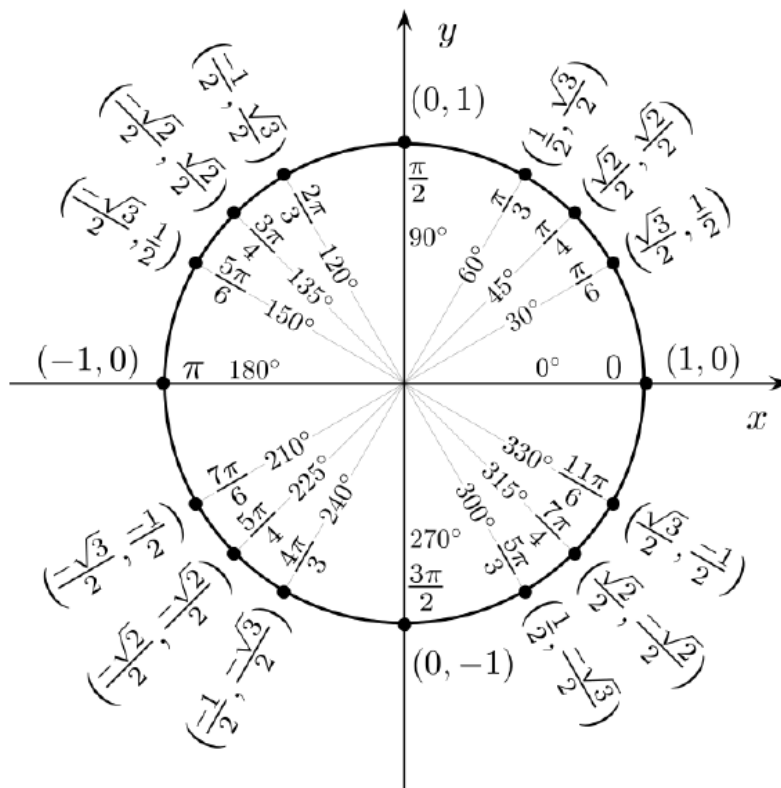


TRIGONOMETRY REVIEW

The Cosines and Sines of the Standard Angles



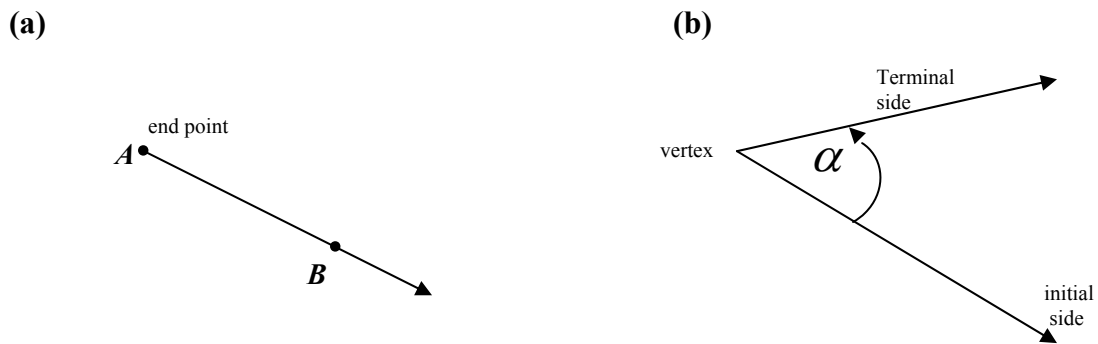
$$P_\theta = (\cos \theta, \sin \theta)$$

1. ANGLES AND THEIR MEASURE

In order to define the trigonometric functions so that they can be used not only for triangular measurement but also for modeling periodic phenomena, we must give a definition of an angle that is somewhat more general than a vertex angle of a triangle.

If A and B are distinct points, the portion of the straight line that starts at A and continues indefinitely through B is called a **ray** with **endpoint** A (Figure 1.1a). An **angle** is determined by rotating a ray about its endpoint. This rotation can be indicated by a curved arrow as in Figure 1b. The endpoint of the rotated ray is called the **vertex** of the

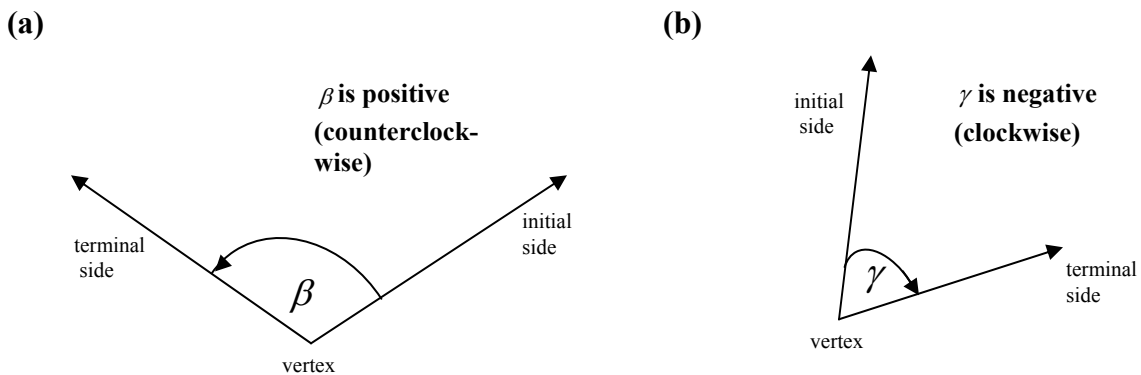
Figure 1.1



angle. The position of the ray before the rotation is called the **initial side** of the angle, and the position of the ray after the rotation is called the **terminal side**. Angles will often be denoted by small Greek letters, such as angle α in Figure 1b (α is the Greek letter *alpha*).

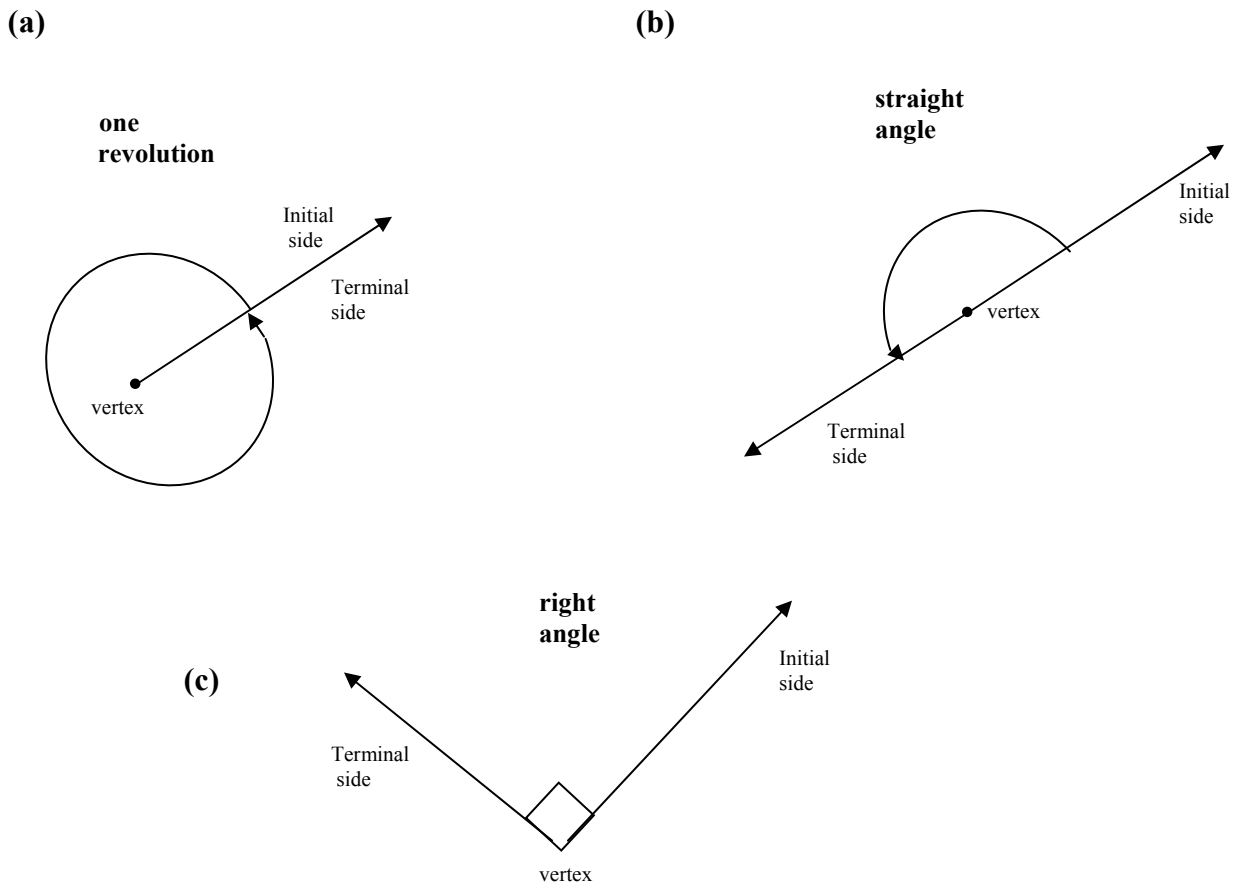
Angles determined by a clockwise rotation are said to be **positive** and angles determined by a counterclockwise rotation are said to be **negative**. In Figure 1.2a, angle β is positive (β is the Greek letter *beta*); in Figure 1.2b, angle γ is negative (γ is the Greek letter *gamma*).

Figure 1.2



One full turn of a ray about its endpoint is called **one revolution** (Figure 1.3a); one-half of a revolution is called a **straight angle** (Figure 1.3b); and one-quarter of a revolution is called a **right angle** (Figure 1.3c). The initial and terminal sides of a right angle are perpendicular; hence, in drawing a right angle, we often replace the usual curved arrow by two perpendicular line segments (Figure 1.3c).

Figure 1.3



*Angles that have the same initial sides and the same terminal sides are called **coterminal angles**. For instance, Figure 1.4 shows three coterminal angles α , β and γ .*

Because different angles can be coterminal, an angle is not completely determined merely by specifying its initial and terminal sides. Nevertheless, angles are often named by using three letters such as ABC to denote a point A on the initial side, the vertex B , and a point C on the terminal side (Figure 1.5). Usually, the subtended angle is the smallest rotation about the vertex that carries the initial side around to the terminal side. Sometimes, when it is perfectly clear which angle is intended, we refer to the angle by simply naming its vertex. For instance, the angle in Figure 1.5 could be called angle B .

Figure 1.4

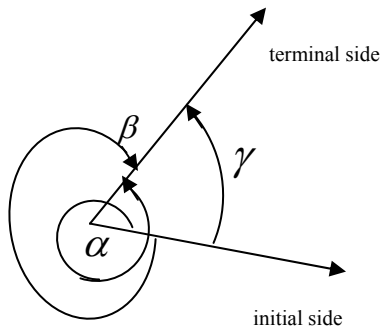
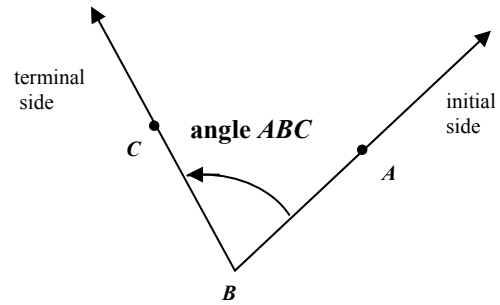


Figure 1.5



There are various ways to assign numerical measures to angles, Of these, the most familiar is the degree measure: **One degree (1°)** is the measure of an angle formed by $\frac{1}{360}$ of a counterclockwise revolution. Negative angles are measured by negative numbers of degrees: for instance, -360° is the measure of one clockwise revolution. A positive right angle is $\frac{1}{4}$ of a clockwise revolution, and it therefore has a measure of $\frac{1}{4}(-360^\circ) = 90^\circ$. Similarly, 180° represents $\frac{180^\circ}{360^\circ}$ or $\frac{1}{2}$ of a clockwise revolution.

What is the degree measure of one-eighth of a clockwise revolution?

$$\frac{1}{8}(-360^\circ) = -45^\circ.$$

Example 1.1

What fraction of a revolution is the angle of 30° ?

$$\frac{30^\circ}{360^\circ} = \frac{1}{12} \text{ of a counter clockwise revolution.}$$

Although fractions of a degree can be expressed as decimals, such fractions are sometimes given in “minutes” and “seconds”. One minute (1') is defined to be $\frac{1}{60}$ of a degree and one second (1'') is defined to be $\frac{1}{60}$ of a minute. Hence, one second is $\frac{1}{3600}$ of a degree. Using the relationships

| |
|---|
| $1' = \left(\frac{1}{60}\right)^\circ, \quad 1'' = \left(\frac{1}{3600}\right)^\circ, \quad 1^\circ = 60', \quad 1' = 60''$ |
|---|

you can convert from degrees, minutes, and seconds to decimals and vice versa.

Although the degree measure of angles is used in most elementary applications of trigonometry, more advanced applications (especially those that involve calculus) require radian measure. **One radian** is the measure of an angle that has its vertex at the center of a circle (that is, a central angle) and intercepts an arc on the circle equal in length to the radius r (Figure 1.6).

A central angle of 2 radians in a circle of radius r intercepts an arc of length $2r$ on the circle, a central angle of $\frac{3}{4}$ radian intercepts an arc of length $\frac{3}{4}r$, and so forth. More generally, if a central angle $\angle AOB$ of θ radians (θ is the Greek letter theta) intercepts an arc AB of length s on a circle of radius r (Figure 1.7), then we have

$$s = r\theta.$$

It follows that $\angle AOB$ has radian measure θ given by the formula

$$\theta = \frac{s}{r}.$$

Figure 1.6

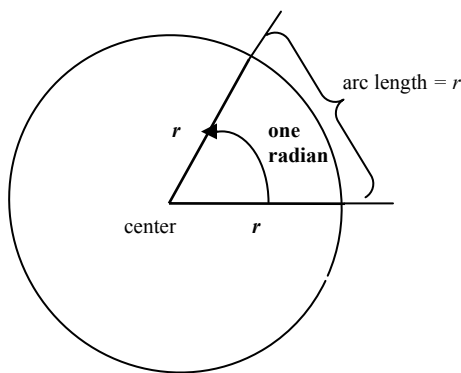
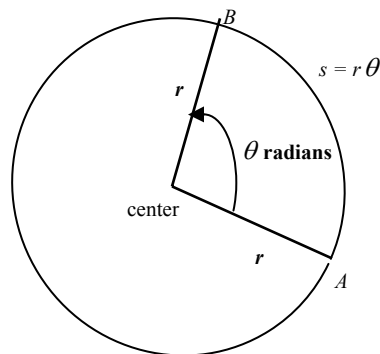


Figure 1.7



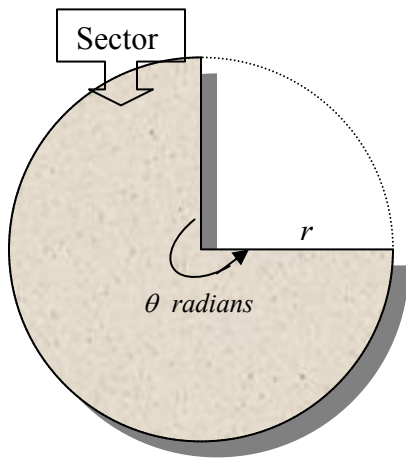
Find the length s of the arc intercepted on a circle of radius $r = 3$ meters by a central angle whose measure is $\theta = 4.75$ radians.

$$s = r\theta = 3 (4.75) = 14.25 \text{ meters.}$$

A central angle in a circle of radius 27 inches intercepts an arc of length 9 inches. Find the measure θ of the angle in radians. Here $s = 9$ inches, $r = 27$ inches, and $\theta =$

$$\frac{s}{r} = \frac{9}{27} = \frac{1}{3} \text{ radian.}$$

Another related formula is that for area of a sector cut off from a circle of radius r by a central angle of radian measure θ .



Note that the area A of this sector of the whole circle as θ is to 2π . That is,

$$\frac{A}{\pi r^2} = \frac{\theta}{2\pi}$$

Thus,

$$A = \frac{1}{2} r^2 \theta$$

A central angle of 360° corresponds to one revolution; hence it intercepts an arc $s = 2\pi r$ equal to the entire circumference of the circle (Figure 8). Therefore, if θ is the radian measure of the 360° angle,

$$\theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi \text{ radians;}$$

that is, $360^\circ = 2\pi$ radians, or

$$180^\circ = \pi \text{ radians.}$$

You can use this relationship to convert degrees into radians and vice versa. In particular,

$$1^\circ = \frac{\pi}{180} \text{ radian} \quad \text{and} \quad 1 \text{ radian} = \left(\frac{180}{\pi}\right)^\circ.$$

Thus, we have the following *conversion rules*:

- (i) Multiply degrees by $\frac{\pi}{180}$ to convert to radians.
- (ii) Multiply radians by $\frac{180}{\pi}$ to convert to degrees.

When no unit of angular measure is indicated, it is always understood that radian measure is intended.

Table 1.1

| Degrees | Radians |
|-------------|-----------------|
| 0° | 0 |
| 30° | $\frac{\pi}{6}$ |
| 45° | $\frac{\pi}{4}$ |
| 60° | $\frac{\pi}{3}$ |
| 90° | $\frac{\pi}{2}$ |
| 180° | π |
| 360° | 2π |

Application: Angular Velocity

A formula closely related to the arc length formula $s = r\theta$ is the formula

$$v = r \omega$$

which connects the speed (velocity) of a point on the rim of a wheel of radius r with the angular velocity ω at which the wheel is turning. Here, ω is measured in radians per unit time.

Example 1.2

Determine the angular velocity in radians per second of a bicycle wheel of radius 16 inches if the bicycle is being ridden down a road at 30 miles per hour.

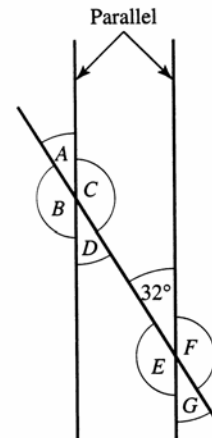
To solve, we must use consistent units

$$\frac{30mi}{hr} = 30 \frac{mi}{hr} \times \frac{5280ft}{1 mi} \times \frac{1 hr}{3600 s} = 44 \frac{ft}{s}$$

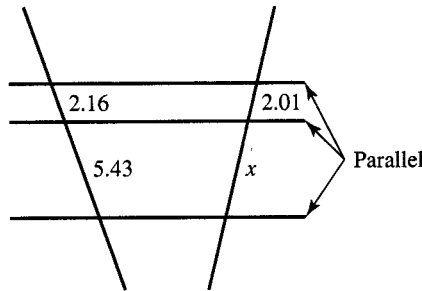
and the radius of the wheel is $\frac{4}{3} ft$. Thus, $44 = \frac{4}{3} \omega$ or $\omega = \frac{3}{4} (44) = 33$ radians per second.

Section 1 Problems

1. Find the angles A , B , C , D , E , and F in the figure shown at the far right.

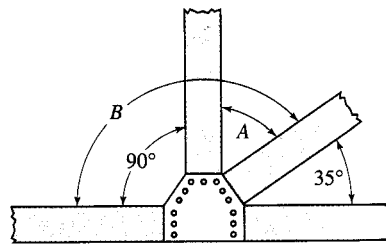


2. Find the distance, x .

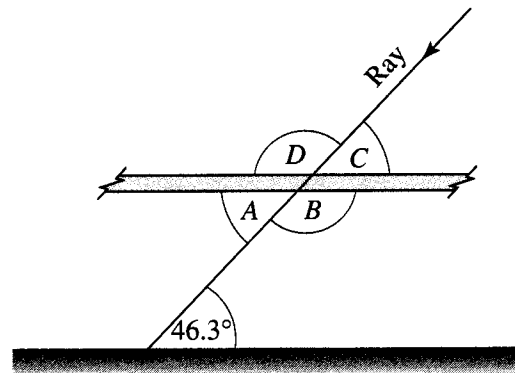


3. Find the angles A and B .

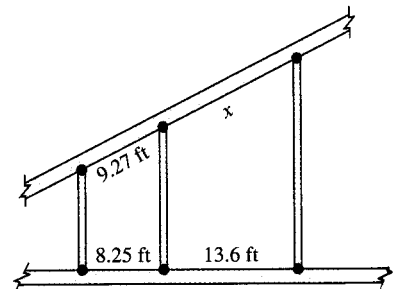
Assume that the beam is of uniform width.



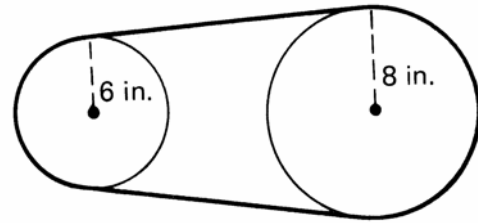
4. On a certain day, the angle of elevation of the sun is 46.3° . Find the angles A , B , C , and D that a ray of sunlight makes with a horizontal sheet of glass. Assume the glass to be so thin as to not bend the ray.



5. Three parallel steel cables hang from a girder to the deck of a bridge. Find the distance x .

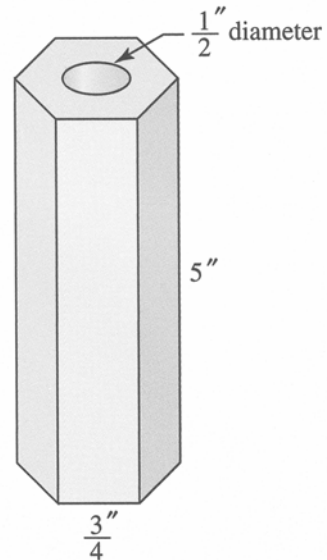


6. A dead fly is stuck to a belt that passes over two pulleys 6 inches and 8 inches in radius, as shown in the figure. Assuming no slippage, how fast is the fly moving when the larger pulley turns at 20 revolutions per minute?



7. How fast (in revolutions per minute) is the smaller wheel in problem 49 turning?
8. Assume that the earth is a sphere of radius 3960 miles. How fast (in miles per hour) is a point on the equator moving as a result of the earth's rotation about its axis?
9. A nautical mile is the length of $1'$ (recall $1 \text{ minute} = \frac{1}{60}$ of a degree) of arc on the equator of the earth. How many miles are there in a nautical mile?

10. Find the volume of the bored hexagonal cylinder shown in the figure.



Hint: Recall that the volume of any cylinder is determined by the formula:
Volume = (Area of Base) x (Height).

Note also that if it were not for the circular hole in the hexagonal base then the base could be partitioned into six congruent triangles.

