HILBERT'S BUILDING BLOCKS

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This paper reports on an ongoing research project on an "nonpencil" approach in generating architectural forms using nontraditional geometries. Space curves are investigated to determine nodal points in 3D space, which are then interpreted into common architectural elements. The nodal points are used in a variety of ways to generate walls, columns, floors, and volumes. The determination of forms is totally under program control without any manual interpretation or intervention. A set of simple rules is used to investigate potential forms. The project further extents my continuing interest in developing software as a method of investigating design concepts and generate the unexpected.

Introduction

Geometry has played a critical roll in the generation of architectural form. From the purely architectural design standpoint, students are introduced early to the organizational framework that geometry gives them. One excellent example is Ching [1]. He begins with a definition of basic architectural elements, point, line, line to plane, plane, and volume; how these can be organized by ordering principles, such as, axis, symmetry, hierarchy, rhythm and repetition, and transformation. Included in the description of these principles is a variety of examples throughout architectural history.

From the analysis viewpoint, March and Steadman [2] include examples from a variety of sources that include concrete block by Frank Lloyd Wright to city and campus planning by Le Corbusier and Mies van der Rohe. Ghyka [3] includes the mathematical basis of forms in nature. With the emerging application of computers to assist in the placement of geometries, Mitchell [3] notes the architectural compositions of Nicholas-Louis Durand and how simple rule-based procedures can be developed to investigate the combinations of architectural elements.

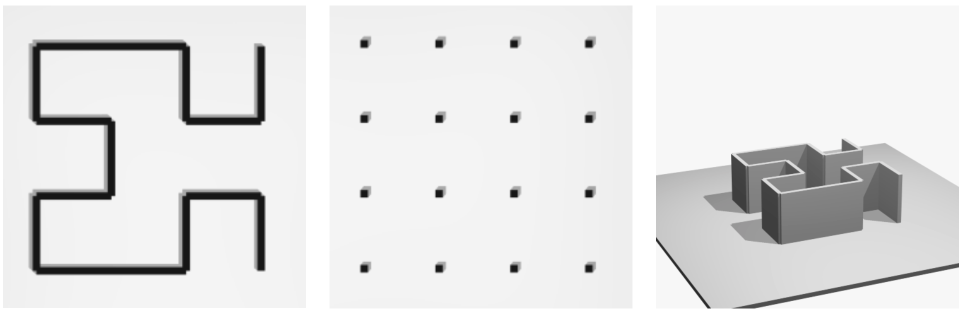
Investigating forms based on traditional geometry using automated methods has clearly been demonstrated and the results are well known, or at least, predictable. My interest is in investigating more geometries that are nontraditional if possible or ones that could yield unexpected results when generated by some computer-based procedure. Yessios [5] in one his studios investigated the use of fractals as a means to generate architectural forms. What I found intriguing about this particular approach was the unexpected results that seemed to be obtained, the architectural forms they suggested, and that such non-traditional geometries showed potential for architectural design.

Fractals and Space Curves

My own investigation into the use of fractals as a means of form generation began with the use of commercial software. I quickly determined that the recursive, replacement, and repetitive nature of fractals did not give me forms that were very interesting architecturally. The other limitation was that I was confined to two dimensions. As I progressed, I began to write my own fractal generating routines. I could clearly see that the recursive mature of this geometry was interesting. Yessios [5] in his fractal studio was able to develop forms in three dimensions and control the evolution of the form by replacing, deleting and inserting generators. He was also able to step through the generative process, backward and forward. He also concluded that if fractals applied in their "pure" fashion could never produce a building.

My investigation continued into other recursive designs and curves represented by nested triangles, snowflakes, and dragon curves, as explained in Abelson [6]. Of special interest were the space filling designs, in particular the most famous of them all, the one developed by the German mathematician David Hilbert. Figure 1a shows the Hilbert Curve for level two.

With the Hilbert Curve, I began to see the possibility of suggesting architectural form. I started to analyze the curve by looking at its component parts. The first were the lines and points that defined it, see Figure 1a and 1b. From the lines, I was able to add the third dimension by simply converting them to vertical planes, Figure 1c. The points and lines were then converted to vertical and horizontal members to make a frame structure. The maximum extent of the curve was converted to a horizontal plane and a volume.



a. Curve as lines *b. Curve as points* *c. Curve as planes*

Figure 1a-c. Hilbert curve

From the beginning, the intention was to convert the defining components of the curve to architectural elements. This conversion or interpretation was to be defined in the program generating the curve; I did not want any manual intervention during the curve generation. The program included a simple set of rules to develop the architectural elements from the basic geometry of the curve. I also understood that the program's interpretation of form was fully based on my instructions and the program itself could not develop any on its own.

Á space-filling curve by definition repeats along a two dimensional horizontal plane filling a rectangular boundary. My interest was to introduce an extension into the third dimension beyond the simple vertical planes I already had. Using the Hilbert Curve as a model, I investigated what would happen if I redefined the turning angle from 90 degrees to some other angle but left the repeating form the same. As I expected, the form began to overlap itself. I used this overlap, the point at which lines intersected, as a means to define vertical "levels" within the curve. When the overlap occurred, the overlapping line was raised above the preceding ones.

The same types of architectural elements were generated as before. The curves began to more closely suggest architectural forms. Instead of investigating the variety of turning angles, I concentrated on the interpretation of the geometry and potential architectural components.

Hilbert's Building Blocks

The next stage in the investigation centered on furthering the architectural interpretation of the geometry being developed.

A formal search of possible architectural components was undertaken. Referring to Figure 2, Hilbert's variation at 120 degree, the following were developed:

a. Lines; outline of the curve

b. Points; vertices of the line segments in the curve

c. Walls; transforming the lines into individual vertical planes at each level

d. Floors; transforming the minimum and maximum extent of each level into a horizontal plane

e. Floors; transforming the area underlying each level into a horizontal plane

f. Columns; transforming the vertices into vertical supports rising up from the base level , from the corners of each floor, and from the start and end on each level

g. and h. Volumes; transforming the floors at each level into a volume

i. Beams; transforming the lines at each level into beams

j. Walls to ground; transforming lines into vertical planes which extend from each individual level to the base

k. Walls to top; transforming lines into vertical planes which extend from each individual level to the top

l. and m. Walls; transforming the walls at each level to curves and splines

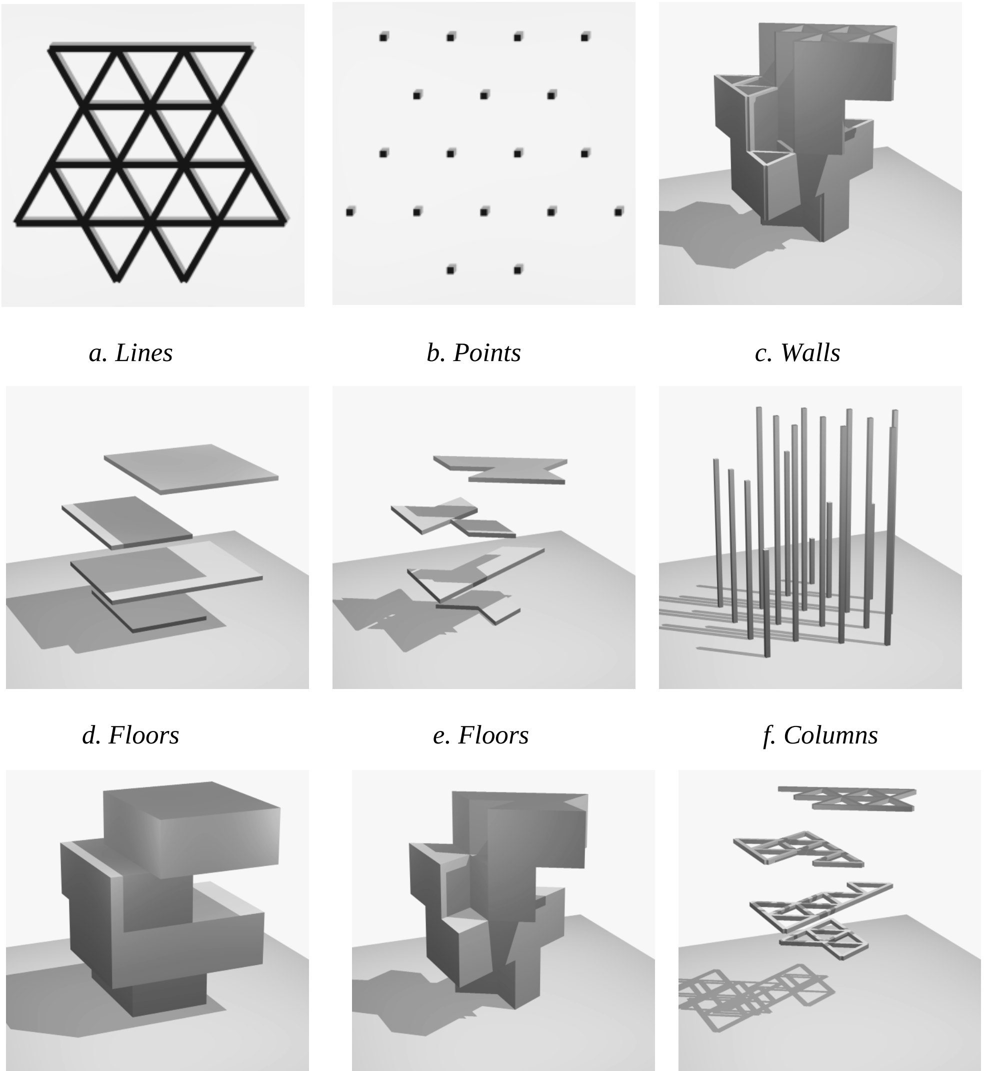
n. and o. Beams; transforming the beams at each level to curves and splines

p. Walls as splines; transforming the walls extending from the base to each level as splines

q. and r. Walls; transforming the walls extending from the top to each level as curves and splines

All the interpretations were defined within the program. As a series was developed, one set of rules defined another. Being in the position to evaluate each set of rules enabled new ones to be defined and others modified. The program being the documentation for the interpretation allowed for the consistent evaluation of each variation. As these were being developed, the focus moved to creating new interpretations, rather than, new curves.

Once a set of basic components were created, these were then placed in combination to attempt to develop complete structures. Figure 3a-r. displays one such series of forms.

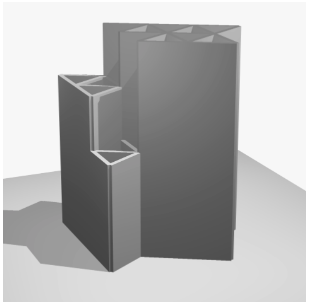


a. Lines *b. Points* *c. Walls*

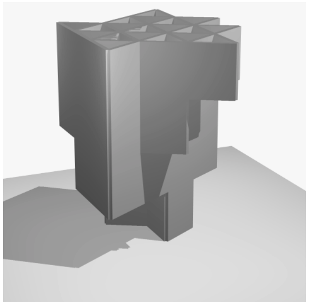
d. Floors *e. Floors* *f. Columns*

g. Volumes *h. Volumes* *i. Beams*

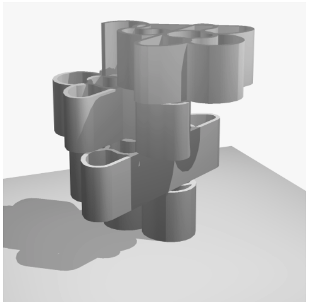
Figure 2a-i. Basic architectural components



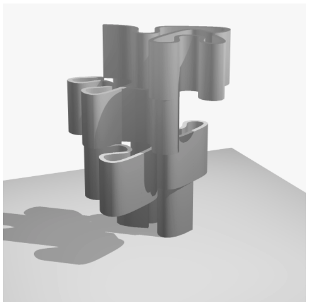
j. Walls to ground



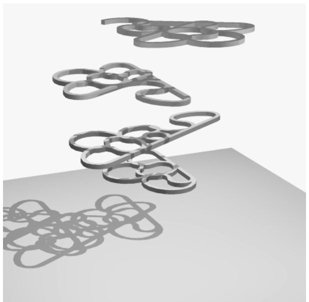
k. Walls to top



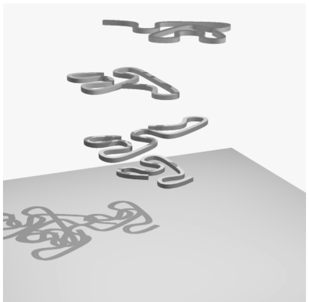
l. Walls as curves



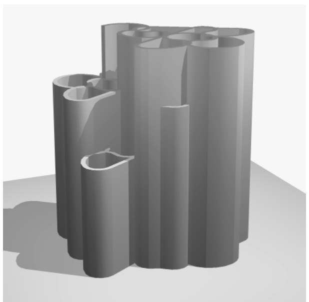
m. Walls as splines



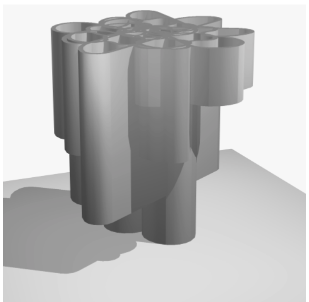
n. Beams as curves



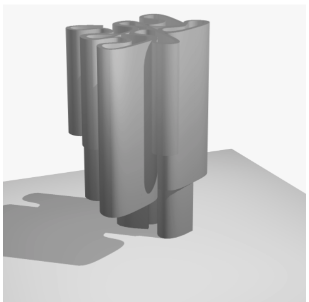
o. Beams as splines



p. Walls as splines

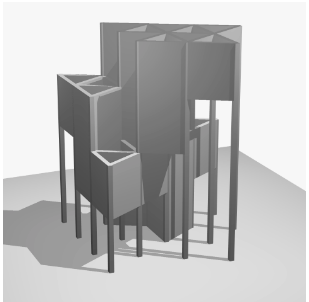


q. Walls as curves

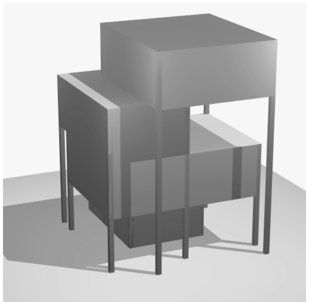


r. Walls as splines

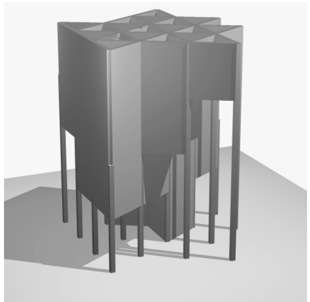
Figure 2j-r. Basic architectural components



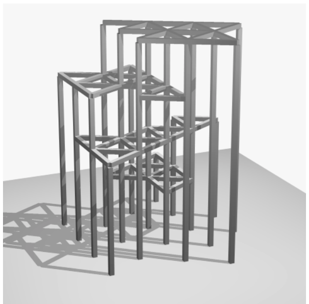
a. Walls and columns



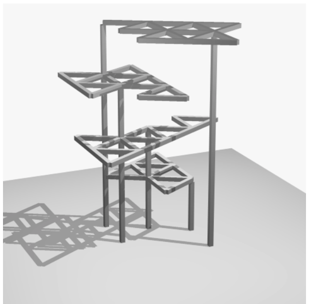
b. Volumes and columns



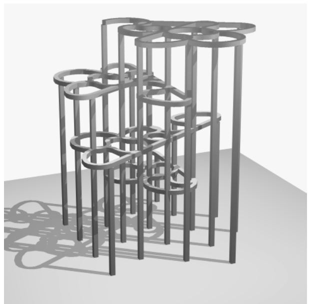
c. Walls and columns



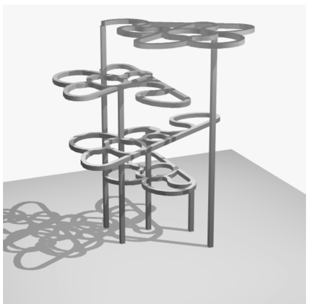
d. Beams and columns



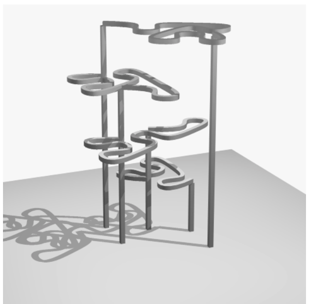
e. Beams and columns



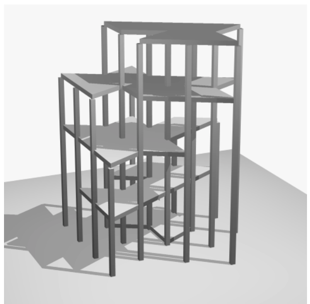
f. Beams and columns



g. Beams and columns

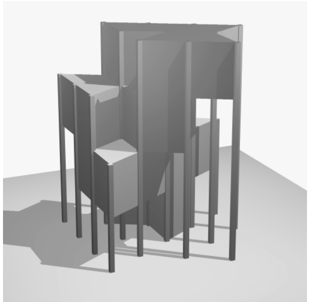


h. Beams and columns

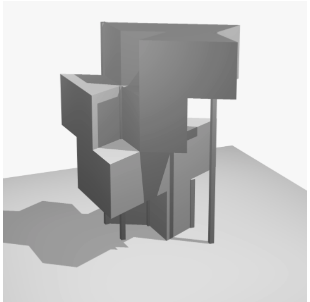


i. Floors and columns

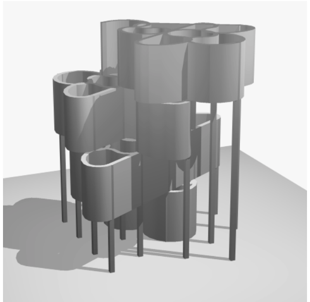
Figure 3a-i. Combining architectural components



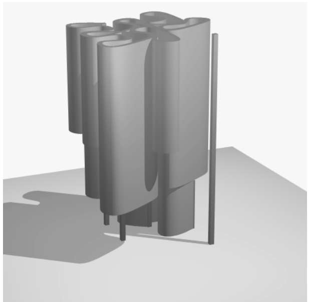
j. Volumes and columns



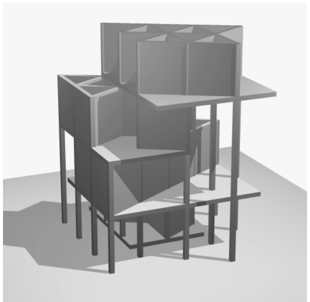
k. Volumes and columns



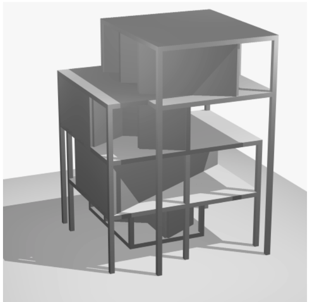
l. Walls and column



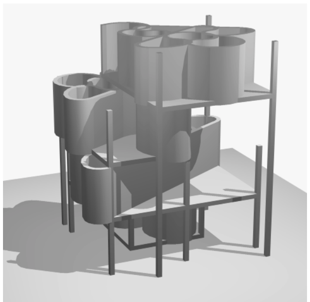
m. Walls and columns



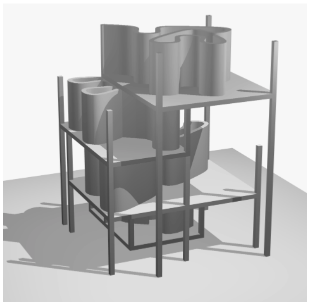
n. Walls, floors, columns



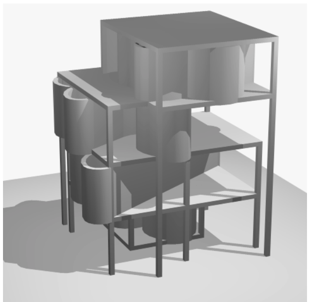
o. Walls, floors, columns



p. Walls, floors, columns



q. Walls, floors, columns



r. Walls, floors, columns

Figure 3j-r. Combining architectural components

Further Areas of Investigation

My intention is to return to the interpretation of the architectural elements as demonstrated with the Hilbert series and investigate if other elements or other related interpretations are possible. There is also the possibility of introducing variation within the curve itself by modifying the lengths and heights being used. This could be done by a range or sequence of values or ones randomly selected. In addition, other types of space-filling curves could be investigated that used a different set of turns.

The programs used in this investigation were written in AutoCAD AutoLisp. I am investigating the possibility of writing these in C++ with OpenGL so that they may be used within a virtual reality environment or converting the geometry to a VRML format for interactive viewing on the Web.

Another area of interest is the physical creation of these forms. Some initial research has been started to see if these programs can develop descriptions that are compatible with rapid prototyping systems. This would also allow me to consider these forms at the scale of sculptural objects.

Conclusion

This investigation has given me a starting point to further develop geometries that could suggest architectural forms. It has helped me to further my understanding how software can be used to investigate abstract design concepts and how it can be used to interpret these concepts. I am quite satisfied that the unexpected and the interesting is possible, even though the interpretation of forms is based on current architectural expression and experience. The greatest value may not be the actual results obtained by any one of these variations, but the "spark" of inspiration that may come when seeing something new and what was discovered in the process of creating them.

Acknowledgment

The programming and development of an early set of results was the work of Amy Ferguson an Independent Study student at IIT.

References

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