

Math 152: Final Exam

Name:.....

1. (10 points) Radium-226 has a half-life of 1590 years. If a sample has a mass of 100 mg as of today, find the mass after 1000 years.

2. (15 points)

(a) Evaluate the integral  $\int_0^1 \ln(x+1) dx$  by Integration by Parts.

(b) Determine if the improper integral  $\int_{-2}^0 \frac{1}{(1+x)^{3/5}} dx$  converges. If yes, find its value.

3. (15 points) For the curve  $x(t) = \cos t, y(t) = t + \sin t, 0 \leq t \leq \pi$ ,

- (a) find an equation for the tangent line to the curve at the point when  $t = \pi/4$ ;
- (b) find the length of the curve.

4. (10 points)

- (a) Find the Cartesian coordinates of the point  $(3, 4\pi/3)$  given in polar coordinates.
- (b) Find a Cartesian equation for the curve  $r = 4 \sin \theta$  given in polar equation.

5. (10 points) Find if the infinite series  $\sum_{n=1}^{\infty} \frac{1}{n \ln n}$  converge or diverge. Explain why. (Hint: you can use integral test.)

6. (20 points) For the power series  $\sum_{n=1}^{\infty} \frac{(x-1)^n}{\sqrt{n}}$ ,

- (a) find the radius and interval of convergence.
- (b) for what values of  $x$  does the series converge absolutely or conditionally?

7. (20 points)

(a) Find the third-degree Taylor polynomial  $T_3$  for  $f(x) = \ln(1 + x)$  at  $a = 0$ .

(b) What is the maximum error possible replacing  $\ln(1 + x)$  by  $T_3$  for  $-0.1 \leq x \leq 0.1$ ?

8. (Bonus, 20 points) Find the Taylor series for  $\tan^{-1} x$  at  $a = 0$ , i.e., the Maclaurin series. For what values of  $x$  does the series converge to  $\tan^{-1} x$ ?