Exam 3: Topics: Solving Systems of DEs

Section 4.8: Method of Elimination

1. May be applied to systems of linear DEs having constant coefficients (of order 1 or higher).
2. Write the system using differential operator notation.
3. Eliminate one of the dependent variables.
4. Find the general sol. for the remaining dep. variable.
   
   \[ x(t) = x_1 t + x_0 \]

   (Assume \( x = e^{\lambda t} \), use annihilators)

5. Use your knowledge of \( x(t) \) and the equations in the system to determine \( y(t) \).

Section 7.7: Laplace Transform Method

1. May be applied to systems of linear DEs having constant coefficients (of order 1 or higher).
2. Transform the given system of DEs to a linear system of \( \mathcal{L} \).
   
   \[ a_{11}(s) X(s) + a_{12}(s) Y(s) = F_1(s) \]
   \[ a_{21}(s) X(s) + a_{22}(s) Y(s) = F_2(s) \]

3. Eliminate one of the dependent variables and use techniques including partial fractions to find the inverse transform of the remaining dependent variable.

4. Use your knowledge of \( x(t) \) and the original equations in the system to help you determine \( y(t) \). If necessary, you may eliminate \( x(s) \) from the transformed system and solve it for \( Y(s) \).
Appendix II: Sections 8.1–8.3: Matrices / Determinants / and
solutions to first order linear systems with constant coefficients

Eigenvalue method for solving $\dot{x} = Ax$

1. All solutions have the form $x = e^{At}k$, where $\lambda, k$ are eigenpairs
   for the constant coefficient matrix $A$.

2. In case there are less than $n$ linearly independent eigenvectors,
   then we construct the generalized eigenvector $v$:
   $(A - \lambda I)^2 v = 0$ but $(A - \lambda I) v \neq 0$. Then
   $e^{At} [I + t(A - \lambda I)] v$ is an additional solution to $\dot{x} = Ax$.

Variation of Parameters: applied to finding a particular solution $x_p(t)$:

$x_p(t) = v(t) \int e^{At} f(t) dt$ which is a special solution to the
system $\dot{x}$ in case $\dot{v}(t) = (x_1 \dot{x}_2 \ldots x_n)$

is a fundamental matrix of the system. (i.e. $k, x_1, x_2 \ldots x_n$ are linearly independent solutions)

Then

$x = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n + x_p$

is the general solution to $\dot{x} = Ax + f(t)$. 