SPIROLATERALS, COMPLEXITY FROM SIMPLICITY

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This paper investigates spirolaterals for their beauty of form and the unexpected complexity arising from them. From a very simple generative procedure, spirolaterals can be created having great complexity and variation. Using mathematical and computer-based methods, issues of closure, variation, enumeration, and predictictability are discussed. A historical review is also included. The overriding interest in this research is to develop methods and procedures to investigate geometry for the purpose of inspiration for new architectural and sculptural forms. This particular phase will concern the two dimensional representations of spirolaterals.

Introduction

Spirolaterals were first encountered while investigating space curves and fractals in Abelson [1]. What was intriguing about them was the simple procedure to generate them and the great variety that could result from modifying a small set of parameters. As with researching space curves and fractals the focus was to develop computer-based methods to investigate these two-dimensional forms and begin to suggest three-dimensional versions of them. Quickly discovering that spirolaterals can generate an infinite number of variations, the focus changed to investigate the rules of generating them that would result in visually interesting designs.

Further research uncovered what seems to be the first description of this geometrical form by Frank C. Odds, a British biochemist [2]. A spirolateral is created by drawing a set of lines; the first at a unit length, then each additional line increasing by one unit length while turning a constant direction. Figure 1 shows the systematic generation of an order 3 spirolateral; one that consists of 3 segments at turns of 90 degrees.

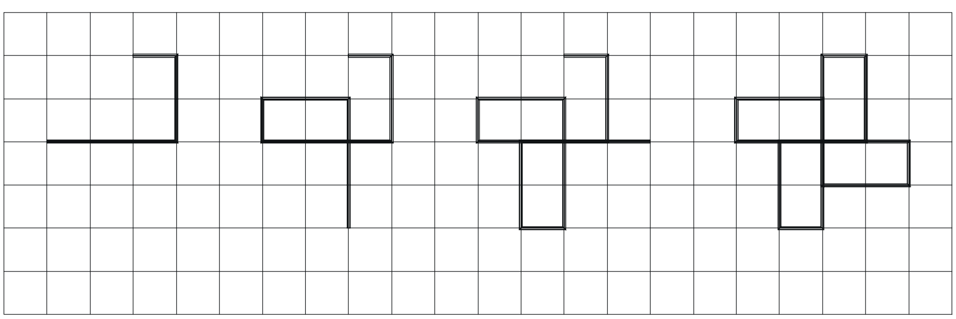

Figure 1. Generation of an order 3 spirolateral

Step 1: turn 90 degrees, draw one unit segment, turn 90 degrees, draw two unit segment, turn 90 degrees, draw three unit segment

Step 2, 3 and 4: repeat Step 1

In this example by repeating the initial three-segment construction four times, the spirolateral closes on itself.

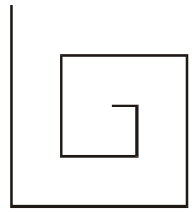

Figure 2. A "square spiral"

Odds writes that name *spirolatera*l is derived from two roots: *lateral*, referring to a flat surface, and *spiro*, since the original series of spirolaterals was generated from the "square spiral" as shown in Figure 2. In this drawing, each segment is one unit longer than the one drawn before it, and each segment turns a constant 90 degrees from its predecessor. To complete a spirolateral, it is necessary to only to repeat the "square spiral" design, until the starting point is reached.

Properties of Spirolaterals

From the introductory example, the spirolateral is defined by three basic factors: the turning angle, the number of segments or turns, and the number of repetitions, which create a closed figure. To investigate the closing property of a spirolateral, Figure 3 shows a 90 degree spirolateral from 1 to 10 turns.

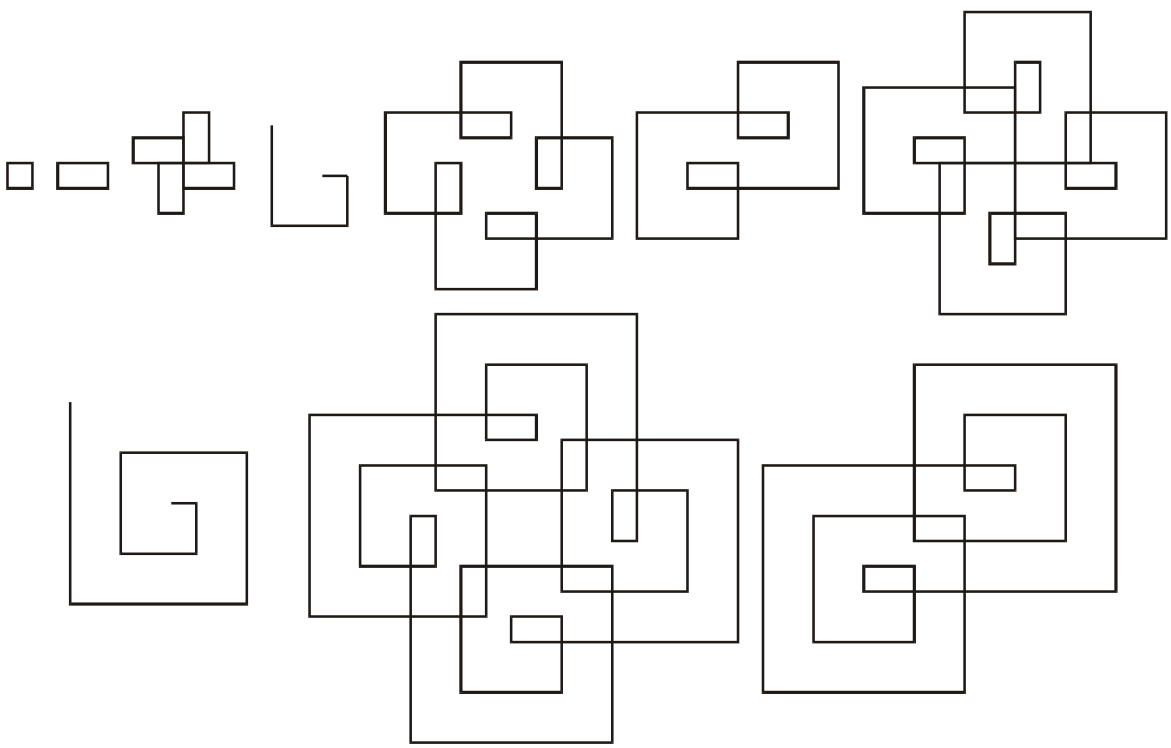

Figure 3. Spirolateral of 90 degrees, from 1 to 10 turns

From the example, the spirolaterals of turns 4 and 8 do not close, all the others do. To predict which parameters generate closed spirolaterals, we see from the following analysis that when the total turning angle is modulo 360 degrees, the spirolateral will be closed.

Total turning angle = angle of turns x number of turns x repetitions

a. 90 x 1 x 4 = 360 b. 90 x 2 x 2 = 360 c. 90 x 3 x 4 = 1080 d. 90 x 4 x 1 = 360

e. 90 x 5 x 4 = 1800 f. 90 x 6 x 2 = 1080 g. 90 x 7 x 4 = 2520 h. 90 x 8 x 1 = 360

i. 90 x 9 x 4 = 3240 j. 90 x 10 x 2 = 1800

The exception is that when only one repetition become 360 degrees, the spirolateral will not be closed, or if repeated further it will just meander outward without ever closing on itself.

Odds developed a notation to describe each of the spirolaterals. The number of turns, *n*, referred to as the "order" of the spirolateral, is the base number. The angle of turn is written as a subscript; thus, 5_{90} would define a spirolateral with five turns each through 90 degrees.

Odds suspected that any angle that is an exact divisor of 180 degrees would generate a spirolateral, Figure 4 shows a series based on those angles. Gardner [3] also agreed with Odds without investigating any other possible angles.

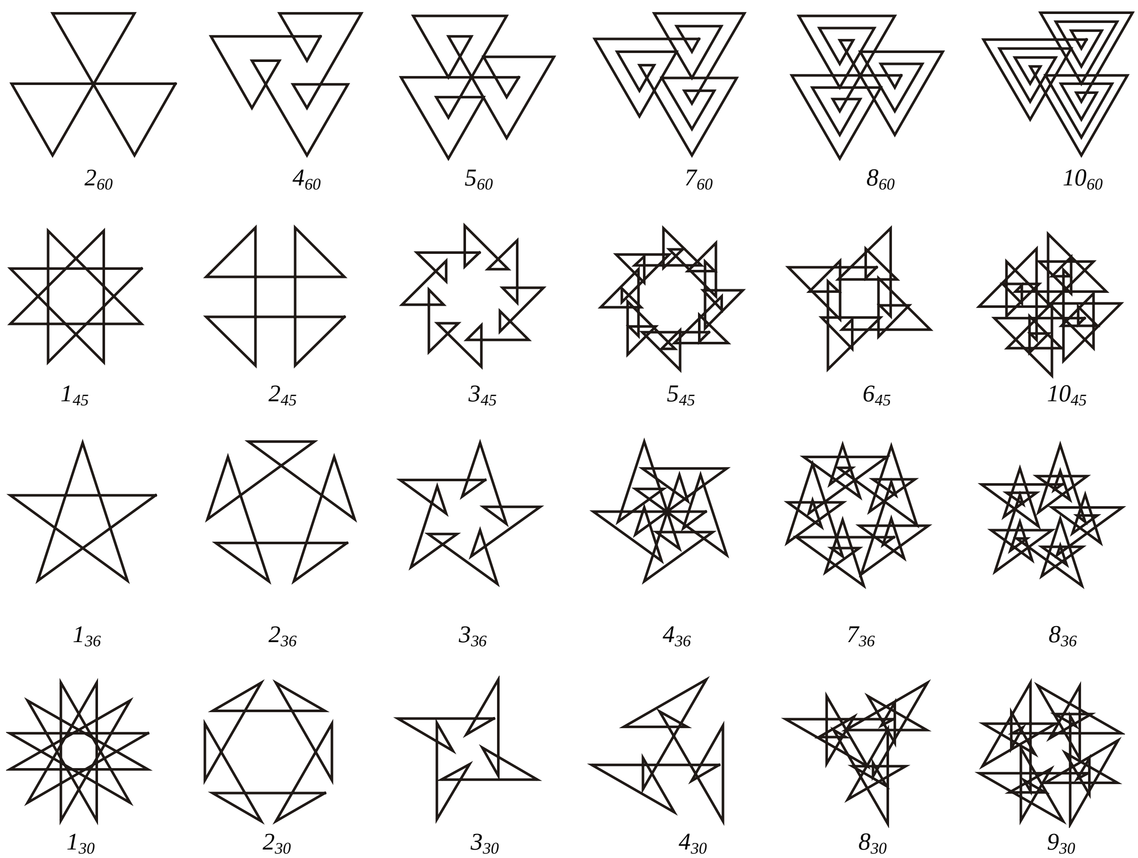

Figure 4. Spirolaterals based on an angle 180/n

To test the 180/n observation, a computer program was written to generate all the possible spirolaterals of angles from 180/2 to 180/30 restricting them to a maximum of 10 turns and 10 repeats. Of these 290 possible spirolaterals, 143 were closed by the 360 degree rule; of the 143, 10 were not actually closed.

Another computer program was written to enumerate other spirolaterals. The enumeration included all the angles from 1 to 180, incremented by 1 degree, and number of turns from 1 to 10 were attempted to generate a closed spirolateral, with a maximum of 10 repeats. From 1800 possible spirolaterals, 561 were closed by the 360 degree rule; of the 561, 113 were not actually closed. Of that gallery of 448 images, Figure 5 exhibits a representative selection.

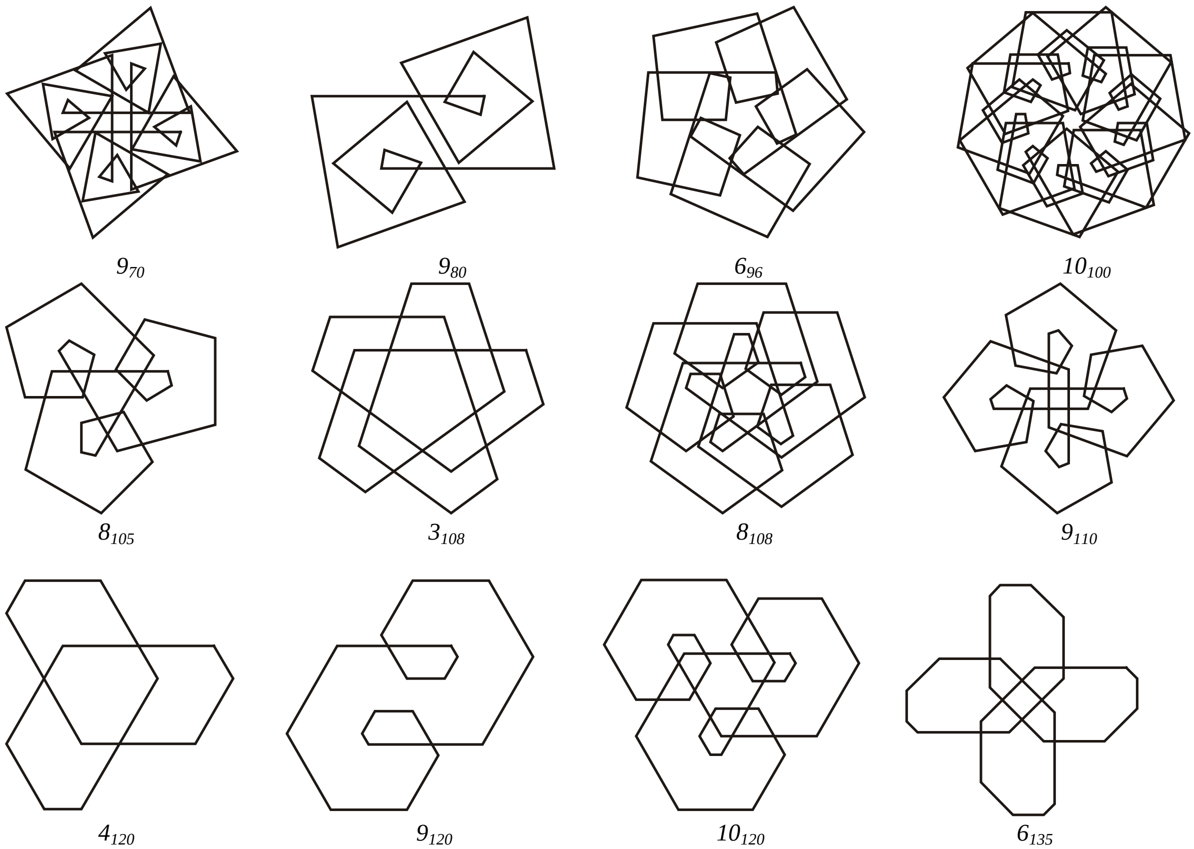

Figure 5. Spirolaterals based on angles not 180/n

Odds in further exploring the possible configurations for spirolaterals introduced the concept that not all the turns need to be in the same direction. For any series of segments, some of the turns can be to the right and others can be to the left. The notation he suggests is: $7_{90}^{4,6}$ for a spirolateral of 7 turns at 90 degrees in which the 4th and 6th turns are in the opposite direction. Figure 6 shows a series of reverse turn spirolaterals based on the angle of 90 degrees.

For a specific order there are 2^n possible turns that generate spirolaterals, half of those are mirror images; so 2^{n-1} unique figures are possible in each order. From the analysis of the closure relationship previously described it has been found that spirolaterals which close for all turns in the same direction will also close for any reversed turns given the same number of turns and repetitions.

The designs generated by reversing turns create an amazing series of figures that are nearly impossible to predict and very difficult to enumerate and evaluate. They are also more suggestive of architectural forms given that the generated spiral quality of each is hidden by the reversal of turns and they tend to create distinct bounded areas within the figure, which can be interpreted as volumes.

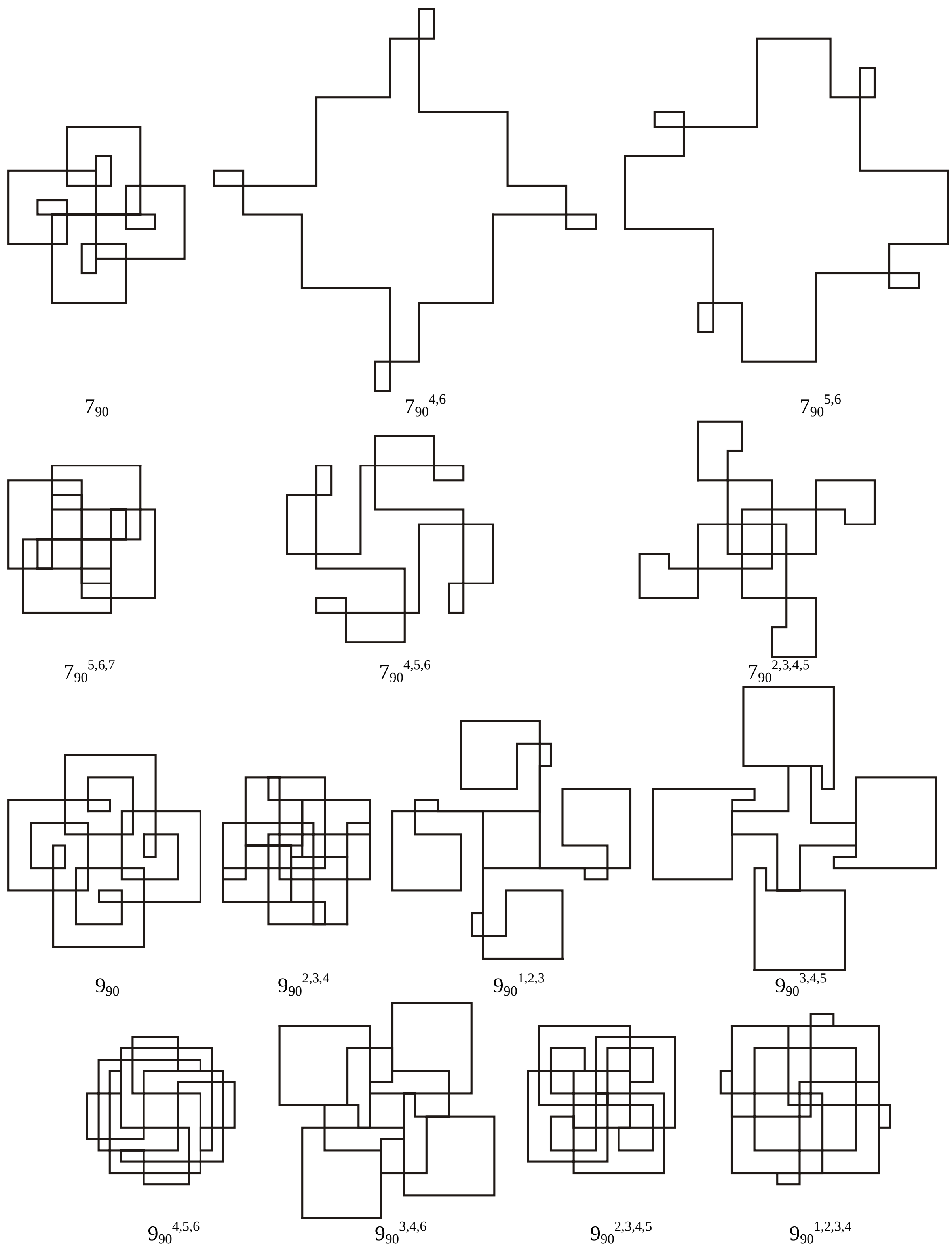

Figure 6. Spirolaterals with reverse turns

A mathematical relationship was never developed by Odds to account for the unclosed spirolaterals, nor was there one found for the prediction of closing reverse turned spirolaterals. Abelson suggests a method to generate unexpectedly closed spirolaterals by enumeration, but not in a predictive mathematical form. This author was also unable to discover this relationship.

Spirolaterals as Form Generators

Spirolaterals can be used explicitly to generate designs. Individual spirolaterals can be replicated into patterns or used singly for many two-dimensional designs in a variety of media. The spirolateral can also be used as an axis or spine for other design elements. For example, the vertices that compose a spirolateral can be used as control points for circular arcs or splines. The interpretation of the line segment can be varied to include unit arcs and zigzags. Figure 8 demonstrated these concepts on a previously generated spirolateral. This is the approach that would be followed to create a three-dimensional version of spirolaterals.

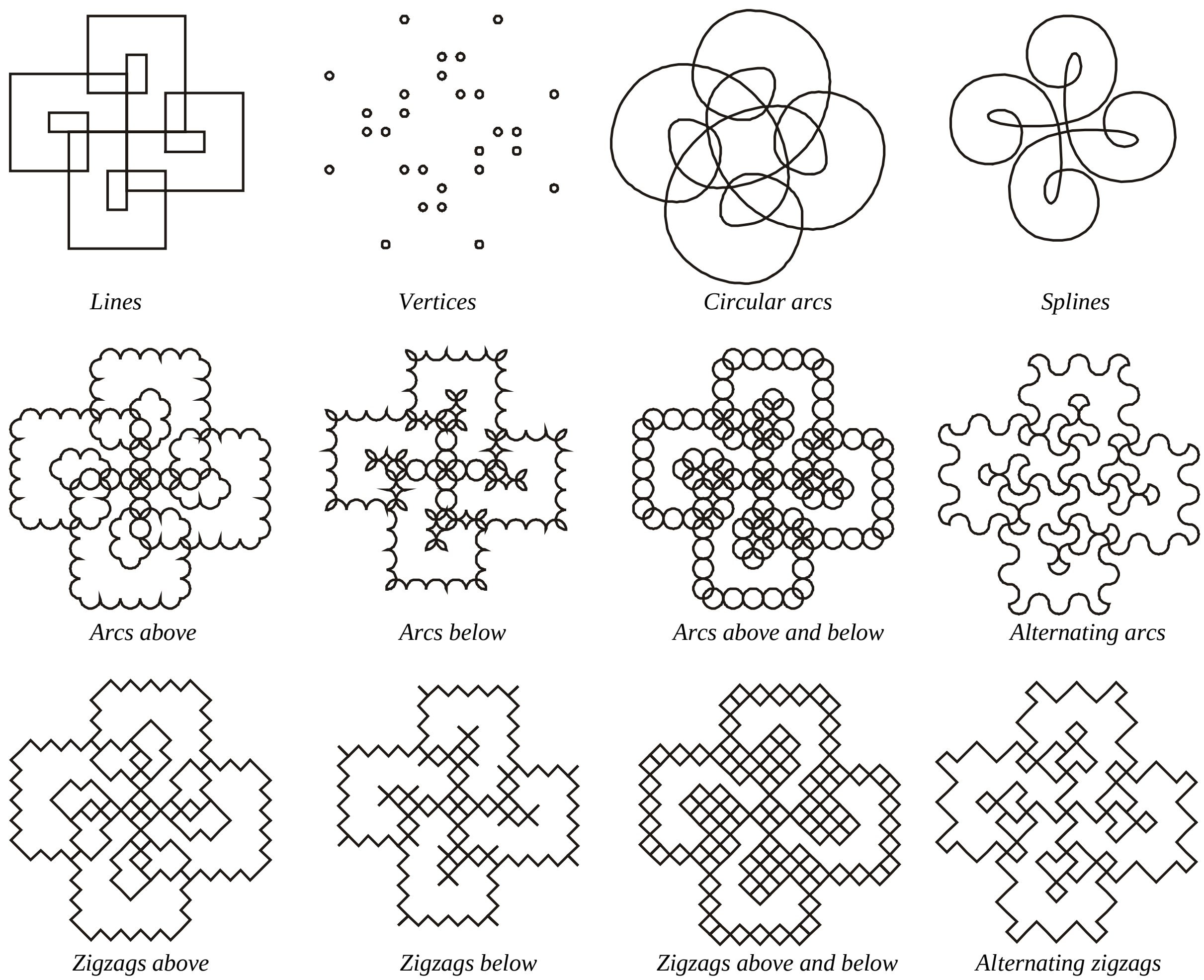

Figure 8. Designs based on the 7_{90} spirolateral

Spirolaterals as Artwork

A number of different attempts have been made to use the forms generated by the spirolaterals in two-dimensions to create actual artwork. Having the design generated within a CAD package allows for the simple drawing of it using a variety of line thickness; no different then drafting the same manually. No attempt has been made to replicate an individual form for the use as a pattern in fabric design or in a similar application. That possibility could still be investigated. A second version was tried using a continuous piece on wire, bent according to a diagram of the design. Somewhat more successful, it gave the design a physical depth but due to overall scale and some of the turning angles, the angles were difficult to reproduce or the smallest length became too small to handle well. In addition, the spiraling and overlapping nature of the designs caused a layering of the wire that made it difficult to work with on complex forms. A third version used a continuous strip of putty on glass to replicate the design. This gave results that were somewhat more interesting. The problem with the turns and overlapping was solved by the softer material; and the "three-dimensionality" added to the drawing quality of the work. Work continues using this method.

Since the underlying concept is to have these design be able to be generated by anyone and then produced into some type of physical form, a series of JAVA programs have been written to generate spirolaterals. You can generate your own spirolaterals at *http:\\www.iit.edu\~krawczyk*. Eventually the results of the JAVA programs could be send to some CAD software which would create a version of the design that could be produced on a rapid prototyping system. Such an approach would allow for a variety of possible forms; from a simple thick tubular type version to one that gently increases in height, similar to a ramp, as the design spirals towards its terminal point.

The second phase of this research is to review the three-dimensional components generated in Krawczyk [4] using the Hilbert Curve and create full three-dimensional sculptures or designs that begin to suggest architectural forms. From the preliminary investigation of these forms, there does exist a great variety which has the potential to suggest three-dimensional forms.

Even with the possibility of enumerating a finite number of forms based on a selected angle scheme, when you include turn reversal, there still exists a great number of unexpected designs. The unpredictable, under controlled conditions, is what makes the spirolateral of continuing interest.

References

[1] Abelson, Harold, diSessa, Andera, 1968, Turtle Geometry, MIT Press, pp.37-39, 120-122

[2] Odds, Frank, “Spirolaterals”, Mathematics Teacher, February 1973, pp.121-124

[3] Gardner, Martin, 1986, Knotted Doughnuts and Other Mathematical Entertainments, W. H. Freemand and Company , pp. 205-208

[4] Krawczyk, Robert, 1998, “Hilbert’s Building Blocks”, in Mathematics & Design 98, edited by J. Barrallo, The University of the Basque Country , pp. 281-288