Seashell Interpretation in Architectural Forms

From the study of seashell geometry, the mathematical process on shell modeling is extremely useful. With a few parameters, families of coiled shells were created with endless variations. This unique process originally created to aid the study of possible shell forms in zoology. The similar process can be developed to utilize the benefit of creating variation of the possible forms in architectural design process.

The mathematical model for generating architectural forms developed from the mathematical model of gastropods shell, however, exists in different environment. Normally human architectures are built on ground some are partly underground subjected different load conditions such as gravity load, wind load and snow load.

In human architectures, logarithmic spiral is not necessary become significant to their shapes as it is in seashells. Since architecture not grows as appear in shells, any mathematical curve can be used to replace this, so call, growth spiral that occur in the shells during the process of growth. The generating curve in the shell mathematical model can be replaced with different mathematical closed curve. The rate of increase in the generating curve needs not only to be increase. It can be increase, decrease, combination between the two, and constant by ignore this parameter. The freedom, when dealing with architectural models, has opened up much architectural solution. In opposite, the translation along the axis in shell model is greatly limited in architecture, since it involves the problem of gravity load and support condition.

Mathematical Curves

To investigate the possible architectural forms base on the idea of shell mathematical models, the mathematical curves are reviewed. As in shell curve, the study in this step will focus on utilizing every mathematical curves property. Most influent works reviewed in this area are *A Handbook on Curves and Their Properties* by Robert C. Yates 1947, *A Book of Curves* by Edward Harrington Lockwood 1961, *A Catalog of Special Plane Curves* by J. Dennis Lawrence 1972, *Concise Encyclopedia of Mathematics (CD-ROM)* by Eric W. Weisstein 1999, and some related woks such as *Wobbly Spiral* by John Shape and mathematical related website.

In this chapter, all mathematical curves are divided into four curve groups in order to facilitate a further development of their property.

1. Close Curves

The mathematical plane curve that always complete or close itself at the 360-degree. Close curves are the followings:

- Circle
- Ellipse
- Piriform
- Bicorn
- Astroid
- Deltoid
- Eight
- Bifolium
- Folia
- Lemniscate of Bernoulli
- Epicycloid
- Hypocycloid
- Epitrochoid
- Hypotrochoid
- Cardioid
- Nephroid of Freeths
- Cayley's Sextic
- Lissajous
- Nephroid
- Rhodonea
- Limacon of Pascal
- Hippopede

2. Open Curves

The mathematical plane curve that never complete or close itself. It represents a single body of curve in one loop (360-degree). This group is divided into three minor group bases on the curve characteristic.

2.1 Increasing Curves

This type of open curve creates only a single curvature without repeating. Increasing curves are the followings:

- Hyperbola
- Parabola
- Pedals of Parabola
- Semi-Cubical Parabola
- Catenary
- Kappa
- Kampyle of Eudoxus
- Bullet Nose
- Serpentine
- Cross
- Conchoid of Nicomedes
- Tschirnhausen's Cubic
- Witch of Agnesi
- Cissoid of Diocles
- Right Strophoid
- Trisectrix of Maclaurin
- Folium of Descartes
- Tractrix

2.2 Periodic Curves

In opposite of the increasing curve, this type of open curve continuously repeats similar curvature along the increasing angle. Periodic curves are the followings:

- Sine Curve
- Cosind Curve
- Cycloid
- Trochoid

2.1 Spiral Curves

Spiral is open curve that every point in the curve travels around the center axis or pole. Spiral curves are the followings:

- Involute of a Circle
- Fermat's Spiral
- Poinsot's Spirals
- Cochleoid
- Archimedes' Spiral
- Logarithmic Spiral
- Hyperbolic Spiral
- Lituus
- Euler's Spiral

2.2 Special Spiral Curves

These curves are developed from others mathematical curves known for mathematician but interpret them into new mathematical property. Special Spiral curves are the followings:

- Wobbly Spiral
- Rectangular Spiral

3. Three-dimensional Curves

This curve is either close or open. The major difference is that the curve takes the height in z-axis while the others are not considering this axis. Three-dimensional curves are the followings:

- Spherical spiral
- Cylinder
- Helix

Each mathematical function and its property mentioned above can be found in a greater detail in appendix.

Seashell Properties and Architecture Properties

Seashell Properties

Again, base on the study in the previous chapter, these are known parameters in shell mathematical study and modeling. For further study of this research, however, a short terminology is introduced to represent these four parameters originally called after those researchers in biology and zoology. These new terms are called as the followings:

Biology and Zoology Terms		Research Terms
1. The shape of generating curve.	=	Section
2. The rate of increase of the generating curve.	=	Growth
3. The position of generating curve to the axis.	=	Path
4. The rate translation along the axis.	=	Height

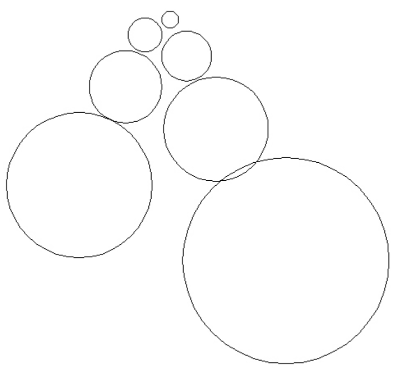


Figure xx: Section
A seashell cross sectional outline of the hollow shell tube.

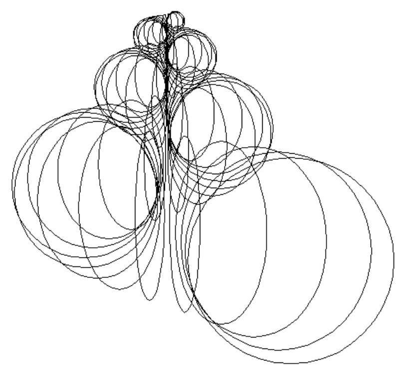


Figure xx: Growth
The rate of increase of the section in size.

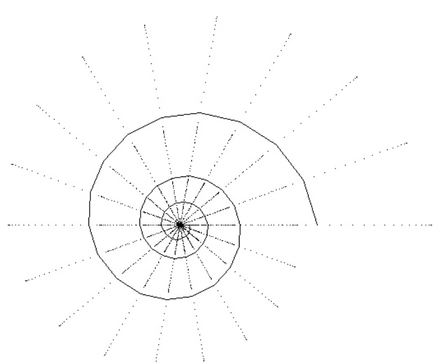


Figure xx: Path
The position of generating curve to the axis.

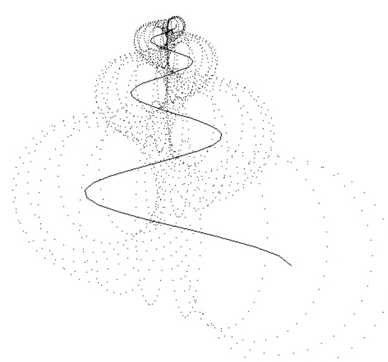


Figure xx: Height
The rate translation along the axis in vertical direction.

Mathematical Curves Properties

A unique property in each parameter can be explored with various mathematical properties. In seashell these relationships are limited depend upon the actual geometry of shells. Here, the limitations are less. The link from seashell properties to mathematical properties is presented in the following table.

Research Terms		Seashell Relative to Its Mathematical Properties		Mathematical Curves Properties
1. Section	=	Circle, Ellipse, Free shape	=	Close Curves
2. Growth	=	Exponential Function	=	Open Curves
3. Path	=	Logarithmic Spiral	=	Open, 3D Curve
4. Height	=	Progressive Function	=	Open Curves

Architectural Properties

In architectural forms, there are some others parameters that architects and engineers have to take into consideration. These architectural parameters came from basic design principle in architecture or the fundamental functions and conditions that make architectural forms work. In this research, the goal is experimenting on architectural forms in general. It is not intended to find architectural form to suite one particular project. Without a specific requirement of the actual site and actual project, the architectural parameters can be set in general as the followings:

1. Ground condition.
2. Human scale and proportion.
3. Wall opening and circulation.
4. Support condition.

Integrating these three unique properties, which are seashell, mathematics, and architecture, certainly yields the result in the mathematical interpretation of architectural forms that maintain the seashell signature. The integration of these three can be done by programming language, similar process in chapter III.

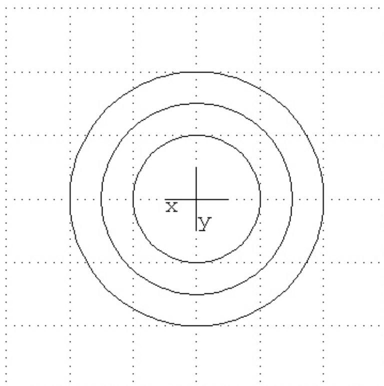
The parameters involved in the program will start from less to a complex one. The mathematical functions behind these architectural forms are reviewed as follows:

Circle:

Mathematical Function:

$x = r \cos t$

$y = r \sin t \quad , \quad - \quad \leq t \leq$



r = 1, 1.5, 2

Factors: r = radius

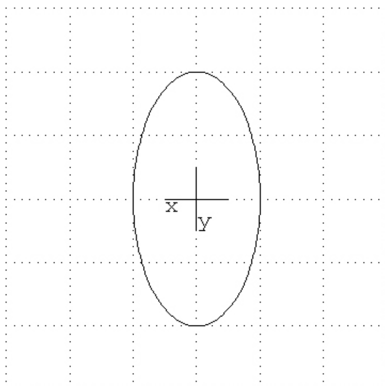
Reference:
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 65)

Ellipse:

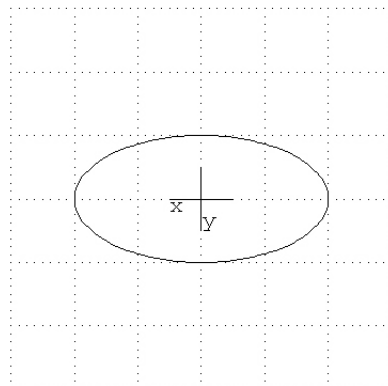
Mathematical Function:

$x = a \cos t$

$y = b \sin t \quad , - \quad \leq t \leq$



$a = 1, b = 2$



$a = 2, b = 1$

Factors: a = radius in x-axis
b = radius in y-axis

Reference:

J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 75)

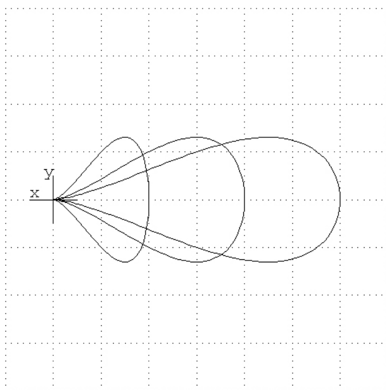
Piriform: (De Longchamps, 1886)

The piriform is also known as the pear-shaped quartic.

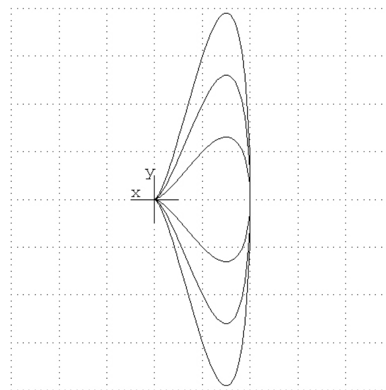
Mathematical Function:

$$x = a\,(1 + \sin t)$$

$$y = b \cos t\,(1 + \sin t) \qquad , \; -\frac{\;}{2} \le t \le \frac{3\;}{2}$$



a = 1, 2, 3
b = 1



a = 1
b = 1, 2, 3

Factors: a = x-axis distance
b = y-axis distance

Reference:

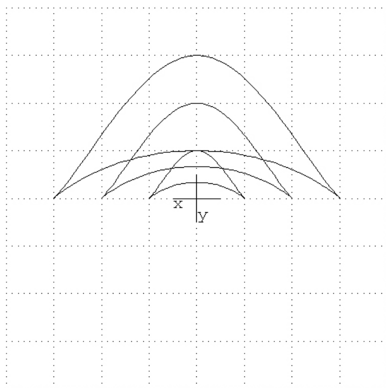
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 149)

Bicorn: (Sylvester, 1864)

Mathematical Function:

$$x = a \sin t$$

$$y = \frac{a \cos^2 t\ (2 + \cos t)}{3 + \sin^2 t} \qquad , \ - \quad \le t \le$$



a = 1, 2, 3

Factors: a = scale of curve

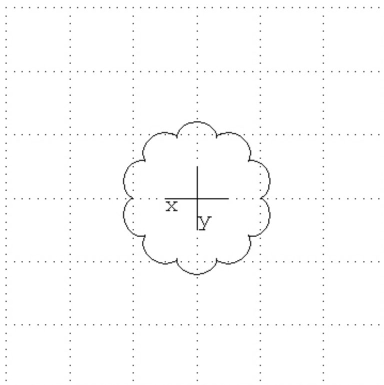
Reference:
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 147)

Epicycloid: (Roemer, 1674)

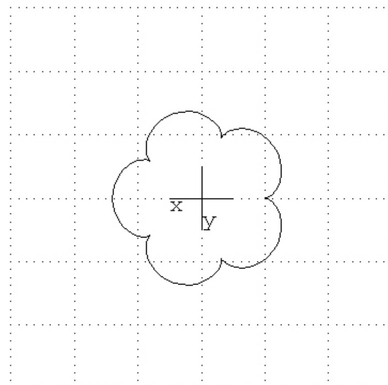
Mathematical Function:

$$x = m \cos t - b \cos \frac{m}{b} t$$

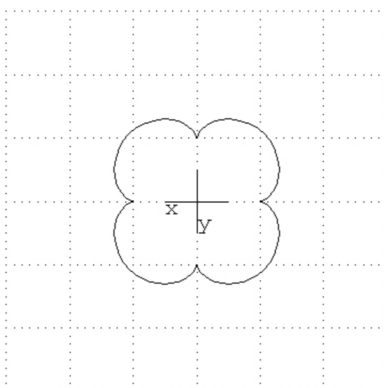
$$y = m \sin t - b \sin \frac{m}{b} t \qquad , \; -\pi \le t \le \pi$$



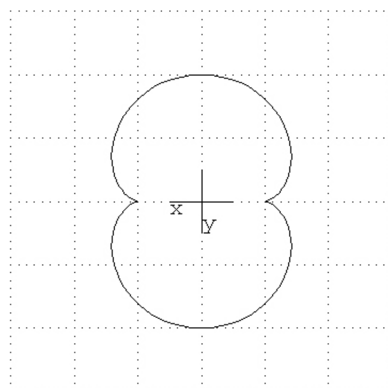
m = 1; b = 0.1



m = 1; b = 0.2



m = 1; b = 0.25



m = 1; b = 0.5

Factors: m = scale of curve
b = number of cusps

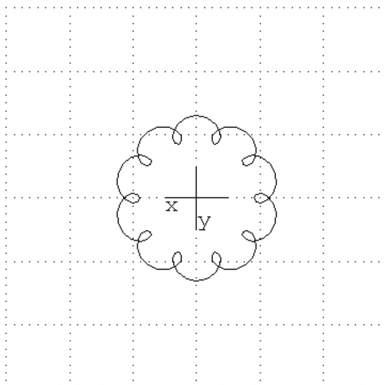
Reference:
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 168)

Epitrochoid:

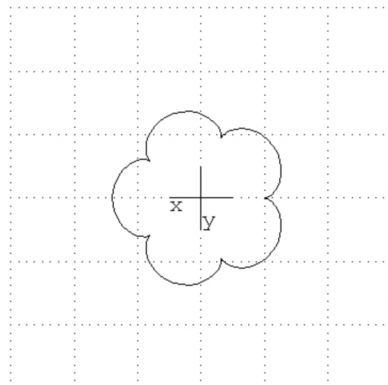
Mathematical Function:

$$x = m \cos t - h \cos \frac{m}{b} t$$

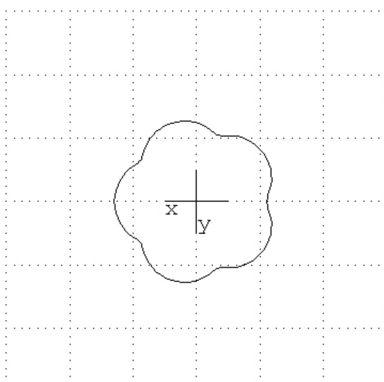
$$y = m \sin t - h \sin \frac{m}{b} t \qquad , \ -\pi \le t \le \pi$$



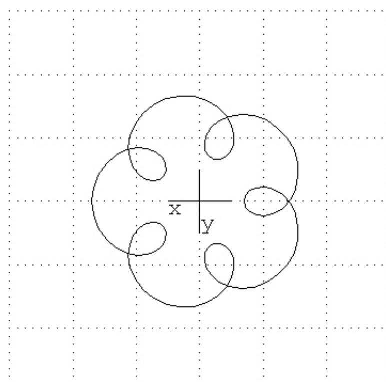
m = 1; b = 0.1; h = 0.2



m = 1; b = 0.2; h = 0.2



m = 1; b = 0.2; h = 0.1



m = 1; b = 0.2; h = 0.5

Factors: m = scale of curve
b = number of cusps
h = overlapping

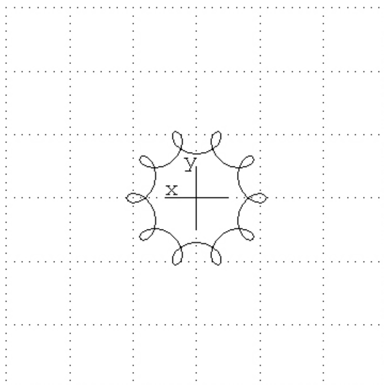
Reference:
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 160)

Hypotrochoid:

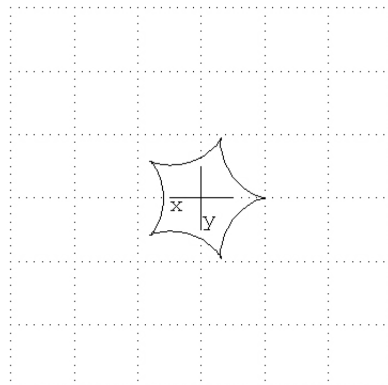
Mathematical Function:

$$x = n \cos t + h \cos \frac{n}{b} t$$

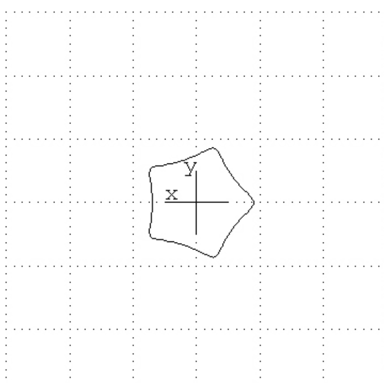
$$y = n \sin t - h \sin \frac{n}{b} t \quad , \; -\pi \leq t \leq \pi$$



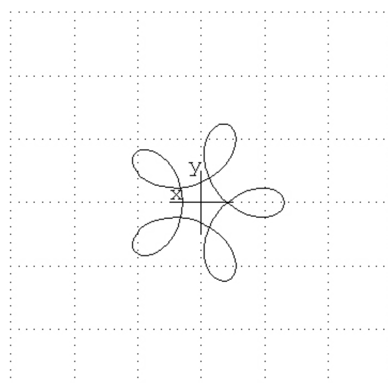
n = 1; b = 0.1; h = 0.2



n = 1; b = 0.2; h = 0.2



n = 1; b = 0.2; h = 0.1



n = 1; b = 0.2; h = 0.5

Factors: n = scale of curve
b = number of cusps
h = overlapping

Reference:

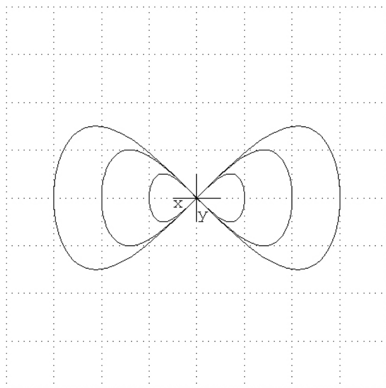
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 165)

Eight:

Mathematical Function:

$x = a \cos t$

$y = a \sin t \cos t \quad , \; -\pi \leq t \leq \pi$



a = 1, 2, 3

Factors: a = scale of curve

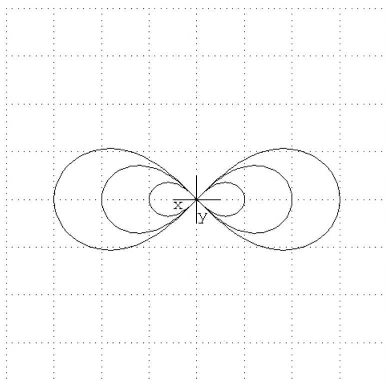
Reference:
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 124)

Lemniscate of Bernoulli:

Mathematical Function:

$$x = \frac{a \cos t}{1 + \sin^2 t}$$

$$y = \frac{a \sin t \cos t}{1 + \sin^2 t} \qquad , \; -\pi \le t \le \pi$$



a = 1, 2, 3

Factors: a = scale of curve

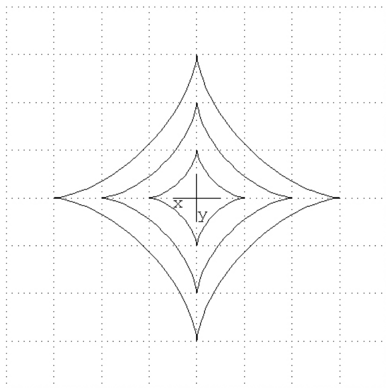
Reference:
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 121)

Astroid: (Roemer, 1674; Bernolli, 1691)

Mathematical Function:

$x = a\ (3 \cos t + \cos 3t)$

$y = a\ (3 \sin t - \sin 3t) \quad , \ -\pi \le t \le \pi$



a = 1, 2, 3

Factors: a = scale of curve

Reference:

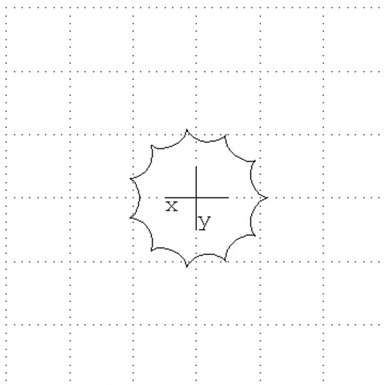
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 173)

Hypocycloid:

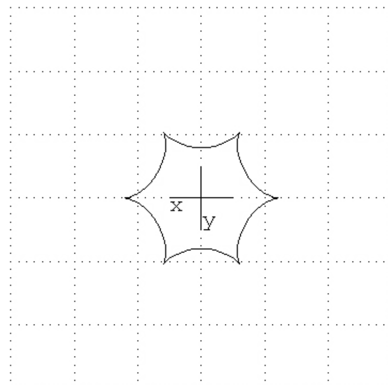
Mathematical Function:

$$x = n \cos t + b \cos \frac{n}{b} t$$

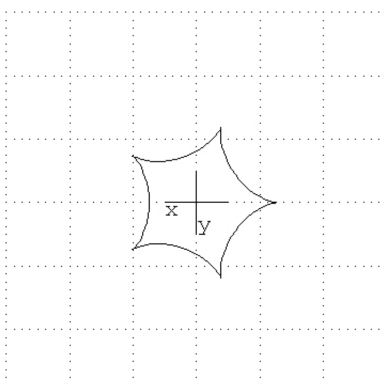
$$y = n \sin t - b \sin \frac{n}{b} t \qquad , \; -\pi \le t \le \pi$$



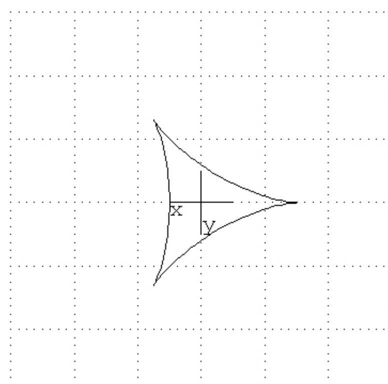
n = 1; b = 0.1



n = 1; b = 0.2



n = 1; b = 0.25



n = 1; b = 0.5

Factors: n = scale of curve
b = number of cusps

Reference:

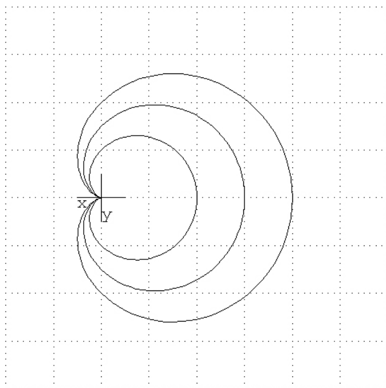
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 171)

Cardioid: (Koersma, 1689)

Mathematical Function:

$x = 2\,a \cos t\,(1 + \cos t)$

$y = 2\,a \sin t\,(1 + \cos t) \quad , \; -\pi \le t \le \pi$



a = 0.5, 0.75, 1

Factors: a = scale of curve

Reference:
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 118)

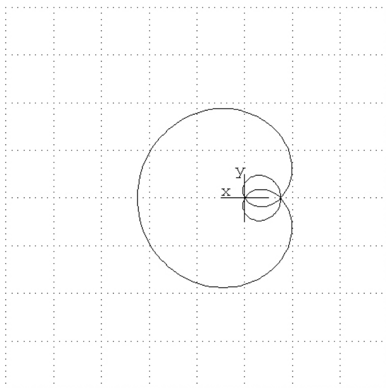
Nephroid of Freeths:

Mathematical Function:

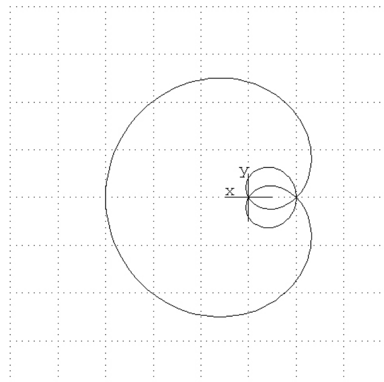
$$r = a\left(1 + 2 \sin \frac{1}{2} \theta\right)$$

$$x = r \cos t$$

$$y = r \sin t \quad , \; -\pi \leq t \leq \pi$$



a = 0.75



a = 1

Factors: a = scale of curve

Reference:

J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 175)

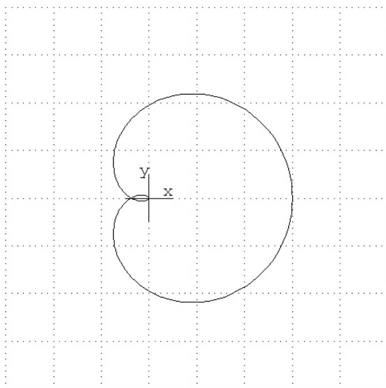
Cayley's Sextic: (Maclaurin, 1718)

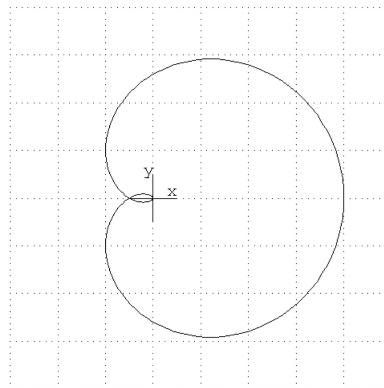
Mathematical Function:

$$r = a \cos^3 \frac{1}{3} \theta$$

$$x = r \cos t$$

$$y = r \sin t \qquad , \; -\pi \le t \le \pi$$





a = 0.75 a = 1

Factors: a = scale of curve

Reference:

J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 178)

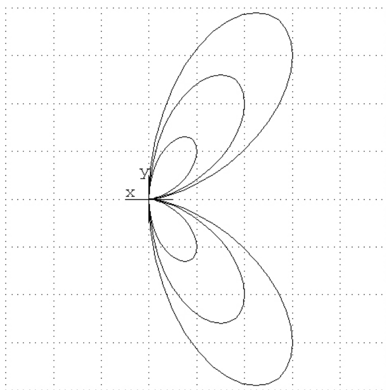
Bifolium:

Mathematical Function:

$$r = 4\, a \sin^2\theta \cos\theta$$

$$x = r \cos t$$

$$y = r \sin t \qquad , \quad -\pi \le t \le \pi$$



$a = 1, 2, 3$

Factors: a = scale of curve

Reference:
Eric W. Weisstein, *Concise Encyclopedia of Mathematics*, CRC Press, CD-ROM Edition, 1999

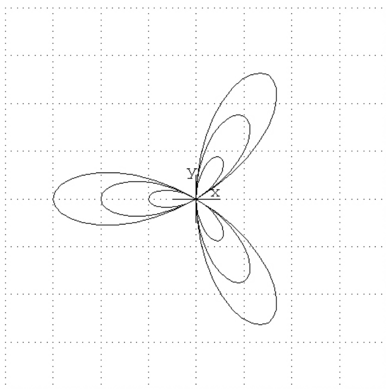
Folia: (Kepler, 1609)

Mathematical Function:

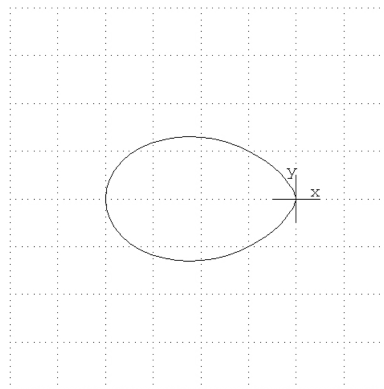
$$r = \cos\theta\,(4\,a\,\sin^2\theta - b)$$

$$x = r\cos t$$

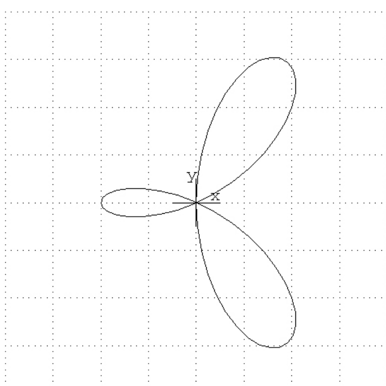
$$y = r\sin t \qquad , \; -\pi \le t \le \pi$$



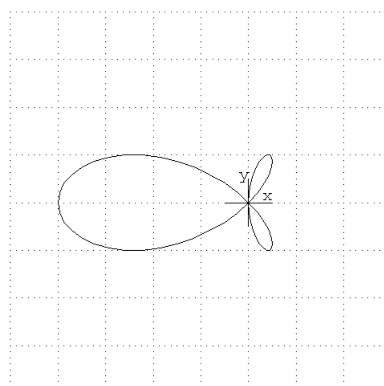
a = 1, 2, 3; b = 1, 2, 3



a = 1, b = 4



a = 3, b = 2



a = 2, b = 4

Factors: a = scale of curve
b   4 a, Single folium
b = 0, Bifolium
$0 < b < 4\,a$, Trifolium

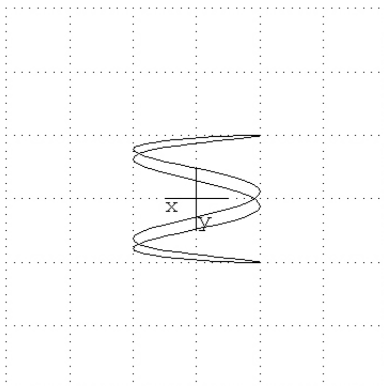
Reference:
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 151)

Lissajous:

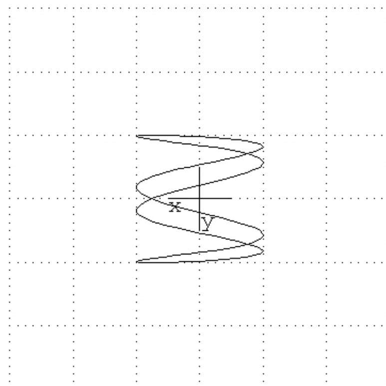
Mathematical Function:

$$x = a \sin(n\,t + c)$$

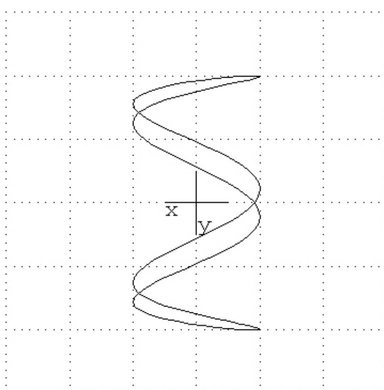
$$y = b \sin t \qquad , \quad -\pi \le t \le \pi$$



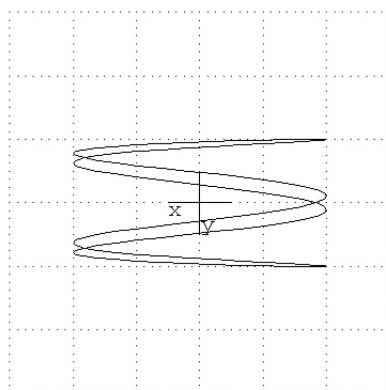
a = 1, b = 1, c = 2, n = 4



a = 1, b = 1, c = 4, n = 4



a = 1, b = 2, c = 2, n = 4



a = 2, b = 1, c = 2, n = 4

Factors: a = x-axis distance
b = y-axis distance
c = x-axis factor
n = number of loop

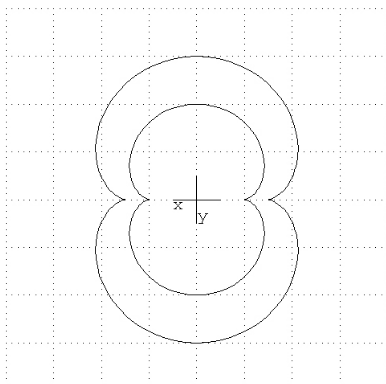
Reference:
Eric W. Weisstein, *Concise Encyclopedia of Mathematics*, CRC Press, CD-ROM Edition, 1999

Nephroid: (Huygens, 1678)

Mathematical Function:

$x = a\ (3 \cos t - \cos 3\ t)$

$y = a\ (3 \sin t - \sin 3\ t)$, $-\pi \le t \le \pi$



a = 0.5, 0.75

Factors: a = scale of curve

Reference:

J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 170)

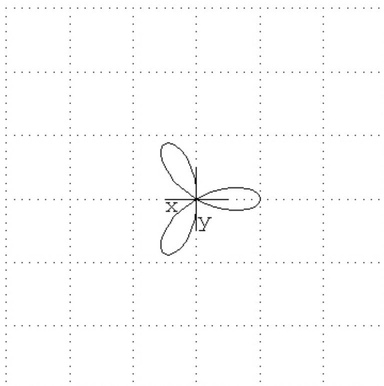
Rhodonea: (Grandi, 1723)

Mathematical Function:

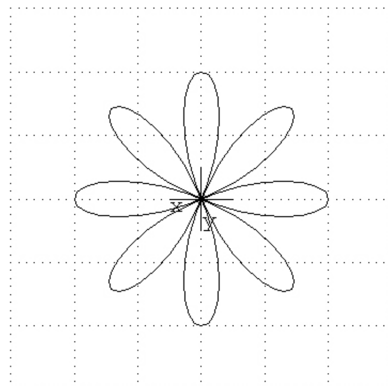
$r = a \cos m\theta$

$x = r \cos t$

$y = r \sin t \quad , \; -\pi \le t \le \pi$



a = 1, m = 3



a = 2, m = 4

Factors: a = scale of curve
m = number of loop

Reference:
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 175)

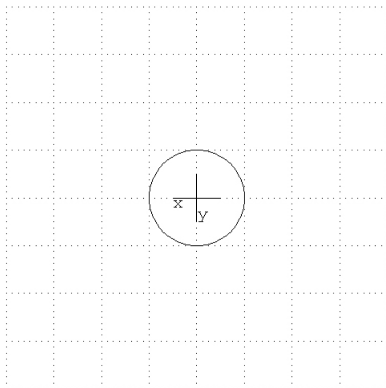
Limacon of Pascal: (Pascal, 1650)

Mathematical Function:

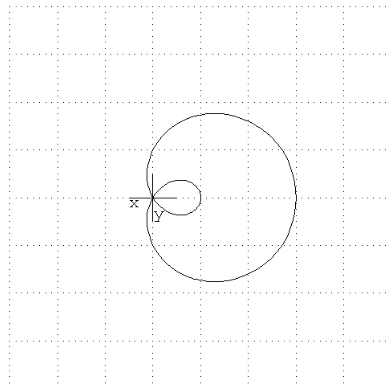
$r = 2\,a \cos\theta + b$

$x = r \cos t$

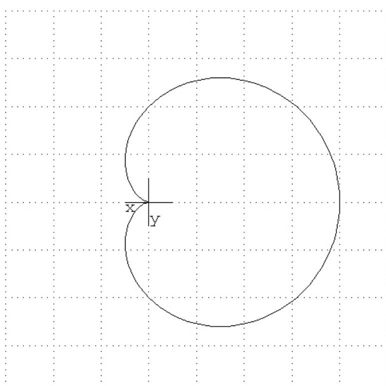
$y = r \sin t \quad , \quad -\pi \le t \le \pi$



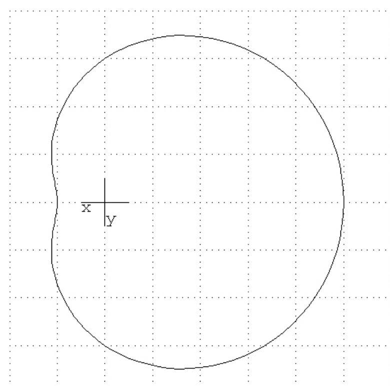
a = 0, b = 1



a = 1, b =1



a = 1, b = 2



a = 1, b = 3

Factors: a = scale of curve; a = 0, Circle
b = a, Trisectrix
b = 2 a, Cardioid

Reference:

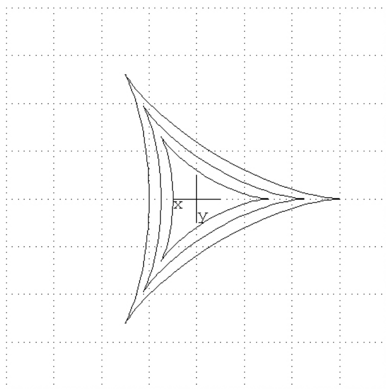
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 113)

Deltoid: (Euler, 1745)

Mathematical Function:

$x = a\ (2 \cos t + \cos 2\ t)$

$y = a\ (2 \sin t - \sin 2\ t) \qquad , \ -\pi \le t \le \pi$



a = 0.5, 0.75, 1

Factors: a = scale of curve

Reference:

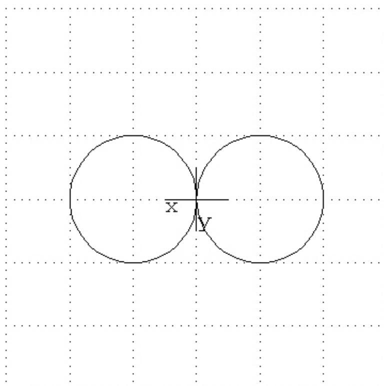
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 131)

Hippopede: (Proclus, ca. 75 B.C.)

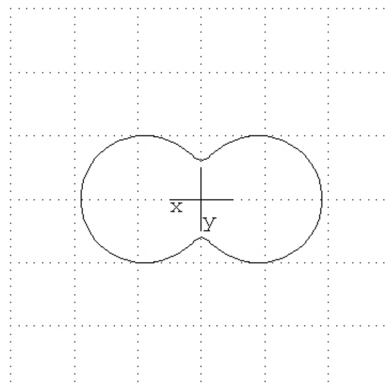
Mathematical Function:

$$x = 2 \cos t \ \mathrm{sqrt}(a\, b - b^2 \sin^2 t)$$

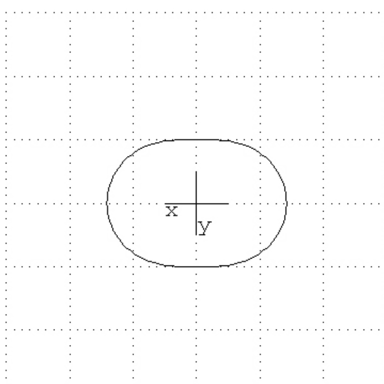
$$y = 2 \sin t \ \mathrm{sqrt}(a\, b - b^2 \sin^2 t) \qquad , \ -\pi \le t \le \pi$$



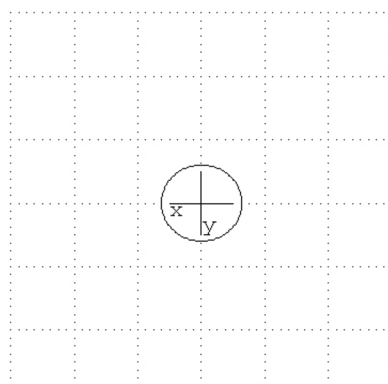
a = 1, b = 1



a = 1, b = 0.9



a = 1, b = 0.5



a = 1, b = 0.1

Factors: a = scale of curve
$b < a$

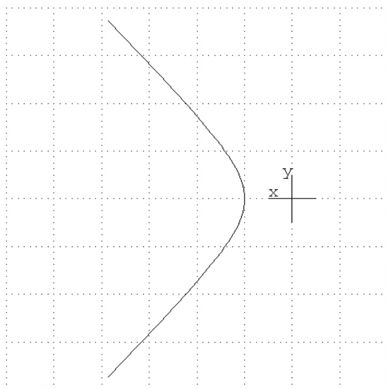
Reference:
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 145)

Hyperbola:

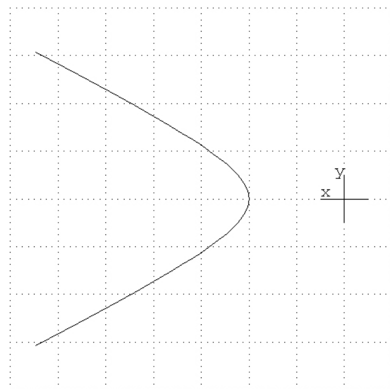
Mathematical Function:

$x = a \sec t$

$y = b \tan t \quad , \quad -\pi \leq t \leq \pi$



a = 1, b = 1



a =2, b = 1

Factors: a = x-axis distance
b = y-axis distance

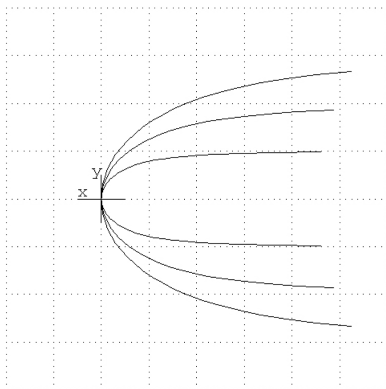
Reference:
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 79)

Kappa: (Gutschoven's curve)

Mathematical Function:

$x = a \cos t \cot t$

$y = a \cos t \quad , \quad 0 < t < 2\pi$



a = 1, 2, 3

Factors: a = scale of curve

Reference:

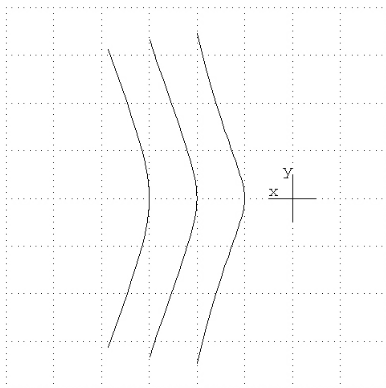
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 139)

Kampyle of Eudoxus:

Mathematical Function:

$$x = a \sec t$$

$$y = a \tan t \sec t \qquad , \; -\frac{\pi}{2} < t < \frac{3\pi}{2}$$



a = 1, 2, 3

Factors: a = scale of curve

Reference:

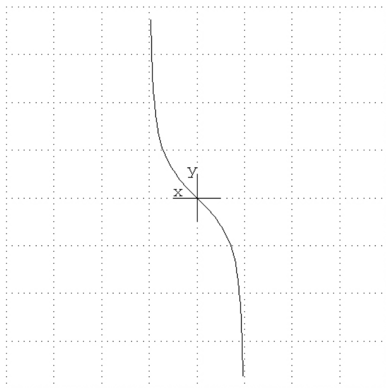
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 141)

Bullet Nose: (Schoute, 1885)

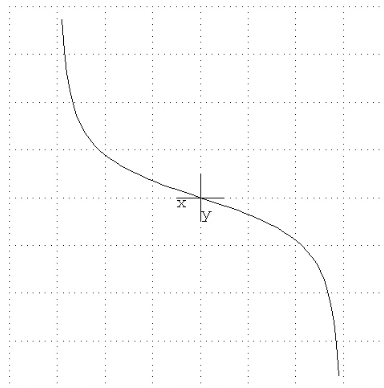
Mathematical Function:

$x = a \cos t$

$y = b \cot t \quad , \; -\pi < t < \pi$



a = 1, b = 1



a = 3, b = 1

Factors: a = x-axis distance
b = y-axis distance

Reference:

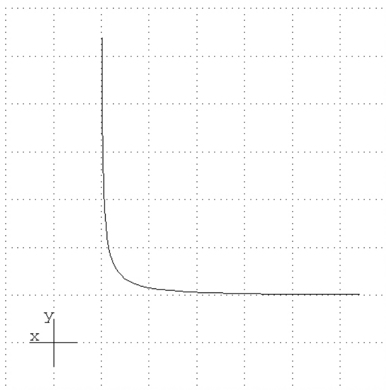
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 128)

Cross:

Mathematical Function:

$x = a \sec t$

$y = b \csc t \quad , \; -\pi < t < \pi$



a = 1, b = 1

Factors: a = x-axis distance
b = y-axis distance

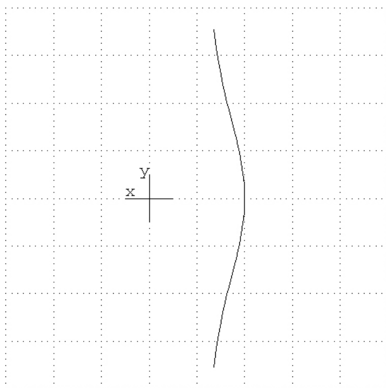
Reference:
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 130)

Conchoid of Nicomedes: (Nicomedes, 225 B.C.)

Mathematical Function:

$$x = b + a \cos t$$

$$y = \tan t\,(b + a \cos t) \qquad , \; -\frac{\pi}{2} < t < \frac{3\pi}{2}$$



a = 1, b = 1

Factors: a = x-axis distance
b = y-axis distance

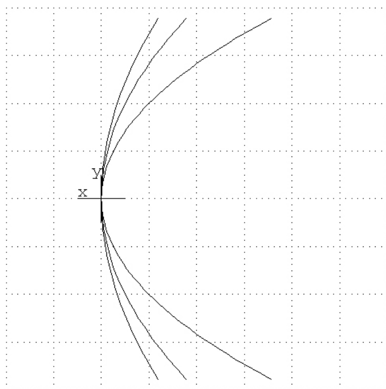
Reference:
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 137)

Parabola:

Mathematical function:

$x = a\,t^2$

$y = 2\,a\,t \qquad , \; -\infty < t < \infty$



$a = 1, 2, 3$

Factors: a = scale of curve

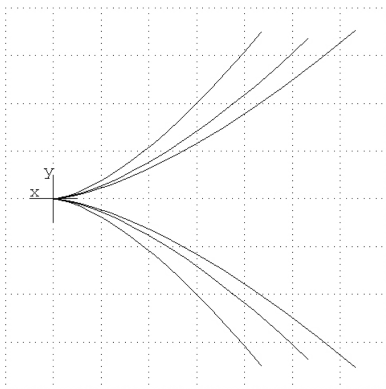
Reference:
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 67)

Semi-Cubical Parabola: (Isochrone)

Mathematical Function:

$x = 3\ a\ t^2$

$y = 2\ a\ t^3 \quad , \quad -\infty < t < \infty$



a = 1, 2, 3

Factors: a = scale of curve

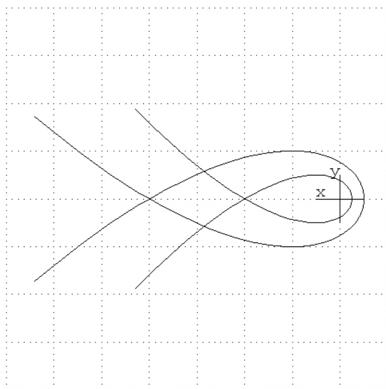
Reference:
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 85)

Tschirnhausen's Cubic:

Mathematical Function:

$x = a\,(1 - 3\,t^2)$

$y = a\,t\,(3 - t^2)$, $-\infty < t < \infty$



a = 0.25, 0.5

Factors: a = scale of curve

Reference:

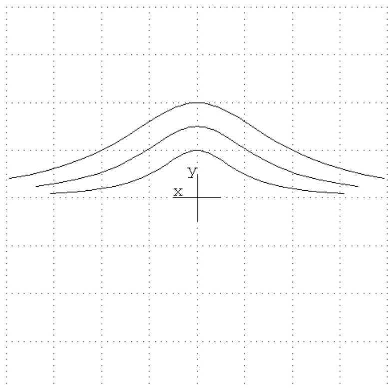
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 88)

Witch of Agnesi: (Fermat, 1666; Agnesi, 1748)

Mathematical Function:

$$x = 2\,a\,\tan t$$

$$y = 2\,a\,\cos^2 t \qquad , \; -\frac{\pi}{2} < t < \frac{\pi}{2}$$



a = 0.5, 0.75, 1

Factors: a = scale of curve

Reference:

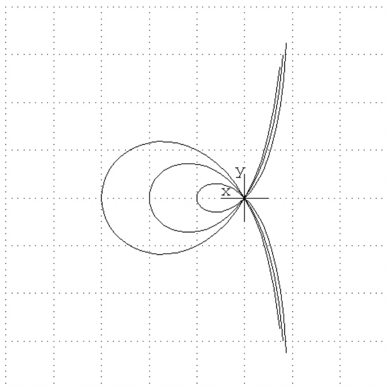
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 90)

Pedals of Parabola:

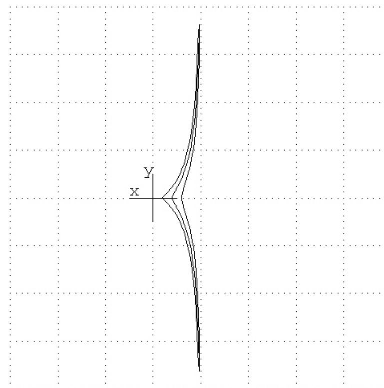
Mathematical Function:

$$x = a\,(\sin^2 t - m \cos^2 t)$$

$$y = a \tan t\,(\sin^2 t - m \cos^2 t) \qquad , \; -\frac{\pi}{2} < t < \frac{\pi}{2}$$



$a = 1;\ m = 1, 2, 3$



$a = 1;\ b = -0.2, -0.4, -0.6$

Factors: a = scale of curve
$m > 0$, one loop
$m \le 0$, no loop

Reference:

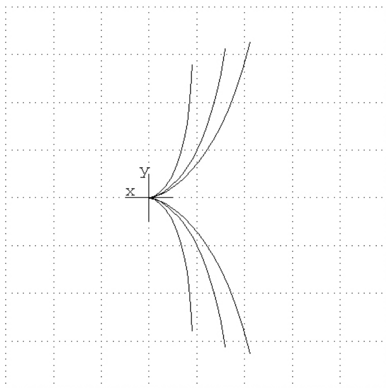
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 94)

Cissoid of Diocles: (Diocles, ca. 200 B.C.)

Mathematical Function:

$x = a \sin^2 t$

$y = a \tan t \sin^2 t \quad , \; -\infty < t < \infty$



a = 1, 2, 3

Factors: a = scale of curve

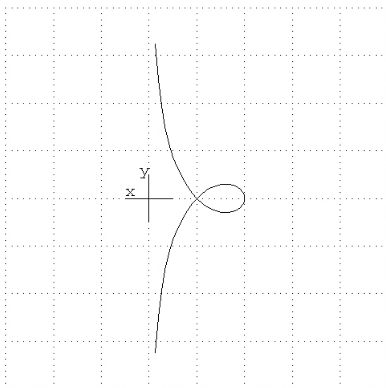
Reference:
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 98)

Right Strophoid: (Barrow, 1670)

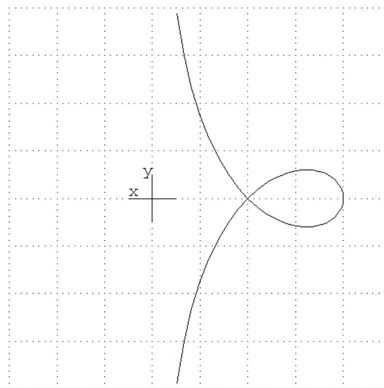
Mathematical Function:

$x = a\,(1 - 2\cos^2 t)$

$y = a\tan t\,(1 - 2\cos^2 t) \qquad , \ -\frac{\pi}{2} < t < \frac{\pi}{2}$



a = 1



a = 2

Factors: a = scale of curve

Reference:

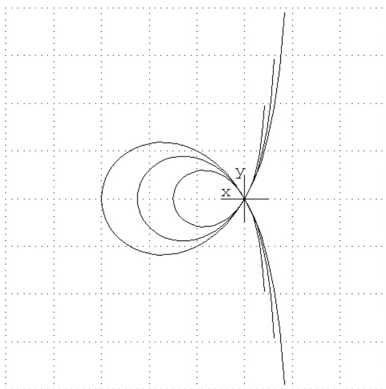
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 100)

Trisectrix of Maclaurin: (Maclaurin, 1742)

Mathematical Function:

$$x = a\ (1 - 4\cos^2 t)$$

$$y = a \tan t\ (1 - 4\cos^2 t) \qquad , \; -\frac{\pi}{2} < t < \frac{\pi}{2}$$



a = 0.5, 0.75, 1

Factors: a = scale of curve

Reference:

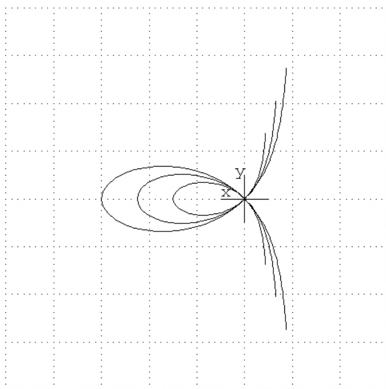
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 104)

Folium of Descartes: (Descartes, 1638)

Mathematical Function:

$$x = \frac{3\ a\ t}{1 + t^3}$$

$$y = \frac{3\ a\ t^2}{1 + t^3} \quad , \ -\infty < t < \infty$$



a = 0.5, 0.75, 1

Factors: a = scale of curve

Reference:

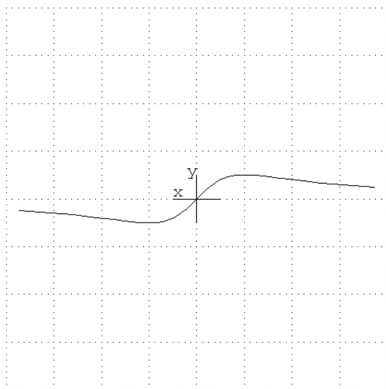
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 106)

Serpentine:

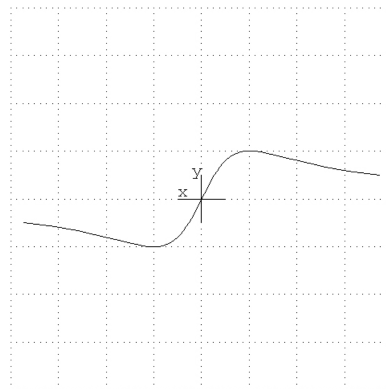
Mathematical Function:

$x = a \cot t$

$y = b \sin t \cos t \quad , \quad 0 < t < \pi$



a = 1, b =1



a = 1, b = 2

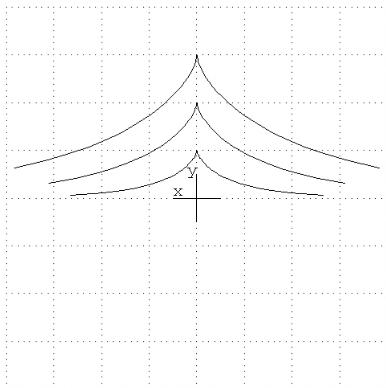
Factors: a = x-axis distance
b = y-axis distance

Reference:
Eric W. Weisstein, *Concise Encyclopedia of Mathematics*, CRC Press, CD-ROM Edition, 1999

Tractrix: (Huygens, 1692)

Mathematical Function:

$$x = a \ln(\sec t + \tan t) - a \sin t$$
$$y = a \cos t \qquad , \quad -\frac{\pi}{2} < t < \frac{\pi}{2}$$



a = 1, 2, 3

Factors: a = scale of curve

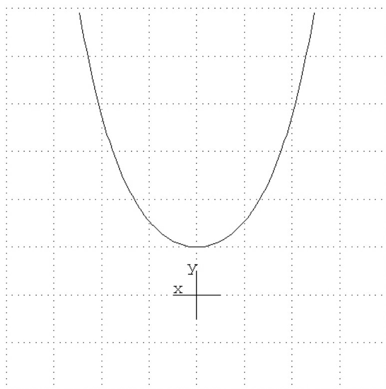
Reference:
J. Dennis Lawrence, _A Catalog of Special Plane Curves_, Dover Publications, Inc., 1972 (p. 199)

Catenary:

Mathematical Function:

$$x = t$$

$$y = a \cosh \frac{t}{a} \quad , \quad -\pi < t < \pi$$



$a = 1$

Factors: a = scale of curve

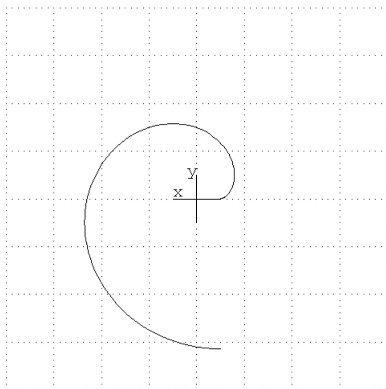
Reference:
Eric W. Weisstein, *Concise Encyclopedia of Mathematics*, CRC Press, CD-ROM Edition, 1999

Involute of a Circle: (Huygens, 1693)

Mathematical Function:

$$x = a\,(\cos t + t \sin t)$$

$$y = a\,(\sin t - t \cos t) \quad , \; -\infty < t < \infty$$



a = 0.5; 0 - 360 degree

Factors: a = scale of curve

Reference:
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 190)

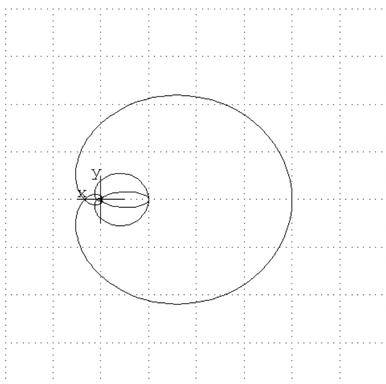
Poinsot's spirals:

Mathematical Function:

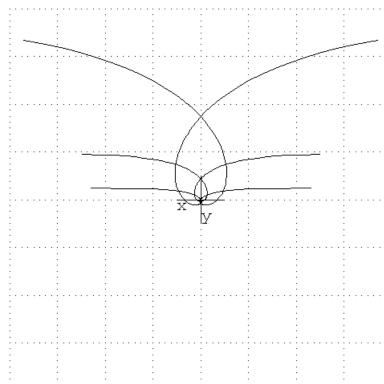
$$r = \frac{a}{\cosh n\,\theta} \quad \text{and} \quad \frac{a}{\sinh n\,\theta}$$

$$x = r \cos t$$

$$y = r \sin t \qquad , \; 0 < t < \pi$$



(a / cosh n θ)



(a / sinh n θ)

a = 1, b = 4
a = 1, b = 1
a = 4, b = 1

a = 1, b = 4
a = 1, b = 1
a = 4, b = 1

Factors: a = x-axis distance
B = y-axis distance

Reference:
J. Dennis Lawrence, _A Catalog of Special Plane Curves_, Dover Publications, Inc., 1972 (p. 192)

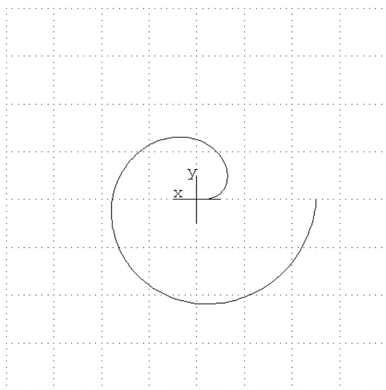
Fermat's Spiral:

Mathematical Function:

$r = a \text{ sqrt } \theta$

$x = r \cos t$

$y = r \sin t \qquad , \quad 0 < t < \pi$



a = 1; 0 – 360 degree

Factors: a = scale of curve

Reference:
Eric W. Weisstein, *Concise Encyclopedia of Mathematics*, CRC Press, CD-ROM Edition, 1999

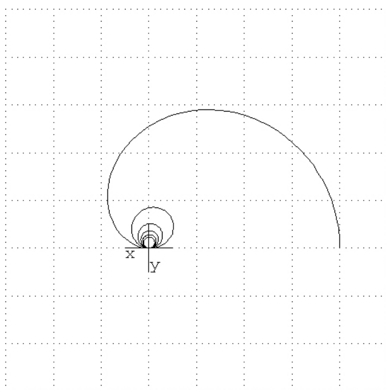
Cochleoid: (Bernoulli, 1726)

Mathematical Function:

$$r = \frac{a \sin \theta}{\theta}$$

$$x = r \cos t$$

$$y = r \sin t \qquad , \; 0 < t < \pi$$



a = 4; 0 – 1080 degree

Factors: a = scale of curve

Reference:
J. Dennis Lawrence, _A Catalog of Special Plane Curves_, Dover Publications, Inc., 1972 (p. 192)

Logarithmic: (Descartes, 1638)

Mathematical Function:

$r = \exp(a\theta)$

$x = r \cos t$

$y = r \sin t \qquad , \ 0 < t < \pi$

(Missing log's image)

Factors: a = scale of curve

Reference:

J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 184)

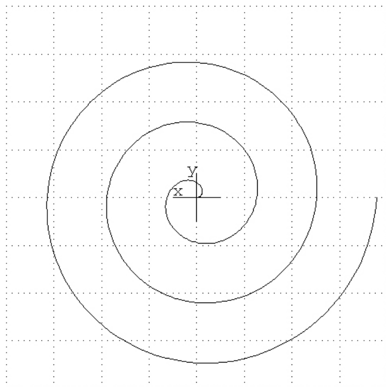
Archimedes' Spiral: (Sacchi, 1854)

Mathematical Function:

$r = a\,\theta$

$x = r \cos t$

$y = r \sin t \qquad , \; 0 < t < \pi$



a = 0.2; 0 – 1080 degree

Factors: a = scale of curve

Reference:
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 186)

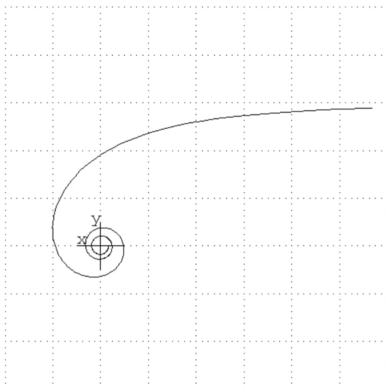
Hyperbolic Spiral:

Mathematical Function:

$$r = \frac{a}{\theta}$$

$$x = r \cos t$$

$$y = r \sin t \qquad , \quad 0 < t < \pi$$



a = 3; 0 – 1080 degree

Factors: a = scale of curve

Reference:
Eric W. Weisstein, *Concise Encyclopedia of Mathematics*, CRC Press, CD-ROM Edition, 1999

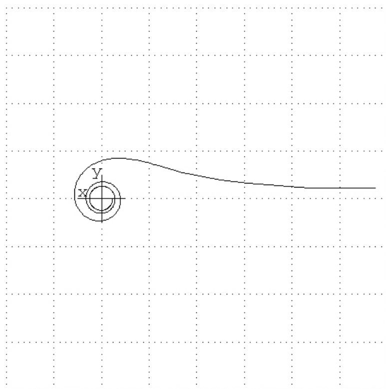
Lituus:

Mathemaical Function:

$$r = \frac{a}{\sqrt{\theta}}$$

$$x = r \cos t$$

$$y = r \sin t \qquad , \quad 0 < t < \pi$$



a = 1; 0 – 1080 degree

Factors: a = scale of curve

Reference:
Eric W. Weisstein, *Concise Encyclopedia of Mathematics*, CRC Press, CD-ROM Edition, 1999

Wobbly Spiral:

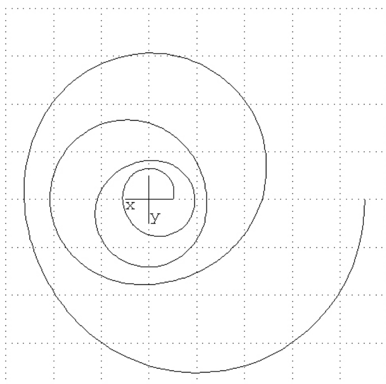
Mathematical Function:

$$r = \left(1 + 2\,k \sin \frac{4\theta}{3}\right) a\, e^{\theta k}$$

$$k = \frac{\ln \phi}{3\,\frac{\pi}{2}} \qquad ; \ \phi = 270$$

$$x = r \cos t$$

$$y = r \sin t \qquad , \quad 0 < t < \pi$$



a = 0.5; 0 - 1440 degree

Factors: a = scale of curve

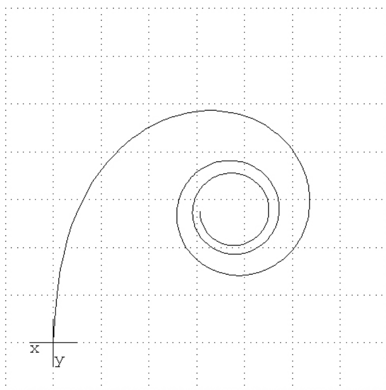
Reference:
John Sharp, *Mathematics in School*, November 1997

Euler’s Spiral: (Euler, 1744)

Mathematical Function:

$$x = a \ \frac{\sin t}{\sqrt{t}} \ d \ t$$

$$y = a \ \frac{\cos t}{\sqrt{t}} \ d \ t \quad , \ 0 \leq t < \infty$$



a = 3, d = 1

Factors: a = scale of curve
d = scale of curve

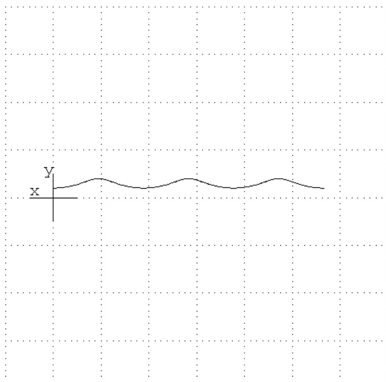
Reference:
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 190)

Cycloid: (Mersenne, 1599)

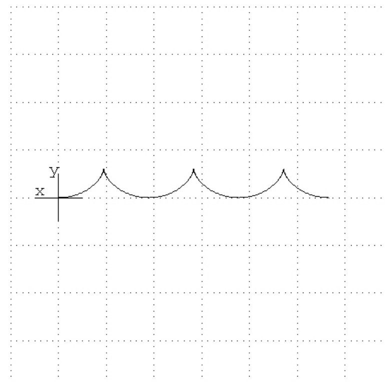
Mathematical Function:

$$x = a\,t + h \sin t$$

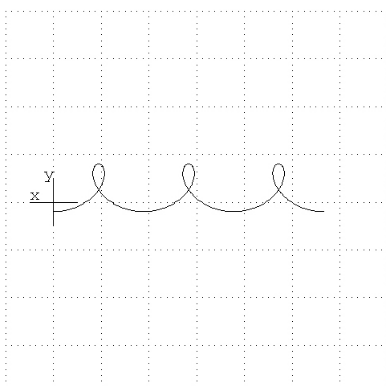
$$y = a - h \cos t \qquad , \quad -\infty < t < \infty$$



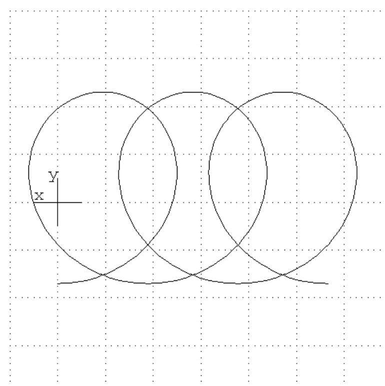
a = 0.3, h = 0.1



a = 0.3, h = 0.3



a = 0.3, h = 0.5



a = 0.3, h = 2

Factors: a = scale of curve
h = overlapping
0 – 1080 degree

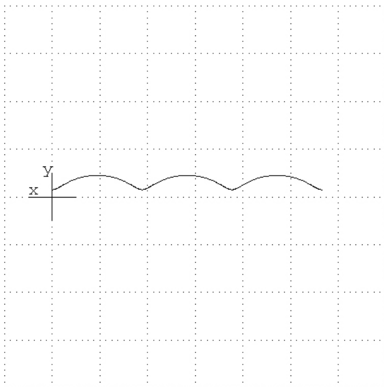
Reference:
J. Dennis Lawrence, *A Catalog of Special Plane Curves*, Dover Publications, Inc., 1972 (p. 192)

Trochoid:

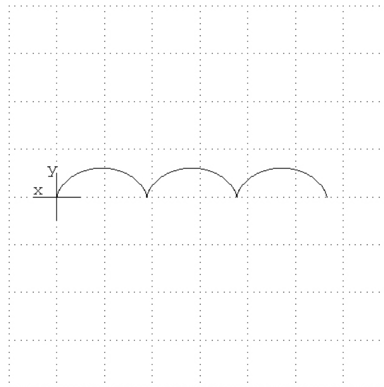
Mathematical Function:

$x = a\,t - h \sin t$

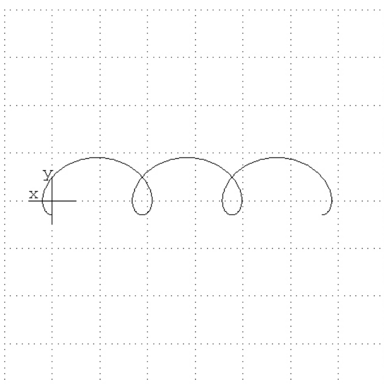
$y = a - h \cos t \qquad , \quad -\infty < t < \infty$



a = 0.3, h = 0.5



a = 0.3, h = 1



a = 0.3, h = 2

Factors: a = scale of curve
h = overlapping
0 – 1080 degree

Reference:
Eric W. Weisstein, *Concise Encyclopedia of Mathematics*, CRC Press, CD-ROM Edition, 1999

Sine and Cosine Curve:

Mathematical Function:

$x = x$

$y = X \sin t \quad , \quad 0 < t < \infty$

(Missing sine and cosine images)