \frac{1}{2}gt^2 \\ t^2 &= \frac{2h}{g} = \frac{2\cdot 2000}{10} = 400 \\ t &= 20\,sec\end{aligned}$$

The small fragment of the rocket thus travels a horizontal distance

$$d = vt = 1500\,m/s \cdot 20\,sec = 30,000\,m = 30\,km$$

Its vertical velocity when it hits the ground is $v_y = -gt = -200\,m/s$, so that its velocity is

$$\vec{v} = (1500\hat{\imath} - 200\hat{j})\,m/s$$

The speed is 1510 m/s, at an angle of about 8^o below the horizontal.

6. [Extra Credit; 10 points] A circular curve of highway is designed for traffic moving at $v = 50$ kilometers per hour. The radius of the curve is $R = 100$ meters. Neglect air resistance and “sideways friction”?

   - What is the proper banking angle for traffic at this speed?

   <u>Solution</u>:

   The only forces acting on the car are its weight $\vec{W} = -mg\hat{j}$, downward, and the normal force $\vec{N} = N\cos\theta\hat{j} - N\sin\theta\,\hat{\imath}$, at an angle θ to the vertical and into the curve. The net force must be $\vec{F} = -\hat{\imath}\,mv^2/R$

   The speed of the car is

$$v = 50\,km/hr \times \frac{1.0\,m/s}{3.6\,km/hr} = 14\,m/s$$

The force components are thus

$$\begin{aligned} N\cos\theta &= mg \\ N\sin\theta &= mv^2/R \end{aligned}$$

Taking their ratio, we get

$$\tan\theta = \frac{v^2}{gR} = \frac{14^2}{100\cdot 10} = 0.2$$

The proper banking angle is thus $\theta \approx 11^o$.

PHYS 123 - 051/052 Final Examination 07 May 2007

1. [25 points] A railroad flatcar is loaded with crates that have coefficient of static friction $\mu_s = 0.1$ with the floor. If the train is initially moving with speed $v_0 = 40$ meters per second, what is the shortest distance D in which the train can be stopped, without having the crates to slide on the floor?

   Solution:

   The forces acting on a crate are its weight $W = m\,g$ (downward), a normal force N from the flatcar (upward), and a static friction force f_s – opposite to the (positive) direction of motion. The weight and the normal force are equal in magnitude and opposite in direction. They cancel, and the net force is that of static friction.

$$f_s = F_{net} = m\,a$$

   In the static case, the static friction force must obey the constraint

$$|f_s| \leq \mu_s N = \mu_s\,m\,g$$

For the critical case just before it slips, we have

$$\begin{aligned} f_s &= -\mu_s\, m\, g = m\, a \\ a &= -\mu_s\, g \approx -1\ m/s^2 \end{aligned}$$

For this uniform acceleration, the initial velocity v_0, the final velocity $v = 0$, the acceleration a, and the distance D traveled are related by the formula

$$v^2 = v_0^2 + 2\, a\, D$$

Thus

$$\begin{aligned} 0 &= (40)^2 + 2 \times (-1) \times D \\ D &= \frac{1600}{2} = 800\ m \end{aligned}$$

Note: The speed is about 75 mph, and the corresponding distance is about half a mile. It typically takes a comparable distance for a fast-moving train to stop at that speed.

distance is about half a mile, and the

2. [25 points] An automobile is traveling at $v_0 = 30$ meters per second, when the driver applies the brakes, bringing it to rest in $T = 8$ seconds. The radius of a wheel is $r = 40\ cm$. Assume that the deceleration is constant, and that the wheels do not slip on the road.

   - Determine the deceleration $-a$ of the automobile.
   - Determine the distance D traveled by the auto during deceleration.
   - Determine the number of revolutions N made by the wheel during the deceleration.

   <u>Solution</u>:

   The distance traveled by the auto satisfies the relation

$$\begin{aligned} D &= \frac{v_0+v_f}{2}\,T \\ &= \frac{30+0}{2}\cdot 8 = 120\;m \end{aligned}$$

Let us determine the acceleration from the formula $v_f = v_0 + a\,T$:

$$\begin{aligned} v_f &= v_0 + aT \\ 0 &= 30 + 8\,a \\ a &= -\frac{15}{4} = -3.75\;m/s^2 \end{aligned}$$

The number of revolutions by the tire can be obtained:

$$N = \frac{D}{2\pi\,R} = \frac{120}{\pi\cdot 0.8} = 47.7\;rev$$

3. [25 points] In a football game, a defensive player of mass $M = 120\;kg$ runs directly toward the ball carrier at $V = 7$ meters per second. The ball carrier has a mass of $m = 80\;kg$, and a speed $v = 9$ meters per second, directly at the defensive player. They undergo a perfectly inelastic collision.

   - Determine their final speed V_f , just after the collision.
   - How much mechanical energy $\Delta\mathcal{E}$ is lost in the collision?

   Solution:

   The ball carrier moves in the $+x$ direction and has positive momentum $p = mv$, whereas the defensive player moves in the $-x$ direction with momentum $P = -MV$. After the collision they move with a common speed V_f and have momentum $P_f = (M+m)\,V_f$. Since momentum is conserved in the collision, we have

$$\begin{aligned} m\,v - M\,V &= (M+m)\,V_f \\ 80\cdot 9 - 120\cdot 7 &= (120+80)\,V_f \\ 720-840 = -120 &= 200\,V_f \\ V_f &= -0.6\;m/s \end{aligned}$$

The total initial kinetic energy of the two players is

$$K_i = \frac{1}{2}\, m\, v^2 + \frac{1}{2} M\, V^2 = \frac{1}{2} 80 \times 81 + \frac{1}{2}\, 120 \times 49 = 3240 + 2940 = 6180\, J$$

The final kinetic energy is

$$K_f = \frac{1}{2}(M+m)\, V_f^2 = \frac{1}{2}\, 200 \times 0.36 = 36\, J$$

As a consequence, a total of 6144 J is converted into thermal energy.

4. [25 points] A girl of mass $m = 50\ kg$ sits on a merry-go-round (carousel) of radius $R = 10\ m$. She does not hold on, and sits facing outward. The tangential speed of the merry-go-round is steadily increased to a value, $v_T = 4\ m/s$, at which point the girl slides off. Determine the coefficient of static friction μ_s between the girl and the merry-go-round.

   Solution:

   The following forces act on the girl:

   - Her weight $w = m\, g = 500\ N$, down
   - The normal force n of the platform pushing on the girl, upward
   - Static Friction f_s, tangent to the edge of the platform

   The net force is the same as the static friction force, which must act toward the center of the platform. Thus, for the critical case,

$$f_s = m\, a = \frac{m\, v^2}{R} = 80\ N$$

   The coefficient of static friction is

$$\mu_s = \frac{f_{s\ max}}{N} = 0.16$$

5. [25 points] A hollow ball of mass m and radius R (with a moment of inertia about the center of mass $I = 2mR^2/3$) rolls without slipping up an plane that is inclined at an angle θ to the horizontal. At the beginning of the inclined

plane its translational speed is v. Determine the height H to which the ball travels before coming to rest on the inclined plane.

Solution:

The ball ends with gravitational potential energy $m\,g\,H$ relative to the bottom of its path. That energy is equal to the kinetic energy of motion of the ball at the bottom, since kinetic friction is absent, and rolling friction is a form of static friction. The kinetic energy can be written as the sum of the translational kinetic energy of the center of mass and the rotational kinetic energy about the center of mass:

$$\begin{aligned} m\,g\,H &= K_{trans} + K_{rot} \\ &= \frac{1}{2}\,m\,v_{cm}^2 + \frac{1}{2}\,I_{cm}\,\omega_{cm}^2 \end{aligned}$$

Because the ball rolls without slipping, the translational speed and rotational angular speed are related by the formula $v_{cm} = \omega_{cm}\,R$, so that

$$\begin{aligned} m\,g\,H &= \frac{1}{2} m\,v_{cm}^2 + \frac{1}{2} \times \frac{2}{3}\,m\,R^2 \times \frac{v_{cm}^2}{R^2} \\ &= \frac{1}{2} m\,v_{cm}^2 + \frac{1}{3} m\,v_{cm}^2 = \frac{5}{6} m\,v_{cm}^2 \end{aligned}$$

As a consequence

$$H = \frac{5\,v^2}{6\,g}$$

6. [25 points] A bullet of mass of $m = 25$ grams is fired into a ballistic pendulum of mass $M = 1\ kg$, made of wood. The wood pendulum, with the bullet embedded in it, swings and comes momentarily to rest at an altitude $H = 25\ cm$ above its initial location. What is the initial speed v_0 of the bullet?

   Solution:

   The initial collision between the bullet and block is completely inelastic, so that we may use momentum conservation to relate v_0 to v_1, the speed of the bullet-block system just after the collision:

$$(m+M)\, v_1 = m\, v_0$$

The corresponding kinetic energy of the bullet-block system just after the collision is

$$K = \frac{1}{2}(m+M)\, v_1^2$$

The increase in gravitational potential energy when the pendulum has swung to a height H (where it comes to rest) is equal to that energy:

$$\frac{1}{2}(m+M)\, v_1^2 = (m+M)\, g\, H$$

Thus

$$\begin{aligned} v_1^2 &= 2gH = 2\,(10)\,(0.25) = 5 \\ v_1 &= 2.24\, m/s \end{aligned}$$

and

$$v_0 = \frac{m+M}{m}\,(2.24) = 92\; m/sec$$

7. [Extra Credit; 10 points] A spherically symmetric asteroid has a mass $M = 4\times 10^9$ kilograms and radius $R = 1\; km$. Determine the minimum rotational period, if a person of mass $m = 60\; kg$ can stand at rest upon its surface at its equator.

   Solution:

   The gravitational force on the person standing on the asteriod is given through Newton's Law of Universal Gravitation. It acts toward the center of the asteroid, with magnitude

$$F = \frac{G\,M\,m}{R^2}$$

The only other force, the contact force N of the asteroid on the body, acts radially outward. Thus, the net (radially outward) force is $N - F$. According to Newton's Second Law, that force must be equal to the mass of the person times the centripedal (inward) acceleration:

$$N - F = m\, a_r = -\frac{m\, v_T^2}{R} = -m\, \omega^2\, R$$

where $v_T = \omega\, R$. For the critical case, the contact force must vanish, and

$$\begin{aligned}
\frac{G\, M\, m}{R^2} &= m\, \omega^2\, R \\
\omega^2 &= \frac{G\, M}{R^3} = \frac{6.67 \times 16^{-11} \cdot 4 \times 10^9}{10^9} = 1.63 \times 10^{-5}\, \frac{rad}{sec} \\
T &= \frac{2\,\pi}{\omega} = 3.85 \times 10^5\, sec \cdot \frac{1\, day}{86400\, sec} \approx 4.45\, days
\end{aligned}$$