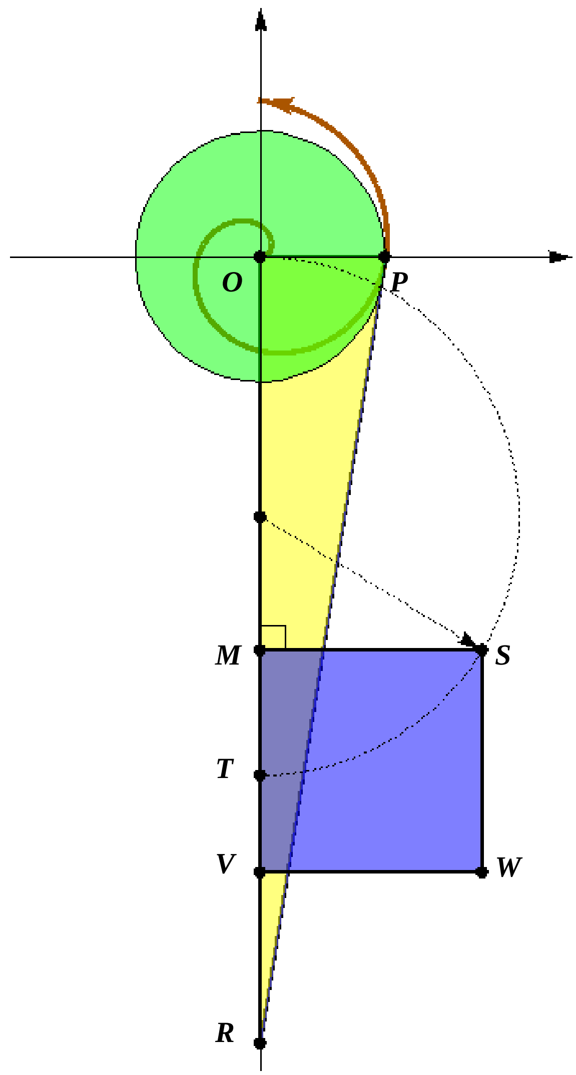
SQUARING THE CIRCLE WITH AN ARCHIMEDEAN SPIRAL

Definition: *Archimedean Spiral*
If a straight line, one extremity of which remains fixed, is made to revolve at a uniform rate in a plane until it returns to the position from which it started, and if, at the same time as the straight line is revolving, a point, P, moves at a uniform rate along the straight line, starting from the fixed extremity, then the path traced by the point P will describe a **spiral** in the plane.



Coordinatizing the Spiral
In the Cartesian plane, let the fixed extremity on the rotating line be at $O = (0 , 0)$. Then at the time that the line has rotated from the positive horizontal axis through the angle θ (radians) the point P tracing the Archimedean spiral will have the rectangular coordinates:

$$P = (k\theta \cos \theta , k\theta \sin \theta)$$

for some positive constant k for all $\theta > 0$.

Archimedes' Theorem:
Let P denote the point on the spiral when it has completed one turn. Let the tangent line at P cut the line perpendicular to OP at R. Then the area of the of the circle with radius OP is equal to that of ΔPOR.

Thus, the area of the circle with radius OP is the same as that of the square $MSWV$ where

$$| OM | = \frac{| OR |}{2}$$

and $$| MT | = | OP |.$$