

# THE ART OF SPIROLATERAL REVERSALS

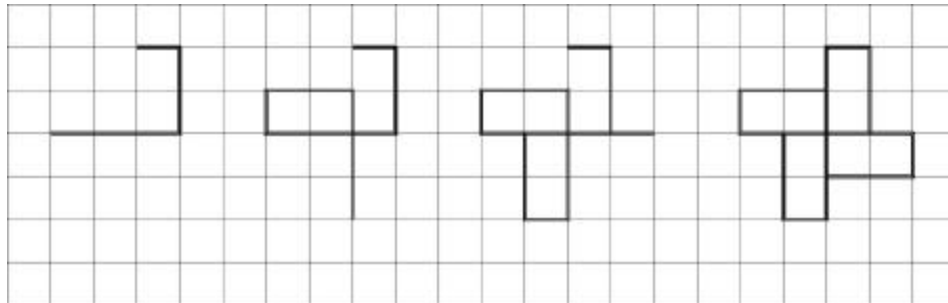
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*This paper continues an investigation into spirolaterals as geometry to generate artistic forms of unexpected complexity and beauty. This particular phase will concern the two dimensional representations of spirolaterals and its forms based on reversals of turns. Using mathematical and computer-based methods, issues of closure, variation, enumeration, and predictability are discussed. The overriding interest in this research is to develop methods and procedures to investigate geometry for the purpose of inspiration for new architectural and sculptural forms.*

## Introduction

The first apparent description of spirolaterals was written by Frank C. Odds, a British biochemist [4]. Further research uncovered additional description by Abelson [1]. A spirolateral is created by drawing a set of lines; the first at a unit length, then each additional line increasing by one unit length while turning a constant direction. Figure 1 shows the systematic generation of an order 3 spirolateral; one that consists of 3 segments at turns of 90 degrees.



*Figure 1. Generation of an order 3 spirolateral*

Step 1: turn 90 degrees, draw one unit segment, turn 90 degrees, draw two unit segment, turn 90 degrees, draw three unit segment

Step 2, 3 and 4: repeat Step 1

Odds writes that name *spirolateral* is derived from two roots: *lateral*, referring to a flat surface, and *spiro*, since the original series of spirolaterals was generated from the “square spiral”. Each segment is one unit longer than the one drawn before it, and each segment turns a constant 90 degrees from its predecessor. To complete a closed spirolateral, it is necessary to only to repeat the “square spiral” design, until the starting point is reached.

## Properties of Spirolaterals

From the introductory example, the spirolateral is defined by three basic factors: the turning angle, the number of segments or turns, and the number of repetitions, which create a closed figure. To predict the closing property of a spirolateral, Krawczyk [3] developed the following relationship. If the total turning angle is modulo 360, then the spirolateral is closed:

$$\text{Total turning angle} = \text{angle of turns} \times \text{number of turns} \times \text{repetitions}$$

The exception is that when only one repetition become 360 degrees, the spirolateral will not be closed, or if repeated further it will just meander outward without ever closing on itself.

Odds developed a notation to describe each of the spirolaterals. The number of turns,  $n$ , referred to as the “order” of the spirolateral, is the base number. The angle of turn is written as a subscript; thus,  $5_{90}$  would define a spirolateral with five turns each through 90 degrees.

Odds suspected that any angle that is an exact divisor of 180 degrees would generate a spirolateral. Gardner [2] also agreed with Odds without investigating any other possible angles. Figure 2 demonstrates closed spirolaterals based on an angle of  $180/n$ .

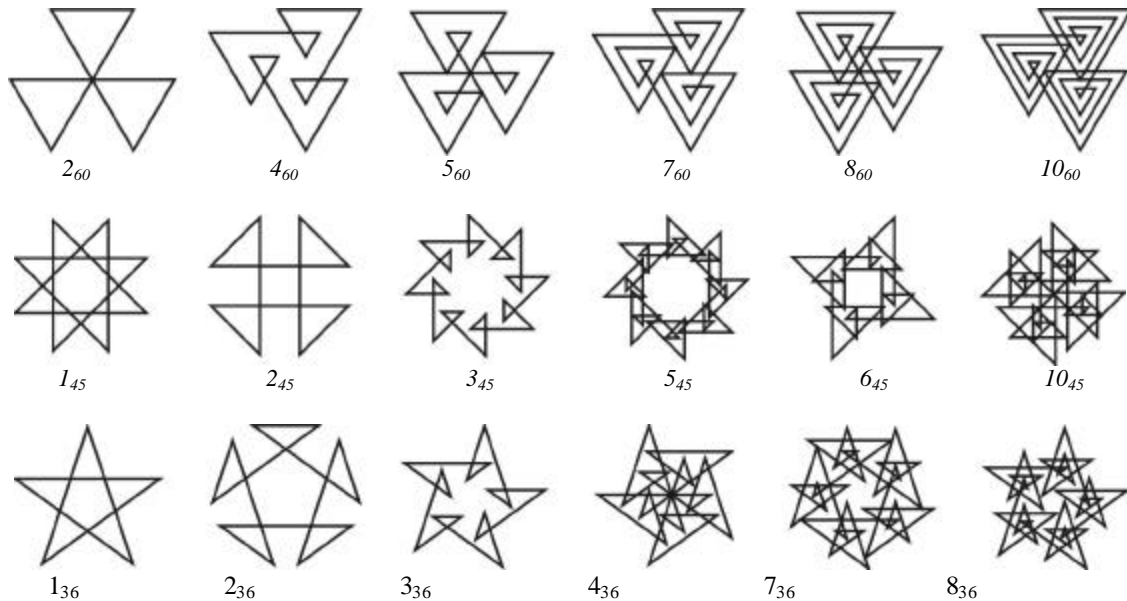


Figure 2. Spirolaterals based on an angle  $180/n$

## Reversing Turns in Spirolaterals

Odds, in further exploring the possible configurations for spirolaterals, introduced the concept that not all the turns need to be in the same direction. For any series of segments, some of the turns can be to the right and others can be to the left. The notation he suggests is:  $7_{90}^{4,6}$  for a spirolateral of 7 turns at 90 degrees in which the 4<sup>th</sup> and 6<sup>th</sup> turns are in the opposite direction. Figure 3 shows a series of reverse turn spirolaterals based on the angle of 90 degrees.

For a specific order  $n$  there are  $2^n$  possible turns that generate spirolaterals, half of those are mirror images; so  $2^{n-1}$  unique figures are possible in each order. From the analysis of the closure relationship previously described it has been found that spirolaterals, which close for all turns in the same direction, will also close for any reversed turns given the same number of turns and repetitions.

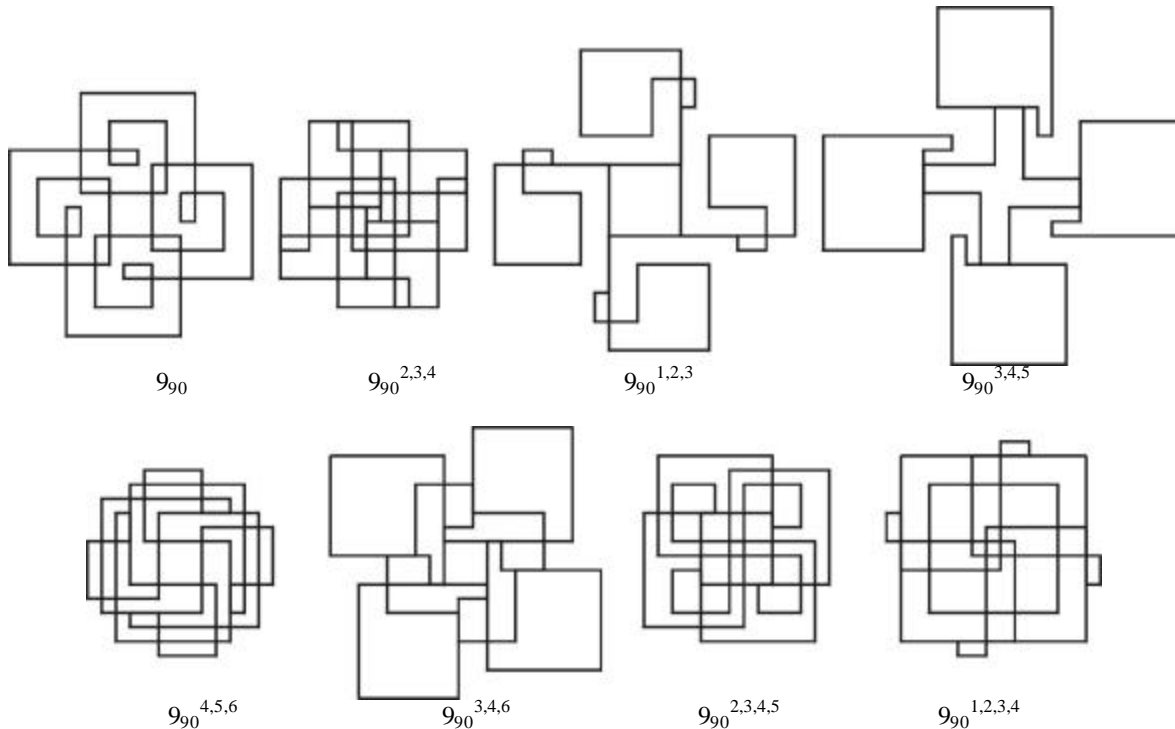


Figure 3. Spirolaterals with reverse turns

A mathematical relationship was never developed by Odds to account for the unclosed spirolaterals, nor was there one found for the prediction of closing reverse turned spirolaterals. Abelson suggests a method to generate unexpectedly closed spirolaterals by enumeration, but not in a predictive mathematical form.

To be able to investigate the variety of forms generated by reversals, the enumeration method suggested by Abelson was developed. Figure 4a and 4b demonstrate all the reversals for a  $7_{60}$  spirolateral. The figures indicate the reversed turns taken. The total number of reversals was found to be 127. After examination of a few sets of spirolateral reversals, exclusion was predicted by the following methods:

1. Only generate half of the possible reversals to eliminate symmetrical forms
2. If the sum of the left turn angles minus the sum of the right turn angles is equal to zero, then the spirolateral is symmetrical to the original one with all one directional turns
3. If the sum of the left turn angles is equal to the sum of the right turn angles then the spirolateral is open, not closed

Using these rules, a unique set of spirolaterals can be generated. Figure 5 displays the 42 unique reversals of the  $7_{60}$  spirolateral.

Expanding the enumeration procedure to include all the spirolaterals possible using the  $180/n$  rule, the following was found:

1. Of the 116 possible spirolaterals that could be closed using 10 or less repeats and with 10 or less turns, 100 were capable of reversals. The others were either not closed or only had one turn so a reversal was not possible.
2. The 100 spirolaterals generated 23,288 possible reversals. The count includes the original of each.
3. Of the 23,288 reversals, 4,966 were closed.

Another example is given in Figure 6 of a  $6_{45}$  spirolateral.

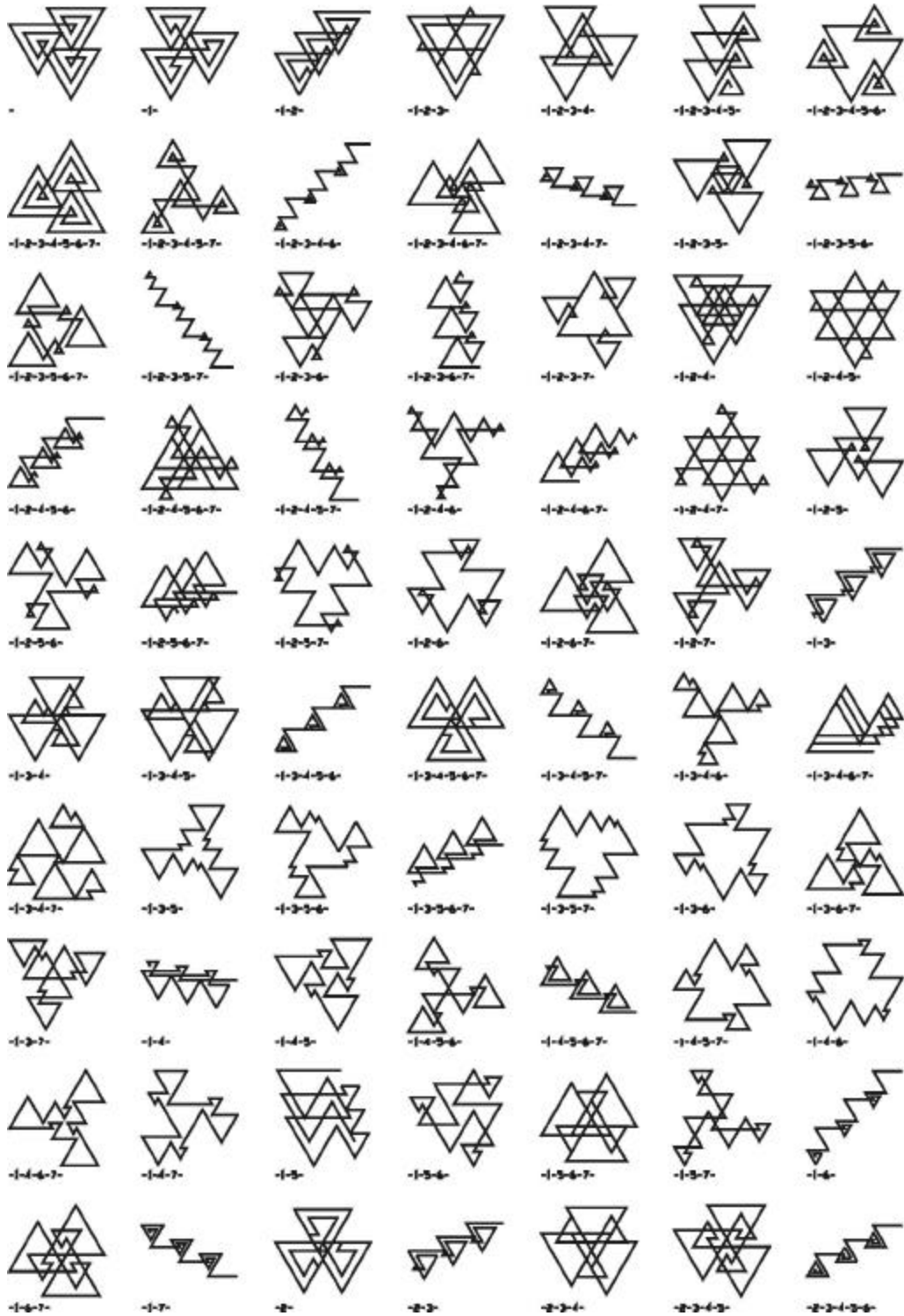


Figure 4a. 7<sub>60</sub> Spirolateral with all reverse turns

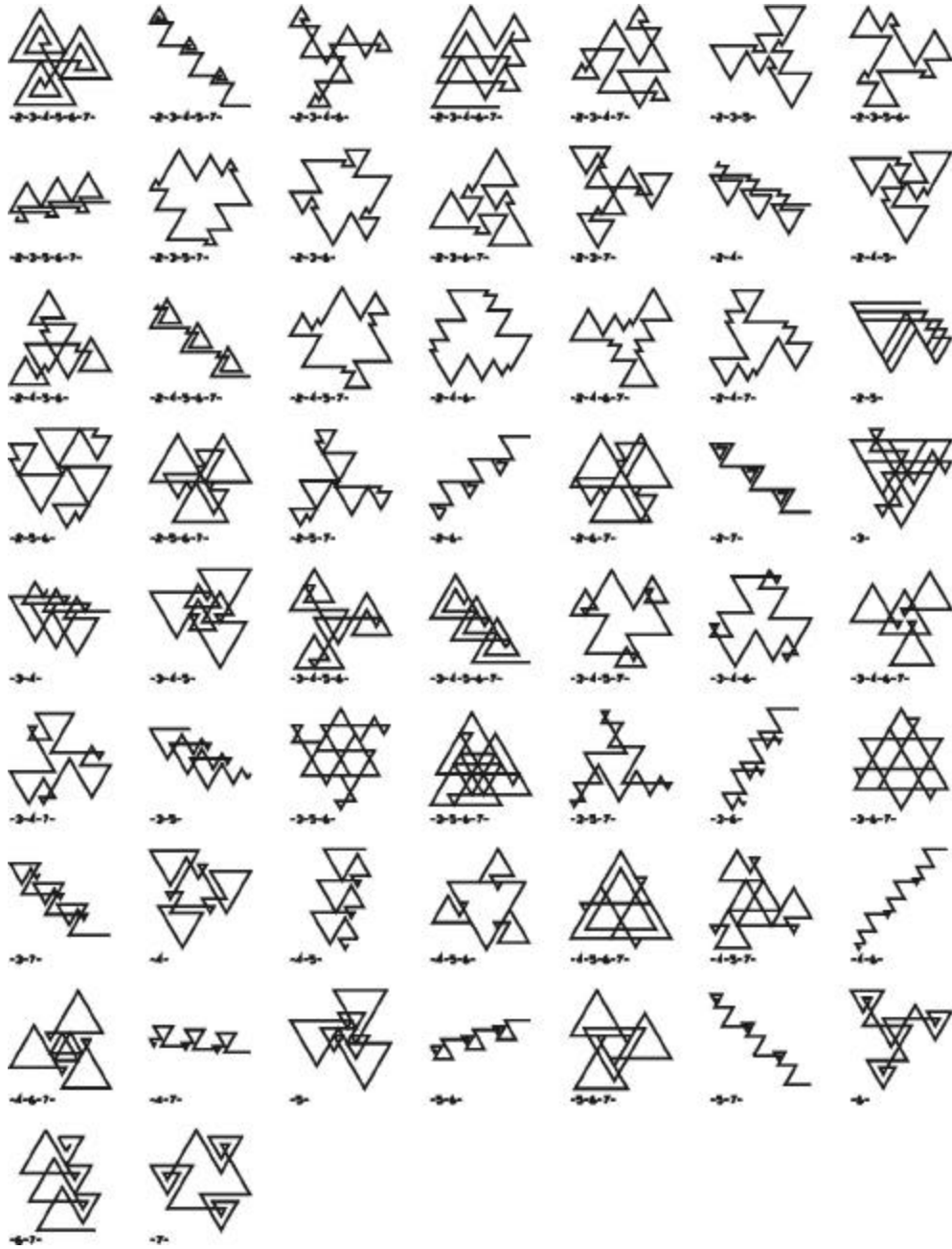


Figure 4b.  $7_{60}$  Spirolateral with all reverse turns

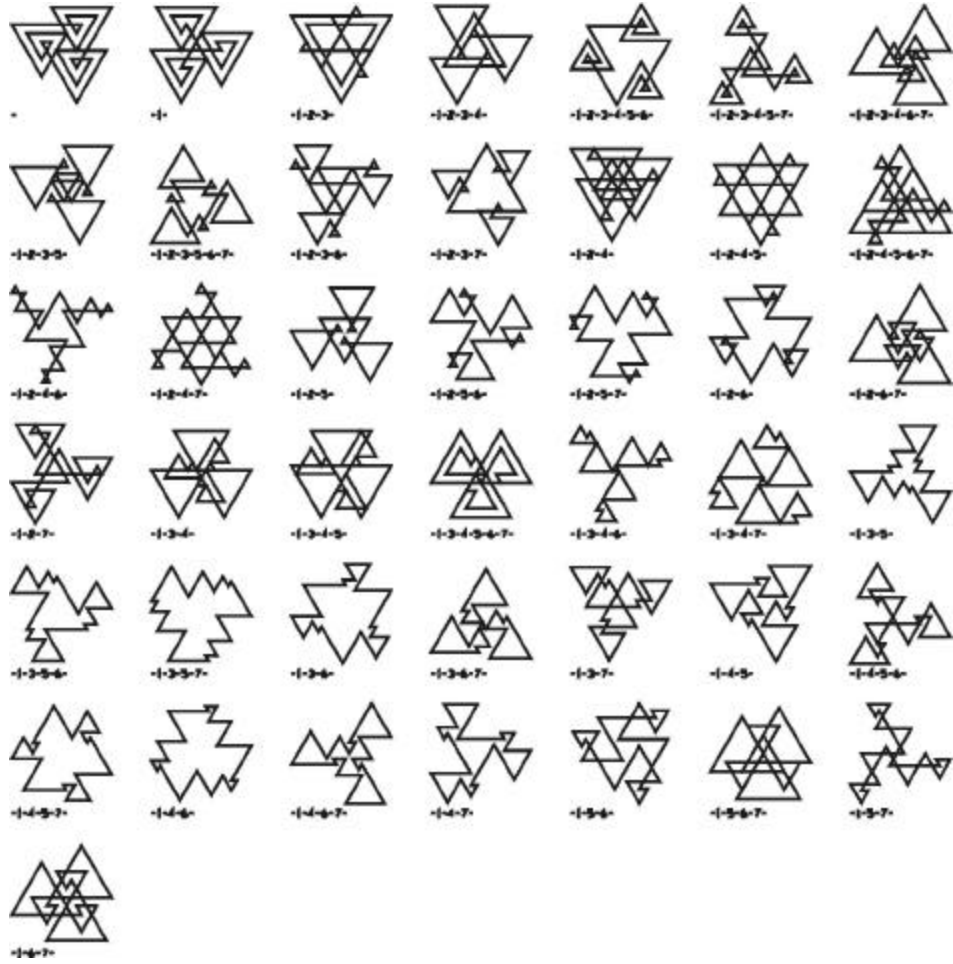


Figure 5. 7<sub>60</sub> Spirolateral with only unique reverse turns

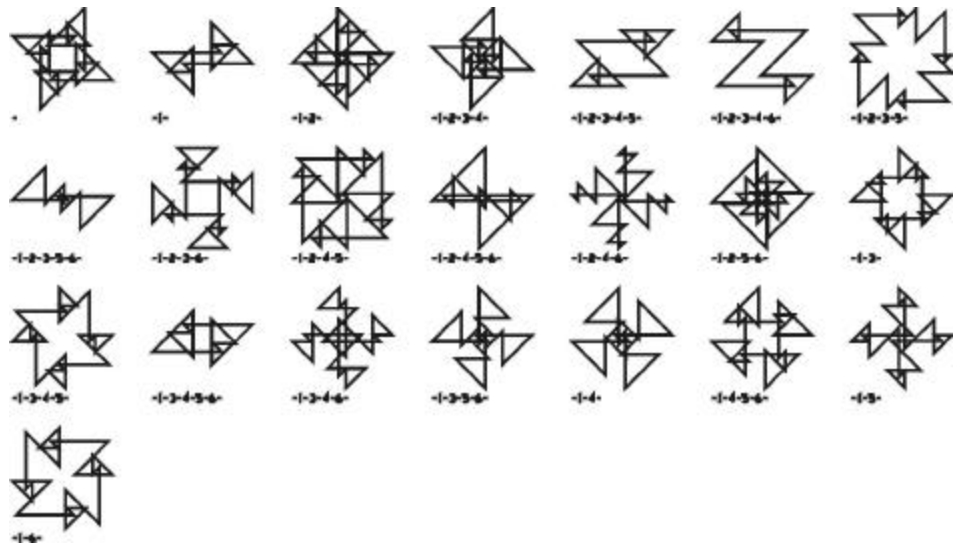


Figure 6. 6<sub>45</sub> Spirolateral with only unique reverse turns

## Art of the Reversals

In support of the investigation of the properties of the spiroilateral, a program was first written in Microsoft QuickBASIC for computational inquiry and AutoCAD's AutoLISP for visualization. The first version simply used a single line to represent each spiroilateral. Visually it was very weak. A line thickness was added. The forms turned from simple line drawings to iconic symbols. As the line thickness was increased, the open areas between the lines began to close and a totally different form emerged. Once the basics were understood a JAVA version as written to display the spirolaterals on the web. Figure 7 displays the JAVA version. This version can be found at: [www.iit.edu/~krawczyk/start.htm](http://www.iit.edu/~krawczyk/start.htm).

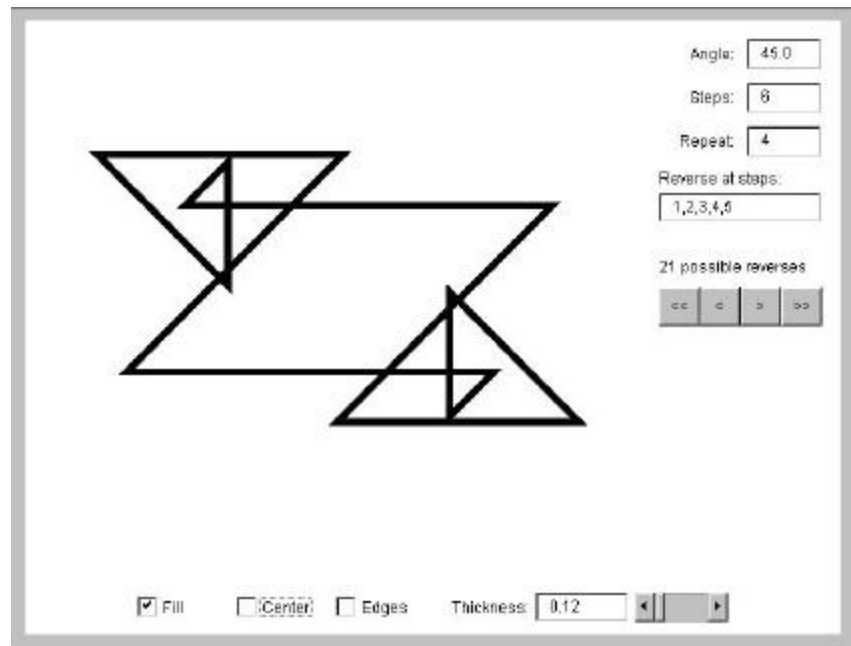


Figure 7. JAVA program to generate spirolaterals

The current web version includes two JAVA modules. The first has a selected group of spirolaterals that generate their reversals as a continuous display. Each is created line-by-line, erased, and then another is generated. The second module, used the same set of spirolaterals, but enables the viewer to modify the line thickness and browse through each of the reversals, Figure 8a.

In addition to displaying the line thickness, options are included to add the centerline and edge lines or both for each. Figure 8b, c, d display these variations. These options displayed the internal geometry of the spiroilateral that was somewhat lost to the line thickness.

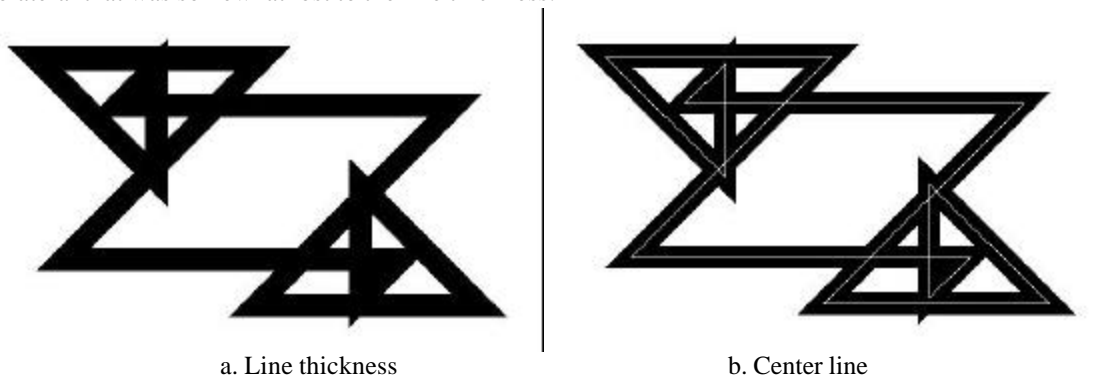
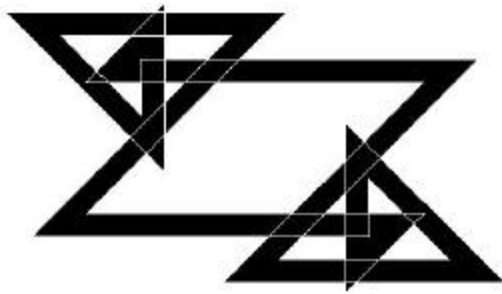
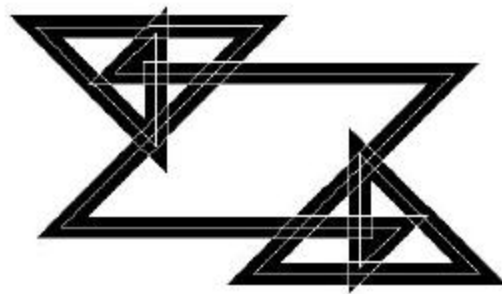


Figure 8. Variation of thickness and lines



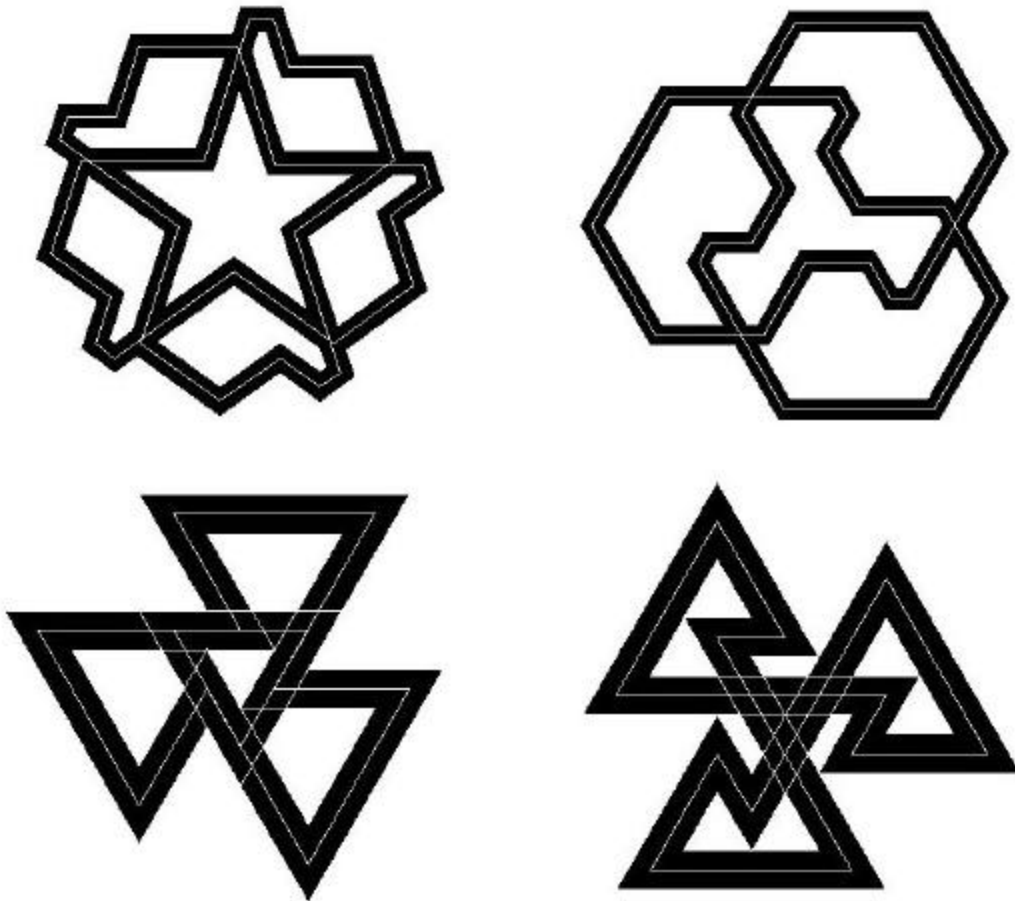
c. Edge lines



d. Center and edge lines

*Figure 8. Variation of thickness and lines*

Figure 9 and 10 display selected examples from the JAVA spiro-lateral-generating module.



*Figure 9. Selected examples*



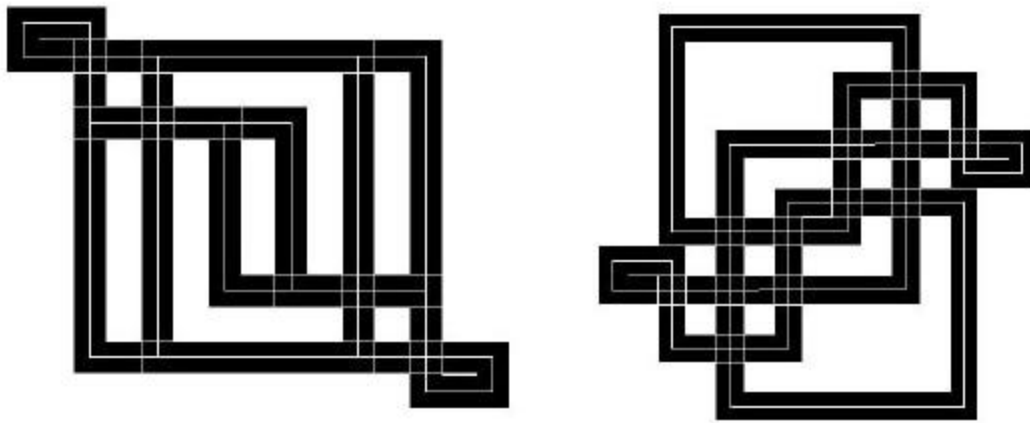


Figure 10. Selected examples

### Web Based Artwork

The development of a web-based version of generating spiroterals advances some very interesting concepts. The first is the distribution of an artistic idea is readily made with very little cost to the artist and it offers the potential for a very large audience. Another is the ability for viewers to actually investigate an idea beyond what was originally developed. Depending on the time spent and the individual interest of the viewer, the viewer is able to see particular instances that the artist never encountered.

The most intriguing aspect is the potential to deliver completed artwork with such a method. One extension to these JAVA modules is to have one ask the viewer for their email address and then email it with the parameters of a selected spiroteral to the artist. The Artist can then offer the viewer a variety of options of size, mounting, and cost for a final version of the design. The artist would be responsible for the environment that the artwork would exist. The viewer could then discover his or her own vision of the artwork.

The next phase of this research is to review methods to create full three-dimensional sculptures or designs that begin to suggest architectural forms. From the preliminary investigation of these forms, there seems to exist a great variety that has that potential.

Even with the possibility of enumerating a finite number of forms based on a selected angle scheme, when you include turn reversal, there still exist a great number of unexpected designs. The unpredictable, under controlled conditions, is what makes the spiroteral of continuing interest.

### References

- [1] Abelson, Harold, diSessa, Andera, 1968, *Turtle Geometry*, MIT Press, pp.37-39, 120-122
- [2] Gardner, Martin, 1986, *Knotted Doughnuts and Other Mathematical Entertainments*, W. H. Freeman and Company, pp. 205-208
- [3] Krawczyk, Robert, 1999, "Spiroterals, Complexity from Simplicity", in *International Society of Arts, Mathematics and Architecture 99*, edited by N. Friedman and J. Barrallo, The University of the Basque Country, pp. 293-299
- [4] Odds, Frank, "Spiroterals", *Mathematics Teacher*, February 1973, pp.121-124