Numerical simulation of finite amplitude standing waves in acoustic resonators using finite volume method

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Abstract

Finite amplitude standing waves in acoustic resonators are simulated. The fluid is initially at rest and excited by a harmonic motion of the entire resonator. The unsteady compressible Navier-Stokes equations and the state equation for an ideal gas are employed. This study extends the traditional pressure based finite volume SIMPLEC scheme for solving the equations without any predefined standing waves. The pressure waveforms are computed in three different closed resonators and two opened resonators. The studied shapes of resonators include cylinder, cone and exponential horn. The numerical results obtained from the proposed finite volume method in the study are in agreement with those obtained with the Galerkin method and the finite difference method. We also investigate the velocity waveforms in the three different kinds of resonators, and finds that the sharp velocity spikes appear at the ends of the resonator when the pressure has shock-like waveform. The velocity in the closed conical resonator displays smooth harmonic waveform. Finally, the pressure waveforms in opened resonators are simulated and analyzed. The results show that the decrease in the ratio of the maximum to the minimum pressure at the small end of the exponential resonator is less than that of cylindrical resonators when the flow velocity at the opened end (relative to the resonator) is same.

Keywords: Nonlinear standing wave; Closed acoustic resonator; Opened acoustic resonator; Numerical simulation; Finite volume method.
1. Introduction

High amplitude pressures in acoustic resonators can be very useful in many engineering applications ranging from acoustic compression\(^1\), microfluidic devices\(^2\), to acoustic seal\(^3\). It has been verified by both experiments and numerical simulations that high finite amplitude standing wave pressures can be generated by oscillating acoustic resonators with axially variable cross-sectional areas (i.e. “shaped resonators”). In the experiments conducted by Lawrenson et al.\(^1\), standing wave over-pressures in excess of 340% ambient pressure were reported. Li et al.\(^4\) numerically optimized the shapes of resonators for improving pressure compression ratio by more than 241% at the same excitation forced by Lawrenson et al.\(^1\)

Studying the standing waves by real physical experiments would be expensive due to the cost of constructing different resonator configurations. Hence, numerical simulation has become an important tool for obtaining the nonlinear standing waves in acoustic resonators. Ilinskii et al.\(^5\) developed a simplified, one-dimensional nonlinear wave equation for the velocity potential in shaped resonators. The velocity potential was expressed in a truncated finite Fourier series in time based on the assumption that the nonlinear wave motion is periodic, and then the nonlinear wave equation was reduced to a two-point boundary value problem. The one-dimensional model was later improved by including the shear viscosity term in the conservation of momentum equation\(^6\).

Erickson and Zinn\(^7\) employed the quasi-one-dimensional model derived in Ilinskii et al.\(^5\) and neglected the third-order nonlinear terms. The velocity potential was expressed with a truncated series of acoustic modes or trial functions with time-dependent amplitudes. Luo et al.\(^8\) also used the Galerkin method to solve the quasi-one-dimensional model, but they modified the momentum equations by inserting the simple forms of the shear viscosity term for a two-
dimensional axisymmetric resonator and a three-dimensional low aspect-ratio rectangular resonator. The Galerkin method was used to solve for the time-dependent amplitudes of the acoustic modes in cylindrical resonators and exponential resonators. The eigenfunctions of the acoustic resonators were chosen as the trial functions. Therefore, the algorithm was limited to certain shaped resonators in which the eigenfunctions can be found analytically.

VanHille and Campos-Pozuelo\textsuperscript{9} derived a second-order wave equation in Lagrangian coordinates for the displacements in cylindrical resonators forced by an oscillating piston. The displacements were assumed to be the sum of two terms, i.e., the linear solution plus a second-order correction, and then solved with a finite difference algorithm. VanHill et al.\textsuperscript{10,11} further employed a finite volume method (FVM) in space and a finite-difference scheme for time derivatives to solve the nonlinear wave equation. Based on the assumption that the fluid is irrotational, VanHille and Campos-Pozuelo\textsuperscript{12,13} extended their finite difference model to the two and three (included axisymmetric) dimensions. Independently, Chun and Kim\textsuperscript{14} solved nonlinear standing waves in cylindrical and shaped resonators with the fourth-order compact finite difference scheme. Based on a linear acoustic theory and fundamental fluid dynamics equations, Hossian et al.\textsuperscript{15} developed a linear theory to estimate the resonant frequency and the standing wave mode in closed exponential resonators and used finite difference MacCormack scheme for simulating the finite amplitude standing waves. The proposed linear theory was limited to the cylindrical and exponential resonators. Based on the linear wave equation, Yu et al.\textsuperscript{16} developed a three-dimensional model for T-shaped acoustic resonators to predict the resonance frequencies and then conducted experiments to validate the model.

Mortell and SeyMour\textsuperscript{17} used a Duffing-type perturbation to find the amplitude-frequency relation, and then obtained shockless pressure waves in closed cone, horn and bulb resonators.
They also found that there are different frequency shift characters in the cone and bulb resonators. Based on Lagrangian mechanics and perturbation for closed cylindrical resonators, Hamilton et al. deduced the Webster horn equation for closed shaped resonators. In their study, the efficiency of second-harmonic generation in modes determined the direction of the resonance frequency shift.

Li and Raman reviewed the detailed information of the computational methods in nonlinear standing waves in acoustic resonators. Most of the previous numerical studies were limited to closed resonators and analyzed standing waves with a simplified approach: the physical variables are assumed as finite sums of basic functions. In current study, we will directly compute the formation of the standing waves in the closed resonators and opened resonators starting from the initial position at rest without any predefined standing waves. To achieve the goal, we use a pressure-based FVM for the unsteady compressible Navier-Stokes equations without any other simplification. The SIMPLE-type method is a pressure-based FVM, and was originally developed for incompressible flows. Karki and Patankar substituted the mass density correction by the pressure correction derived from the state equation, and then successfully extended SIMPLE-type methods for solving steady viscous compressible flows at all speeds. We employ similar idea of Karki and Patankar, and extend the traditional SIMPLEC scheme for solving unsteady viscous compressible flows in acoustic resonators.

This work will pave the way to compute the standing waves in resonators for acoustic seal. Acoustic seal is a new application of the nonlinear standing waves in opened acoustic resonators that requires complex boundary conditions, e.g., inlet flow, outlet flow and central blockages etc. In this case, the standing waves cannot be predefined as a combination of harmonic components.
The study will provide a solid foundation for constructing successful future models for acoustic seal.

The rest of the paper is organized as follows. In Sec. 2, we provide the basic equations for a viscous compressible fluid and the boundary conditions of closed and opened resonators. In Sec. 3, we describe the pressure-based FVM. In Sec. 4, the FVM is used to simulate the standing waves in closed resonators and opened resonators. Finally, we present our conclusions in Sec. 5.

2. Governing equations and boundary conditions

Let us consider nonlinear standing waves in shaped resonators with the excitation of the fluid imposed on the entire resonator. In this study, we restrict ourselves to the pseudo-one-dimensional case, where the resonators have variable cross sections but we ignore the change of variables within the planes perpendicular to the axis of resonators. Note that the cylindrical resonators can be regarded as a special kind of shaped resonators with the constant cross-sectional area. The finite amplitude standing waves generated in shaped resonators can be modeled by the quasi-one-dimensional unsteady compressible Navier-Stokes equations. The mass and momentum conversation equations are given by

\[
\frac{\partial (\rho A)}{\partial t} + \frac{\partial (\rho Au)}{\partial x} = 0, \tag{1}
\]

\[
\frac{\partial (\rho Au)}{\partial t} + \frac{\partial (\rho Au^2)}{\partial x} = -\rho Aa(t) + A \frac{\partial}{\partial x} \left( -p + \mu \left( \frac{1}{A} \frac{\partial (Au)}{\partial x} \right) \right), \tag{2}
\]

where \( \rho \) is the mass density, \( t \) is the time, \( x \) is the coordinate along the axial direction of the resonator, \( A \) is the area of the cross-section which depends on \( x \), \( u \) is the fluid velocity in the \( x \) direction, \( p \) is the pressure, \( \mu \) is the dynamic viscosity of the fluid and \( a(t) \) is the acceleration due to an external force acted on the entire resonator.
The state equation is taken for an ideal gas as

\[ p = \rho RK, \]  

(3)

where \( R \) is the gas constant and \( K \) is the gas temperature.

When finite amplitude standing waves are generated, the changes of gas density and temperature fields in the resonators in time cannot be ignored. The above three equations are insufficient to solve the four physical fields, i.e., the pressure, the velocity, the density and the temperature fields. The another state equation is considered for an ideal gas as

\[ p = p_0 \left( \frac{\rho}{\rho_0} \right)^\gamma, \]  

(4)

where \( p_0 \) and \( \rho_0 \) are the initial value of the pressure and density in the resonator and \( \gamma \) is the ratio of specific heats of the fluid.

The fluid is at complete rest at its initial state and is then excited by the harmonic motion at the entire resonator with the frequency \( f \). The acceleration \( a(t) \) is described as

\[ a(t) = a_0 \cos(2\pi ft), \]  

(5)

where \( a_0 \) is the amplitude of the acceleration \( a(t) \).

At the both end walls of the closed resonator, the fluid velocities equal to zero

\[ u(0,t) = 0 \]

\[ u(L_x,t) = 0, \]  

(6)

where \( L_x \) is the length of the resonator.

At the both end of the opened resonator, the gas will flow into and out of the resonator. The boundary conditions are described as
\[ u(0,t) = u_0 \]
\[ u(L_x,t) = u_{L_x} \] (7)

It should be noted that the above Navier-Stokes equations and boundary conditions are given in the coordinate frame that is fixed with respect to the resonator.

3. Pressure-based finite volume method

The first step for constructing the pressure-based FVM is to divide the spatial domain into a finite number of control volumes or computational cells. To avoid a pressure-checkerboard problem, one-dimensional shaped resonator is divided into control volumes for the velocity \( u \) (called \( u \)-control volumes) and regular control volumes for all other variables in a staggered grid manner as shown in Fig. 1. In this arrangement, the nodes (marked with \( \bullet \) ) are located at the centers of the regular control volumes or the faces of the \( u \)-control volumes. Pressure, mass density and temperature field are evaluated at these nodes. On the other hand, velocities are computed at the centers of \( u \)-control volumes or the faces of regular control volumes, indicated by the horizontal arrows (\( \rightarrow \)). The indices for the centers of regular control volumes are denoted by the upper-case letters \( \ldots, I - 1, I, I + 1, \ldots \) etc., while the indices for the faces of regular control volumes are denoted by the lower-case letters \( \ldots, i - 1, i, i + 1, \ldots \) etc.

The discretized equation for momentum is obtained by integrating the momentum conservation equation, Eq. (2), over each \( u \)-control volume and using the fully implicit backward Euler method in time.

\[
\left(\rho u\right)_i - \left(\rho u\right)_{i-1}^0 \frac{A \Delta x}{\Delta t} + \left[\left(\rho Au^2\right)_i - \left(\rho Au^2\right)_{i-1}\right] = -a(i)\left(\rho A\right)_{i} \Delta x + A_i \left(p_{i+1} - p_i\right) \\
+ \mu \left[\frac{\partial (Au)}{\partial x}_i - \frac{\partial (Au)}{\partial x}_{i-1}\right] \frac{1}{A} \frac{\partial (Au)}{\partial x} \Delta x, \quad (8)
\]

where \( \Delta t \) and \( \Delta x \) are the time and the spatial step sizes, respectively. The quantities with superscripts 0 are at the previous time step and all others have the values at the current time step.
The subscripts $I-1, i$ and $I$ denote the values at the left boundary, the center and the right boundary of the $i$-th the u-control volume, respectively.

In this study, the walls parallel to the $x$-direction are considered as no-slip, so the fluid velocity is equal zero at the walls. Based on the assumption that the velocity varies linearly with distance from the wall in the fluid, the wall shear stress on the fluid near the walls can be described as \(^{22}\)

\[
\tau_w = \mu \frac{u_i}{\Delta y_i},
\]

(9)

where $\Delta y_i$ is the distance of the near wall node $i$ to the wall in two-dimensional grid scheme.

Because the pseudo-one-dimensional model is used in our study, there are no grid nodes along the radical direction. We define $\Delta y_i$ as ten percent of the radius of the resonator at the current position $r_i$ to simulate twenty equidistant grid nodes along the radical direction. We find that the number of nodes is enough for the study.

Based on the wall shear stress, an effective wall shear force is added to the discretized momentum equation, Eq. \((8)\). It can be described as \(^{22}\)

\[
F_i = \tau_w A_{cell},
\]

(10)

where $A_{cell}$ is the wall area of the u-control volume.

We use linear interpolation to calculate the mass density at the centers of the u-control volumes and apply central differencing to compute the term $\frac{\partial (Au)}{\partial x}$ at the faces of the u-control volumes. The results are

\[
\rho_i = \frac{\rho_i + \rho_{i-1}}{2}, \quad \rho_i^0 = \frac{\rho_i^0 + \rho_{i-1}^0}{2},
\]

(11)
The mass flux and the diffusion conductance through the u-control volume face (say \( I \)) are given respectively by

\[
F_i = (\rho Au)_i \quad \text{and} \quad D_j = \frac{\mu A_j}{\Delta x}.
\]  

Substituting Eq. (11), (12) and (13) into Eq. (8), and then employing a power-law scheme described by Patankar\(^{23}\) for calculating velocities at the faces of the u-control volume, we can obtain, after slight rearrangement, the final discretized equation for velocities at the centers of u-control volumes

\[
a_{p_j} u_j = a_{w(i-1)} u_{i-1} + a_{E(i+1)} u_{i+1} + (p_{j-1} - p_i) A_j + b_j,
\]  

where

\[
a_{E(i+1)} = D_j \max \left( 0, (1 - 0.1 |P_j|)^{5} \right) + \max (-F_i, 0),
\]

\[
a_{w(i-1)} = D_{i-1} \max \left( 0, (1 - 0.1 |P_{i-1}|)^{5} \right) + \max (F_{i-1}, 0),
\]

\[
a_{p_j} = a_{E(i+1)} + a_{w(i-1)} + F_i - F_{i-1} + \frac{(\rho_j + \rho_{j-1}) A_i A_{\Delta x}}{2 \Delta t} + \mu \frac{A_{\text{cell}}}{A_j} \Delta y_i,
\]

\[
b_j = \frac{\rho_i^0 \rho_{i-1}^0}{2 \Delta t} u_j^0 - \frac{\rho_j + \rho_{j-1}}{2} a(t) A_i A_{\Delta x} - \frac{\mu}{A_j} \frac{(uA)_j - (uA)_{i-1}}{\Delta x} \left( \frac{\partial A}{\partial x} \right)_i \Delta x.
\]

\( P_j \) is the Peclet numbers at the face of the u-control volume, defined by \( \frac{F_j}{D_j} \).

Equation (14) can be solved only when the pressure field is given or estimated. Unless the correct pressure field is used, the resulting velocity field will not satisfy mass conservation.
equation (1). The approximate velocity field based on a guessed pressure field $p^*$ will be denoted by $u^*$. The discretized equation (14) is rewritten as

$$a_{pi}u_i^* = a_{pi}u_i^* + a_{E(i+1)}u_{E(i+1)}^* + \left(p_{i-1}^* - p_i^*\right)A_i + b_i.$$  \hspace{1cm} (15)

Assume that the correct pressure $p$ can be obtained by adding the pressure correction term $p'$

$$p = p^* + p'.$$ \hspace{1cm} (16)

The corresponding velocity correction term $u'$ can be introduced in a similar manner

$$u = u^* + u'.$$ \hspace{1cm} (17)

Subtracting Eq. (15) from Eq. (14), we have

$$a_{pi}u_i' = a_{pi}u_i^* + a_{E(i+1)}u_{E(i+1)}^* + \left(p_{i-1}' - p_i'\right)A_i.$$ \hspace{1cm} (18)

Based on SIMPLEC scheme, velocity correction term $u_i$ can be described as

$$u_i' = d_i \left(p_{i-1}' - p_i'\right),$$ \hspace{1cm} (19)

where

$$d_i = \frac{A_i}{a_{pi} - a_{pi(i-1)} - a_{E(i+1)}}.$$ \hspace{1cm} (20)

Substituting Eq. (19) into Eq. (17), we can obtain the velocity $u_i$

$$u_i = u_i^* + d_i \left(p_{i-1}' - p_i'\right).$$ \hspace{1cm} (21)

The discretized mass conservation equation around the center of a regular control volume $I$, shown in Fig. 1, can be written as

$$\left(\rho_i - \rho_i^0\right)\frac{A_i\Delta x}{\Delta t} + \left[(\rho Au)_{i+1} - (\rho Au)_i\right] = 0.$$ \hspace{1cm} (22)
For a compressible thermoviscous fluid, the mass density correction can be written based on the state equation, Eq. (3) (a similar idea is used in Karki and Patankar\textsuperscript{21})

\[ \rho' = \frac{1}{RK} \rho' . \]  

(23)

Substituting Eq. (21) and Eq. (23) into Eq. (22) and employing the upwind scheme for the mass density and mass density correction, we can obtain the following discretized equation for \( \rho' \)

\[ a_{pj} \rho'_j = a_{E(i+1)} \rho'_{j+1} + a_{w(i-1)} \rho'_{j-1} + b_j , \]  

(24)

where

\[ a_{E(i+1)} = \left( \rho^* A_d \right)_{i+1} - \frac{A_{i+1}}{RK_{j+1}} \max \left( -u^*_{i+1}, 0 \right) , \]

(26)

\[ a_{w(i-1)} = \left( \rho^* A_d \right)_{i-1} + \frac{A_i}{RK_{i-1}} \max \left( u^*_{i}, 0 \right) , \]

(27)

\[ a_{pj} = \left( \rho^* A_d \right)_{i+1} + \frac{A_{i+1}}{RK_{j}} \max \left( u^*_{i+1}, 0 \right) + \left( \rho^* A_d \right)_{i} - \frac{A_i}{RK_{i}} \max \left( -u^*_{i}, 0 \right) + \frac{A_i \Delta x}{RK_{i} \Delta t} , \]

(28)

\[ b_j = \left( \rho^* u^* A \right)_{i+1} - \left( \rho^* u^* A \right)_{i} - \frac{\left( \rho^* - \rho^0 \right) A_i \Delta x}{\Delta t} . \]

(29)

Note that \( b_j \) is computed from the guessed mass density \( \rho^* \) and velocity \( u^* \) and is also the residual of the discretized mass conservation equation (21). Therefore, we determine whether the guessed mass density satisfies the mass conservation equation by checking the criteria

\[ \| b \| \leq \epsilon , \]  

(25)

where \( \epsilon \) is a specified tolerance and \( b \) is the vector composed by \( b_j \) at all centers of the regular control volumes.
Now we can summarize the pressure-based FVM according to the above descriptions. The
algorithm can be used to solve a compressible thermoviscous fluid in a shaped acoustic resonator.

The sequence of steps is as follows

1. Give the initial pressure \( p_0 \) and temperature \( K_0 \) in the acoustic resonator. The initial
   mass density \( \rho_0 \) is calculated using the state equation, Eq. (3).

2. Let \( p^0 = p_0, u^0 = 0, \rho^0 = \rho_0 \) and \( K^0 = K_0 \) for the first time step.

3. Let \( p^* = p^0, u^* = u^0, \rho^* = \rho^0 \) and \( K^* = K_0 \), and start time loop.

4. Let \( t = t + \Delta t \).

5. Start internal loop for a new time step.

6. Solve the momentum equation, Eq. (15), to obtain the guessed velocity \( u^* \).

7. Solve the pressure correction equation, Eq.(24), to obtain the pressure correction \( p' \).

8. Update the pressure \( p \) from Eq. (16).

9. Calculate the velocity correction \( u' \) from Eq. (19).

10. Update the velocity \( u \) from Eq. (17).

11. Calculate the mass density \( \rho \) from Eq. (4) with the pressure obtained in the step (8).

12. Calculate the temperature \( K \) from Eq. (3) with the mass density and the pressure
    obtained in the step (11) and step (8), respectively.

13. If the iteration criteria Eq. (25) is not satisfied, let \( p^* = p, u^* = u, \rho^* = \rho \) and repeat
    steps (6) through (12); otherwise, go to step (14).

14. Let \( p^0 = p, u^0 = u, \rho^0 = \rho, K^0 = K \) and prepare a new time step.

15. Steps (3) through (14) are repeated until the final time is reached.

4. Numerical results and discussion
The finite volume method described in Sec. 3 is applied to predict the standing waves in both closed resonators and opened resonators. The studied shapes of resonators include cylinder, cone and exponential horn. The geometries of these resonators are shown in Fig. 2. In order to compare with the results obtained by previous methods, the resonators are filled with R-12 refrigerant and air. The cylindrical resonator and the conical resonator are filled with \(306\,\text{kPa}\) of \(27^\circ\text{C}\) R-12 refrigerant (mol. wt=120.09, \(\gamma=1.129\)). The sound speed \(c_0\) of this fluid is \(152.6\,\text{m/s}\). The exponential resonator is filled with \(100\,\text{kPa}\) of \(0^\circ\text{C}\) air (mol. wt=29, \(\gamma=1.4\)). The sound speed \(c_0\) of the air is \(330\,\text{m/s}\).

### 4.1 Closed resonators

Figure 3 shows the pressure waveforms at the left end of the closed cylindrical resonator (precisely, the center of most left computational cell). The cylindrical resonator with circular cross-section has a length of \(0.2\,\text{m}\) and a constant cross-sectional area of \(0.0024\,\text{m}^2\). The resonator is excited at its fundamental natural acoustic frequency calculated according to the relation of \(f_0 = c_0 / 2L_s\) in the cylindrical resonator, resulting in \(f_0 = 381.5\,\text{Hz}\). To compare with the solution from the Galerkin method, the two normalized forcing amplitudes \((F_0)\) of \(8.939\times10^{-5}\) and \(5\times10^{-4}\) employed by Erickson and Zinn\(^7\) are taken to be the amplitude of the acceleration with the relation of \(a_0 = F_0L_s \left(2\pi f_0\right)^2\), resulting in \(100\,\text{m/s}^2\) and \(574.58\,\text{m/s}^2\), respectively. The results are in very good agreement with the Galerkin method solution.

Besides the pressure waveforms, the velocity waveforms are computed directly in our study. Figure 4a shows the velocity waveforms located at three positions in the cylindrical resonator \((a_0 = 574.58\,\text{m/s}^2)\): near the left end \((x = \Delta x)\), the center position \((x = L_s / 2)\) and near the right end \((x = L_s - \Delta x)\) of the resonator. We can clearly observe that the sharp velocity spikes appear...
near the two ends. Comparing the pressure waveforms in Fig. 4b with the velocity waveforms in Fig. 4a, we find that the sharp velocity spikes appear at the ends of the resonator at the times when the pressure has the shock-like waveform or abrupt change. The velocity near the left end has 180-degree phase shift and opposite sign compared with that near right end, while the pressure waveforms near the two ends are the same taking into account of the 180-degree phase shift.

The FVM is also used to simulate the standing wave in a closed conical resonator defined by Chun and Kim\textsuperscript{10}

\begin{equation}
  r(x) = 0.005 + 0.04 \left( \frac{x}{L_s} \right) \quad \text{for } 0 \leq x \leq L_s, \tag{26}
\end{equation}

where $r(x)$ is the radius of the cross section in meters and $L_s$ is 0.2m.

Figure 5 shows the pressure waveform at the small end of the conical resonator, which is in very good agreement with that from the finite difference method solution ($a_0 = 100\text{m/s}^2$, $f = 491\text{Hz}$) of Chun and Kim\textsuperscript{14}. We can observe that the shape of the pressure waveform no longer resembles the shock-like shape found in straight cylindrical resonator, and the amplitude of the pressure reaches a maximum of over $4 \times 10^5 \text{Pa}$, which is significantly larger than the amplitude of the pressure of $3.18 \times 10^5 \text{Pa}$ in the cylindrical resonator at the same amplitude of the acceleration of $a_0 = 100\text{m/s}^2$. At higher amplitudes of acceleration of $a_0 = 200\text{m/s}^2$ and $a_0 = 300\text{m/s}^2$, the maximum amplitudes of the pressure from our results(not shown in here) are smaller than those in Chun and Kim\textsuperscript{14}. The difference is due to the hysteresis effect taking place in the conical resonator at high amplitudes of acceleration\textsuperscript{5}. While our numerical results are obtained from the gas at rest initially, Chun and Kim\textsuperscript{14} swept frequency up for their results.
Figure 6a shows the evolution of the velocity relative to the oscillating resonator near the two ends and at the center position, while Fig. 6b shows the corresponding pressure waveforms at the same locations. Unlike the waveform of the velocity for a straight cylindrical resonator, there are no sharp velocity spikes near the ends of the conical resonator. All the velocity and pressure waveforms for the conical resonator have smooth sinusoidal shapes and those near the two ends have 180-degree phase difference.

Figure 7 shows the pressure waveform at the small end in a closed exponential resonator characterized by

\[
A(x) = A_0 \exp \left( \alpha \frac{x}{L_s} \right), \quad \text{for } 0 \leq x \leq L_s,
\]

where the area of the small end \( A_0 \) is set to \( 6.31 \times 10^{-5} \text{ m}^2 \), the length of the resonator \( L_s \) is 0.224m and the flare constant \( \alpha \) is 5.75.

According to the fundamental natural frequency of a cylindrical resonator of \( \omega_0 = \frac{\pi c_0}{L_s} \) and the relation \( a_0 = F_0 L_s \omega_0^2 \), the amplitude of the acceleration \( (a_0) \) is chosen to be 2.40 \( \times 10^3 \text{ m/s}^2 \) for the normalized forcing amplitude \( (F_0) \) of 5 \( \times 10^{-4} \). As shown in Fig. 7, the obtained pressure from the FVM is in good agreement with that from the Galerkin method solution. Due to the differences between the FVM and the Galerkin method and the different resolutions used in the simulations, the pressure waveform obtained by the FVM have more detailed feature than that obtained by a truncated limited acoustic modes used in the Galerkin method. The pressure waveform is obtained at the frequency of 1005Hz which is higher than the fundamental natural acoustic mode frequency of the exponential resonator of 1000Hz. It shows hardening physical character in the exponential resonator, defined as an increase in the forcing
frequency of maximum response with respect to the fundamental natural acoustic mode frequency\(^7\).

For exponential resonator, the velocity and pressure waveforms near the two ends and the center of the resonator are shown in Fig. 8a and Fig. 8b, respectively. The shapes are no longer sinusoidal and the waveforms near the two ends are dramatically different. At the small end, the pressure has sharp peak at the times when the velocity has sudden changes; at the large end, the relative velocity is almost flat. It is interesting to note that the waveform of the velocity at the center is similar to that of the pressure near the small end.

4.2 Opened resonators

In order to analyze the pressure waveforms in opened resonators, we set the boundary conditions of the above cylindrical and exponential resonators such that the gas can flow into and out of the resonators.

The cylindrical resonator with acceleration \(a_0 = 574.58 \text{m/s}^2\) is opened at both ends, and the gas flows into and out of the cylindrical resonator are specified with the speeds of 1 m/s, 5 m/s and 10 m/s relative to the resonator, respectively. Figure 9 shows that, as the imposed gas speed increases, the ratio of the maximum and minimum pressures decreases and the pressure waveform displays the weaker shock-like shape. Figure 10 shows that the sharp velocity spikes progressively disappear with the decrease of the pressure ratio. The results further support the previous conclusion in Sec. 4.1 that the sharp velocity spikes appear at the ends of the resonator at the times when the pressure has the shock-like waveform.

Due to the different cross-sectional areas at the two ends of the exponential resonator, we specify the relative gas speed at the small end, and then specify the gas speed at the large end according to the mass conservation in the resonator. Figure 11 shows the pressure waveforms of
the opened exponential resonator when the gas flow velocity at the small end relative to the
oscillating resonator is at 10 m/s, 25 m/s and 50 m/s. Comparing with the pressure wave of the
closed exponential resonator, the ratio of the maximum to minimum pressure at the small end of
the exponential resonator decrease only slightly for the relative flow speed at the small end up to
25 m/s. When the relative flow speed is 50 m/s, Fig.11 shows that the pressure waveform
decrease significantly compared with that of the closed resonator and the phase shift is much
stronger. Figure 12 shows the evolution of the pressure waveform in time from initial rest
condition in the closed exponential resonator, the opened cylindrical resonator at flow speed 10
m/s and the opened exponential resonator at flow speed 50 m/s, respectively. Comparing the
three transient processes, we can observe that the high-speed flow in the opened exponential
resonator will result in irregular oscillations during the initial time period, causing the phase
shifts shown in Fig. 11. The stability of the high pressure ratio in the exponential resonator
relative to opening the resonator could be useful for acoustic seal application.

5. Conclusion

To obtain and analyze the standing waves in closed and opened acoustic resonators, we
extended the FVM SIMPLEC scheme to directly solve the unsteady compressible Navier-Stokes
equations without any predefined standing waves. The characteristics of the pressure waveforms
in closed resonators was presented and compared with the results obtained with pervious
numerical methods in particular finite element and finite difference method. Our study shows
that the sharp velocity spikes accompanied by the shock-like pressure waveforms appear at the
end of the resonator. The pressure waveforms in opened cylindrical and exponential resonators
are simulated and analyzed. The results show that the maximum to minimum pressure ratio in the
opened cylindrical resonators decreases quickly when the relative flow speed at the opened ends
increases. In contrast, the pressure ratio at the small end of the opened exponential resonator stays at high values when the relative flow speed at the opened end is not very large. The stability of the high pressure ratio in the opened exponential resonator could be used to design acoustic seal. Due to the complex boundary conditions of acoustic resonators in acoustic seal, the two-dimensional FVM will be developed in the future.

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Figure Captions

Figure 1 The regular control volume (a) and u-control volume (b) in a staggered grid manner.

Figure 2 The geometries of these resonators: (a) cylindrical resonator, (b) conical resonator, and (c) exponential resonator.

Figure 3 Comparison of the shock-like pressure waveforms obtained from the FVM at the left end of the closed cylindrical resonator with those from the Galerkin method (Ref. 7). The comparison results are obtained with the amplitude of the acceleration of 574.58 m/s² and 100 m/s², respectively.

Figure 4 The velocity and pressure waveforms at the three positions in the closed cylindrical resonator (a₀ = 574.58 m/s², f = 381.5 Hz). (a) The velocity waveforms and (b) the pressure waveforms.

Figure 5 Comparison of the pressure waveform obtained from the FVM with that from Chun & Kim (Ref. 9) at the small end (x = Δx/2) of the closed conical resonator (a₀ = 100 m/s², f = 491 Hz).

Figure 6 The velocity and pressure waveforms near the left end (x = Δx), at the center position (x = Lx/2) and near the right end (x = Lx − Δx) in the closed conical resonator (a₀ = 100 m/s², f = 491 Hz). (a) The velocity waveforms and (b) the pressure waveforms.

Figure 7 Comparison of the pressure waveform obtained from the FVM at the small end with that from the Galerkin method (Ref. 7) of the closed exponential resonator (a₀ = 2.40 × 10^5 m/s², f = 1005 Hz).

Figure 8 The velocity and pressure waveforms near the left end (x = Δx), at the center position (x = Lx/2) and near the right end (x = Lx − Δx) in the closed exponential resonator.
\( a_0 = 2.40 \times 10^3 \text{ m/s}^2, f = 1005\text{Hz} \). (a) The velocity waveforms and (b) the pressure waveforms.

Figure 9 Comparison of the pressure waveforms at the left end of the opened cylindrical resonator with different specified gas speeds relative to the oscillating resonator at the opened ends \( a_0 = 574.58\text{m/s}^2, f = 381.5\text{Hz} \).

Figure 10 Comparison of the velocity waveforms near the left end of the opened cylindrical resonator at different gas speeds relative to the oscillating resonator with that of the closed cylindrical resonator \( a_0 = 574.58\text{m/s}^2, f = 381.5\text{Hz} \).

Figure 11 Comparison of the pressure waveforms at the small end of the opened exponential resonator at different gas speeds relative to the oscillating resonator with that of the closed resonator \( a_0 = 2.40 \times 10^3 \text{ m/s}^2, f = 1005\text{Hz} \).

Figure 12 The transient processes of the pressure waveforms measured at the small-end of the resonators starting from initial rest condition: (a) in the closed exponential resonator, (b) in the opened cylindrical resonators at flow speed 10 m/s, and (c) in the opened exponential resonator at flow speed 50 m/s.