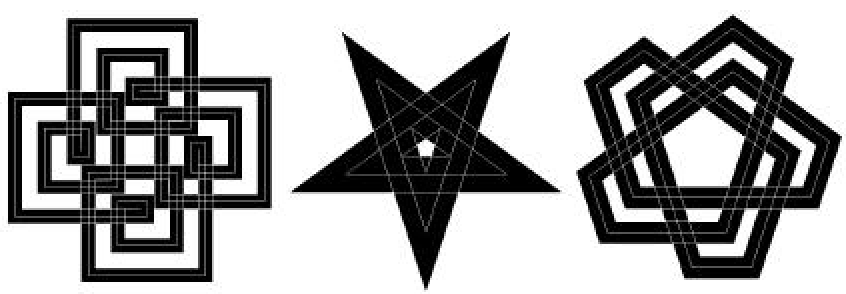
CURVING SPIROLATERALS

Robert J, Krawczyk
College of Architecture
Illinois Institute of Technology
Chicago, IL 60616 USA
krawczyk@iit.edu

Abstract: Spirolaterals are based on the concept of a square spiral constructed of straight lines of increasing length. This paper introduces methods that can be used to transform their straight-line turns into curves. From the tens of thousands of possible spirolaterals which have been found, that are all closed, with and without reversed turns, a series of artwork is developed showing possible curving methods: curve fitting, spline, inversion, antiMercator and circular transformations. These transformations are implemented using CAD methods and custom software. The parameters that are used to generate a variety of artwork are discussed and demonstrated.

Introduction

A spirolateral is created by drawing a set of lines; the first at a unit length, then each additional line is increased by one unit length while turning a constant direction. To complete a closed spirolateral, it is necessary only to repeat this procedure until the starting point is reached. The first apparent reference to this geometric figure was by Odds [9]. Further information can be found at Abelson [1], Gardner [4], and Fisher and Campbell [3]. In addition to the property of closure, spirolaterals need not always turn the same direction. The direction can be reversed at any turn, which makes the total number of possible spirolateral unknown

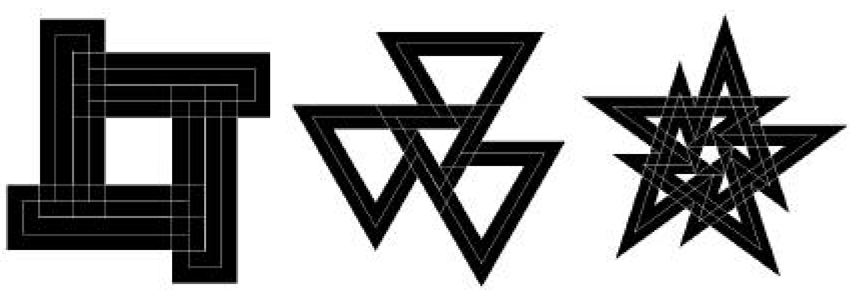


9_{90} 1_{36} 3_{108}

Figure 1: Closed spirolaterals

In this series of investigations, spirolaterals were generated to determine predictable closure and the variety of angles that could be used for turns as in Krawczyk [5]. The image of the spirolaterals remained as single lines. Reversals were the next area of interest. Krawczyk [6] covers the generation of all the unique reversals of any particular spirolateral. While developing software for the generation of spirolaterals as artwork, the single line was re-evaluated as the only representation. The turning line was given a thickness, and options were included to overlay a centerline, edge lines, or corner lines. These options were included in Krawczyk [7] and the web site, www.netcom.com/~bitart, which displays over 200 spirolaterals. The web site also includes a JAVA program to generate your own variations. Figure 1 displays a sample of closed spirolaterals and Figure 2, ones with reversed turns.

The notation for identifying spirolaterals starts with the number of turns, followed by the turning angle, then a list of the reversed turns, if present.



$5_{90}^{1,2,4,5}$ $4_{60}^{1,3,4}$ $3_{36}^{1,3}$

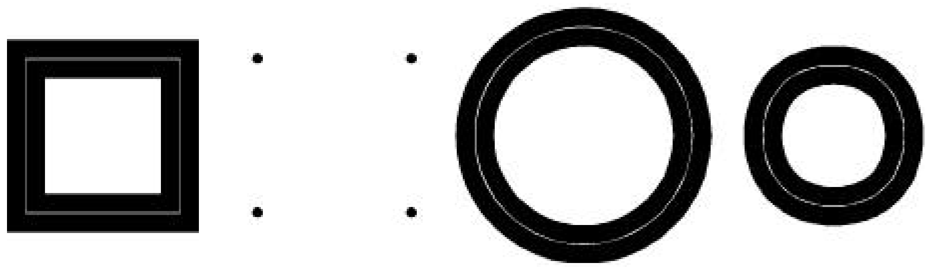
Figure 2: Closed spirolaterals with reversals

Curved Spirolaterals

One of the extensions of using spirolaterals as artwork beyond their native image was first discussed in Krawczyk [6]. A spirolateral consists of lines drawn between two vertices. Instead of using the actual lines, one concept was to use only the vertices. Using CAD-based software, procedures were written to interpret the vertices as control points for curves. Figure 3 displays this concept. The simple 1_{90} spirolateral, a square, consists of four vertices. These vertices can be fitted with arcs at the start and end of adjacent vertices. The vertices can also be control points to generate a spline curve.

Many of the spirolaterals reviewed as curves were found to be interesting but not visually exciting nor did they generate any unexpected images relative to their straight line origins. The first set created were ones that consisted of ten or less turns. Figure 4 displays one of these.

Only when the number of turns increased to a range of 11 to 15, with reversed turns included, did the number of control points increase to begin to generate images of unexpected detail and complexity. Figure 5 displays two such examples.



Original *Vertices* *Curve Fit* *Spline*

Figure 3: Curved spirolateral 1_{90}

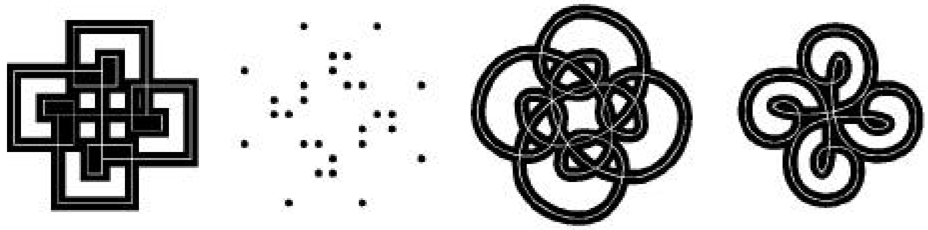


Figure 4: Curved spirolateral 7_{90}

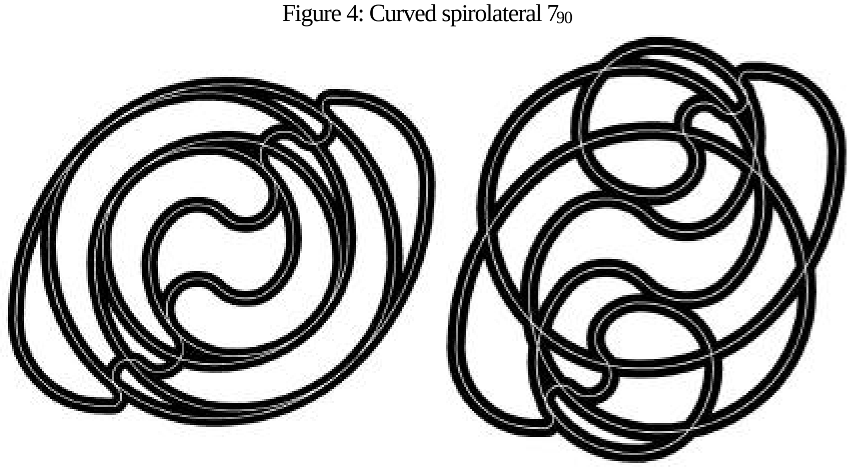


Figure 5: Curved spirolaterals of 14_{90}

Curves by Inversion

Another approach to curving spirolaterals is to apply a transformation, a projection, or mapping from a linear image to a curved one. In researching these possibilities, the first one encountered that was interesting was the concept of inversion, Dixon [2] and Lawerence [8]. This mathematical method turns lines into circles. The transformation based on the image midpoint is as follows:

$$X = (x * r^2) / (x^2 + y^2) \qquad Y = (y * r^2) / (x^2 + y^2)$$

where x and y are the original coordinates and r is the circle radius.

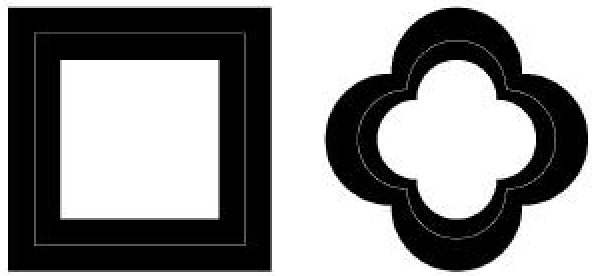
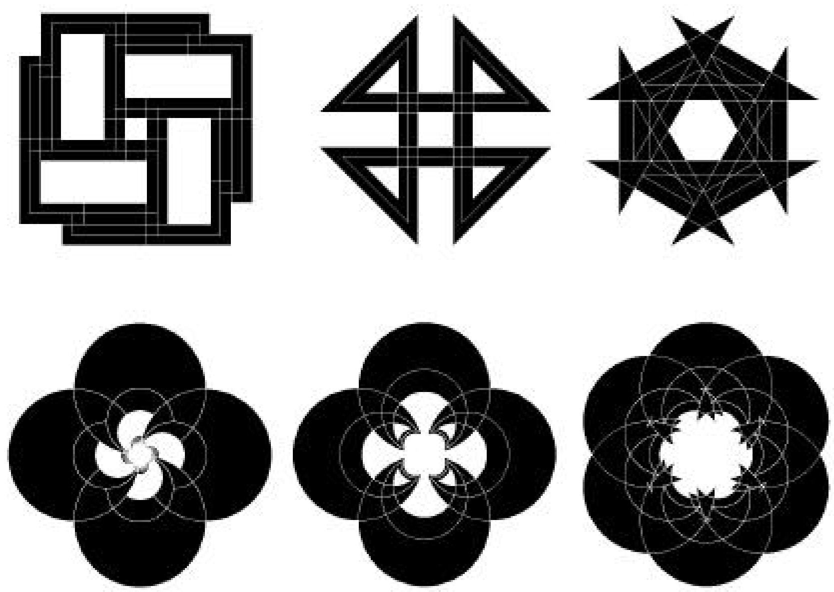


Figure 6: Inversion of spirolateral 1_{90}

Figure 6 demonstrates this transformation on a simple square spirolateral

Inversion by definition reconstructs the spirolateral as curves. After transforming a variety of spirolaterals, it was found that ones that are open in the center generate the most interesting results. Ones that have lines that cross the image midpoint generate very strange results, which are visually not intreresting. Symmetry also added to the quality of the results, as well as, the selection of the line thickness. Figure 7 displays a sample of spirolateral inversions.



$7_{90}^{1,3,4,6,7}$ 2_{45} 2_{30}

Figure 7: Spirolateral inversions

antiMercator and Circular Transformations

Continuing the search for other methods to curve spirolaterals, Dixon [2] in a section on transformations outlines a method he calls anitMercator. Horizontal lines become circles concentric with the coordinate origin, vertical lines become radial, and slanting lines become logarithmic spirals. The transformation in polar coordinates is as follows:

$$A = k * x \qquad R = \exp(k * y)$$

where $k = 2\pi/(xmax-xmin)$, A and R are angle and radius. Figure 8 demonstrates this transformation on a simple square spirolateral.

This transformation is effected by the line thickness and also by the offset from the origin. The origin is positioned in the lower-left corner of the image. This location results in the image being bent clockwise starting with the left corner. Figure 9 displays the effect of increasing the offset. Figure 10 displays a sample of spirolateral transformed by antiMercator.

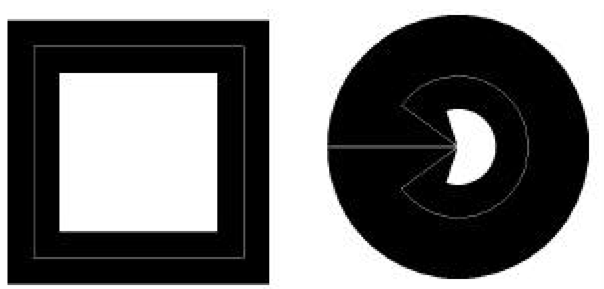


Figure 8: antiMercator spirolateral 1_{90}

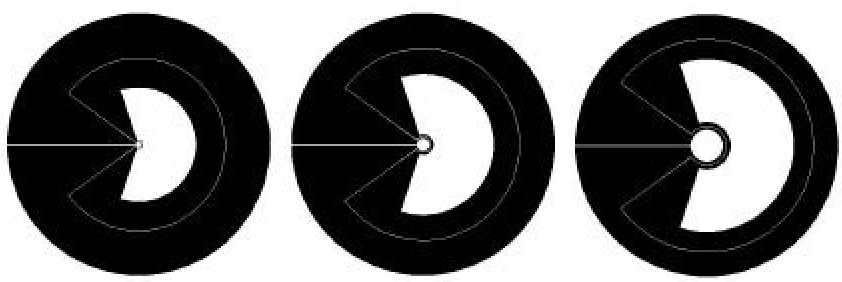
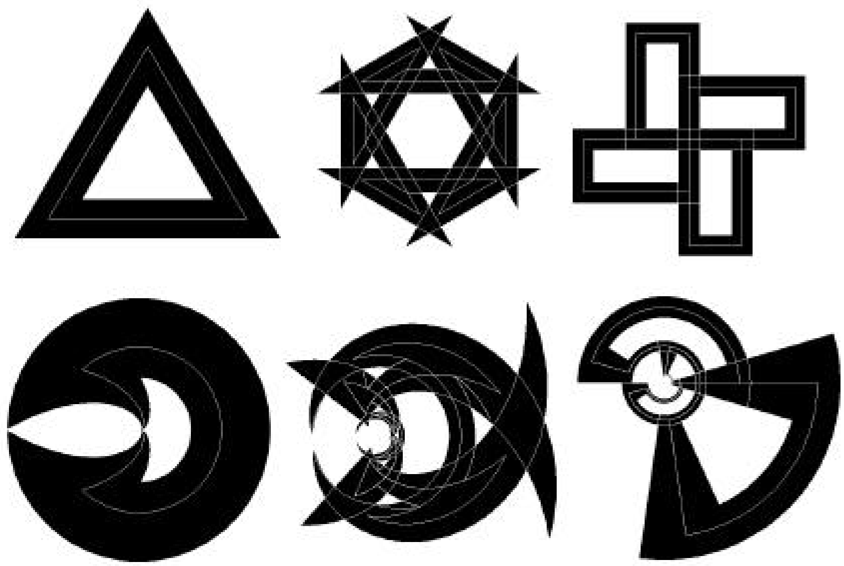


Figure 9: antiMercator offsets on spirolateral 1_{90}



1_{60} 2_{30} 3_{90}

Figure 10**:** antiMercator spirolaterals

While investigating the antiMercator transformation, one alternate method was found by removing the exponential function. The circular form remains without the logarithmic spiral effect. Since no formal name has been found for this transformation, it will be referred to as simply Circular. Figure 11 demonstrates this circular transformation on a simple square spirolateral.

This transformation differs from the antiMercator in that the horizontal line spacing is of a more constant distance for the center, so that original distances are better represented. This transformation also changes as the offset increases. Figure 12 displays the effect of increasing the offset. Figure 13 displays a sample of spirolaterals transformed by the Circular transformation.

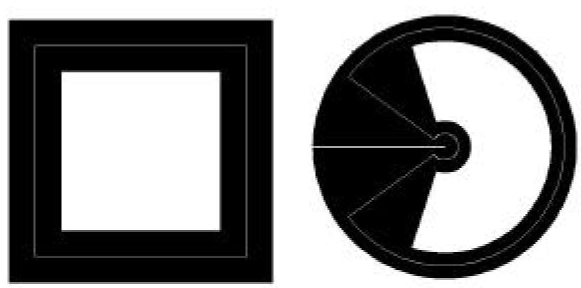


Figure 11: Circular spirolateral 1_{90}

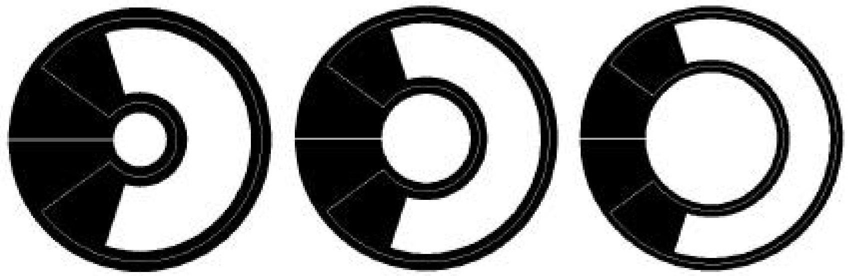
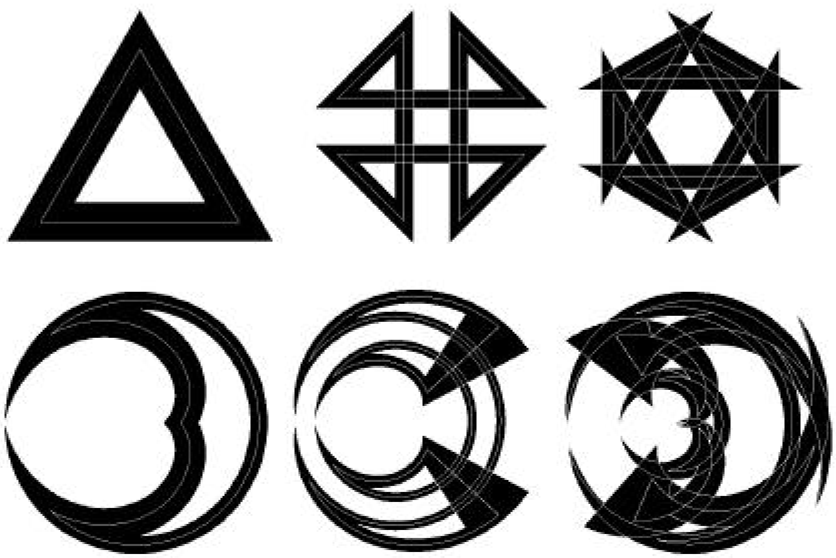


Figure 12: Circular offsets on spirolateral 1_{90}



1_{60} 2_{45} 2_{30}

Figure 13**:** Circular spirolaterals

The initial Circular transformation was based on the origin positioned in the lower-left corner, the minimum point of the image. Another variation is possible if the origin is moved to the center on the image, its midpoint. Figure 14 displays the difference between these positions using the square spirolateral and Figure 15 displays a sample of spirolaterals based on the midpoint origin.

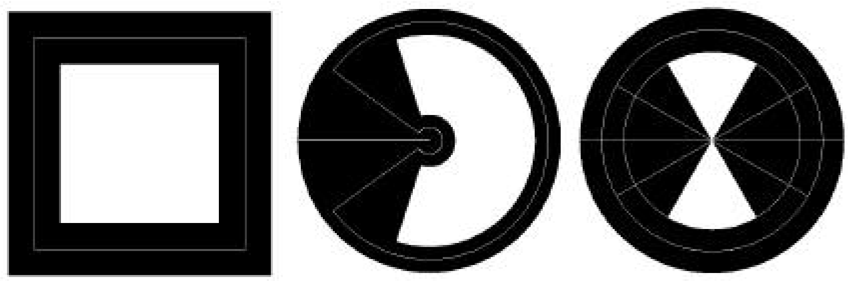
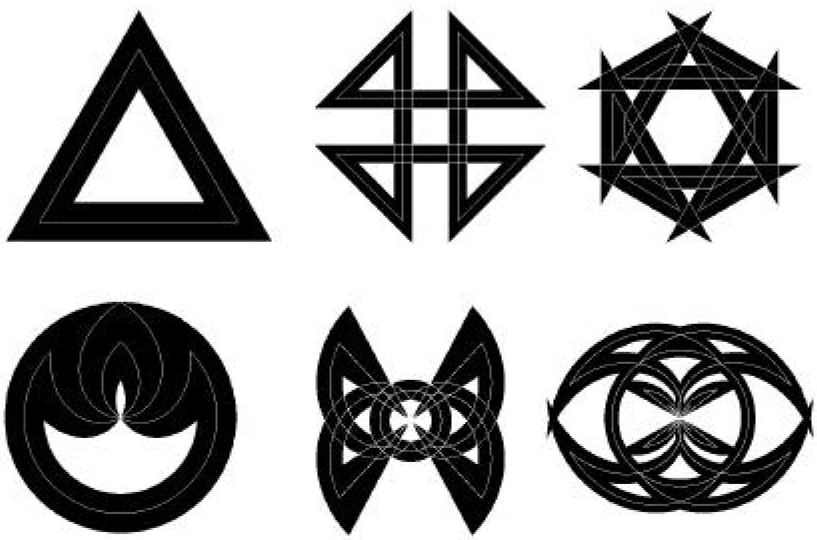


Figure 14: Circular spirolateral 1_{90} min and midpoints



1_{60} 2_{45} 2_{30}

Figure 15**:** Circular spirolaterals based on midpoint

Observations

As the mathematical basis of these forms evolves and image variations are developed, there is a sense that even though it is at times difficult to predict an outcome, the resulting images have a strange familiarity to them, reminiscent of decorative patterns from the ancient world. On the other hand the sharp edges, the lack of color, and the natural mathematical presentation give them a distinctive futuristic look. It may be very difficult to judge totally new forms since the aesthetic we use is based on all of our previous visual experience and training. The potential for the unexpected is clearly there, even from the simplest of beginnings.

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