

Homework Assignments – Math 119-001/003 – Fall 2009

Assignment 1

Due date : Friday, August 28

Section 5.1, Page 155: #1,2,3,5,6.
 Section 5.2, Page 160: #1,2,3,5,6,8,10-15,17,21,23.

Assignment 2

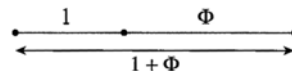
Due date : Friday, September 11

Section 5.3, Page 163: # 1(a)(c), 2(a)(c)(f), 4,7.
 Section 5.4, Page 167: #3,7,8,12,13,15,16,17,19,24.

Additional Exercises

- Carefully prove that the perpendicular bisector of any chord to a circle must necessarily pass thru the center of the circle.
 Support each step in your solution. You should include a figure and invoke an appropriate Euclidean criterion for congruent triangles.
- Prove that the diagonals of a rhombus intersect at right angles and bisect the interior angles of the rhombus. In your argument, you may use the proposition proved in class: *The diagonals of a parallelogram always bisect each other.*
- The **golden** (or extreme) **ratio**, Φ , was defined in *The Elements* by Euclid to be that ratio which divides a whole so that: *the whole is to the greater as the greater is to the less.*

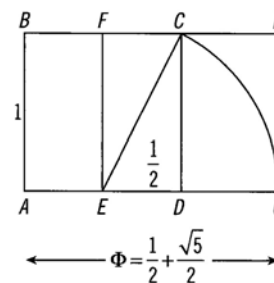
Thus, $1 + \Phi : \Phi = \Phi : 1$ or $\frac{1+\Phi}{\Phi} = \frac{\Phi}{1}$.



(a) Either complete the square or use the quadratic formula (see page 325 of the textbook) to verify that $\Phi = \frac{1+\sqrt{5}}{2}$.

(b) Verify that the construction described below yields the golden ratio: $|AG| = \Phi$.

Start with the square $ABCD$, 1 unit on a side, and bisect it vertically with line EF . Then swing an arc of radius EC to the extension of line AED . Now the length of AG will be Φ .



Assignment 3

Due date : Wednesday, September 23

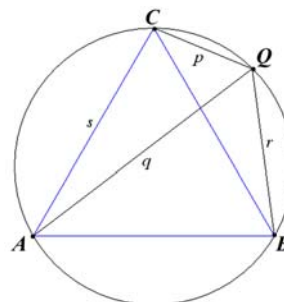
Section 5.5, Page 173: #1,3,5,7,9,11,14,16,19,20,25,28.
 Section 6.1, Page 181: #1(a)(c),2(a)(b),3(a)(h)(p),5,8,9,10,14.
 Section 6.2, Page 185: #1,4,7,11,15,20.
 Section 6.3, Page 187: #2,5,10,16,17,20,21,22,25,26,33.

Additional Exercises

Use **Ptolemy's Theorem**: *If a quadrilateral is inscribed in a circle then the sum of the products of its two pairs of opposite sides is equal to the product of its diagonals.* to prove the following theorems.

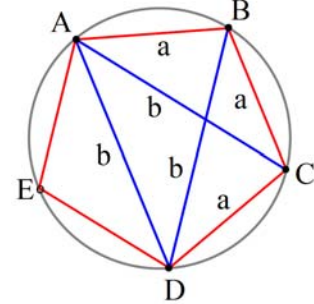
- If equilateral triangle $\triangle ABC$ is inscribed in a circle and Q is a point on the circle then the distance from Q to the most distant vertex of $\triangle ABC$ is equal to the sum of the distances from Q to the two nearer vertices.

So relative to the figure, you must prove that $q = p + r$.



2. In any regular pentagon the ratio of the length of a diagonal to the length of a side is the golden ratio, ϕ .

Hint: Relative to the figure which displays the regular pentagon ABCDE, apply Ptolemy's Theorem to the cyclic quadrilateral ABCD to prove that $\phi = \frac{b}{a} = \frac{1+\sqrt{5}}{2}$



Drawing Project

Due date : Wednesday, September 30

Follow the instructions in the article: <http://mypages.iit.edu/~maslanka/GeoConst.pdf> and reproduce each of its nineteen drawings by using only compass and straight-edge. Display each drawing on a separate 8.5 x 11 inch sheet.

Assignment 4

Due date : Friday, October 2

Section 7.1, Page 204:	#2,4,9,10,18,22,25,46,49,53,60,67,68,69,73,74,75.
Section 7.2, Page 208:	#1,2,3,5,6,7,8,13,14,15,16.
Section 7.3, Page 213:	#1(a)(b),7(c)(d),11,12,13.
Section 7.5, Page 221:	#12,13,14.

..... **Exam 1 –Wednesday, October 7**

Assignment 5

Due date : Monday, October 19

Section 13.1, Page 365:	#1,2,5,7,14,15,19,22,23,29,34,36,41,47,53,57,73,78,79.
Section 13.2, Page 368:	#1,3,8,11,15,16,17,18,21,22,25,28,33,34.
Section 13.3, Page 372:	#4,7,8,15,16,17.

Additional Exercise

Use the distance formula along the surface of the Earth (refer to formula (5) for SD_{PR} , in the table: <http://mypages.iit.edu/~maslanka/3Dcoordinates.pdf>) to find the surface distance between Chicago, IL (Latitude = 41.8795° , Longitude = -87.6243°) and each of the cities:

- i) Cairo, Egypt: Latitude = 30.0571° , Longitude = 31.2272°
- ii) Bogotá, Columbia: Latitude = 4.6473° , Longitude = -74.0963°
- iii) Cape Town, South Africa: Latitude = -33.9160° , Longitude = 18.4222°
- iv) Beijing, China: Latitude = 39.9082° , Longitude = 116.3979°
- v) Sydney, Australia: Latitude = -33.8671° , Longitude = 151.2071°

Assignment 6

Due date : Monday, October 26

Exercises

#1 – 8 (below).

Instructions

In problems 1- 4 plot at least once cycle of each of the sine or cosine waves. Plot all of the curves for each problem on a single graph. Do not use a calculator but rather follow the method outlined in the text, section 14-2, on pages 383-386.

In each problem, you should obtain the graph of equations (b) and (c) from that of equation (a) by using a suitable horizontal or vertical shift of the previous graph in each instance. Be certain to:

- (i) Locate the coordinates of all the **zeros**
(i.e. the points of the form $(x, 0)$) on your plots.
- (ii) Locate the coordinates of all the **extreme points**
(i.e. the points of the form (x, a) and $(x, -a)$ where a = amplitude of the graph).
- (iii) Locate the coordinates of all points of the form:
 $(x, a/2)$ and $(x, -a/2)$ where a = amplitude of the graph.

- | | | |
|--------------------------|-------------------------------|--------------------------------|
| 1. (a) $y = 2 \sin(3x)$ | (b) $y = 2 \sin(3x - \pi)$ | (c) $y = 2 \sin(3x - \pi) - 4$ |
| 2. (a) $y = \cos(x)$ | (b) $y = \cos(x + \pi/3)$ | (c) $y = \cos(x + \pi/3) + 1$ |
| 3. (a) $y = -4 \sin(2x)$ | (b) $y = -4 \sin(2x - \pi/4)$ | |
| 4. (a) $y = -\cos(x/2)$ | (b) $y = -\cos(x/2 + \pi)$ | |

In problems 5 – 8, plot at least two cycles of each periodic function. Plot both of the curves for each problem on a single graph. Use shifting (if relevant) to obtain the graph of equation (b) from that of equation (a). Be certain to:

- (i) Locate the coordinates of all the **zeros** on your plots.
- (ii) Locate all the vertical asymptotes for your plots.

- | | |
|-------------------------|----------------------------|
| 5. (a) $y = \tan(4x)$ | (b) $y = \tan(4x - \pi/2)$ |
| 6. (a) $y = 2 \sin(3x)$ | (b) $y = 2 \csc(3x)$ |
| 7. (a) $y = -\cos(x/2)$ | (b) $y = -\sec(x/2)$ |
| 8. (a) $y = \cot(3x)$ | (b) $y = \cot(3x) + \pi/2$ |

Assignment 7

Due date : Friday, November 6

Section 15.1, Page 419

#1,2,3,5,10,12,17,21,27,33,43.

Section 15.2, Page 425

#1,2,3,5,9,10,11,12,13,14,15.

Section 15.3, Page 431

#1,2.

Additional Exercises

#1 – 12 (below).

For each equation, 1 – 12, do the following:

(a) Graph the equation in the rectangular $r\theta$ - plane over the specified θ range.

(b) Graph the equation in polar coordinates in the xy -plane

where $x = r \cos \theta$ and $y = r \sin \theta$

(c) Convert the equation for the polar graph from (b) to Cartesian coordinates.

- | | | |
|---|---|--|
| 1. $r = 6, 0 \leq \theta \leq 2\pi$ | 2. $\theta = \frac{\pi}{3}, \frac{\pi}{3} \leq \theta \leq \frac{\pi}{3}$ | 3. $r = 8 + 6 \cos \theta, 0 \leq \theta \leq 2\pi$ |
| 4. $r = 7 + 7 \cos \theta, 0 \leq \theta \leq 2\pi$ | 5. $r = 5 + 9 \cos \theta, 0 \leq \theta \leq 2\pi$ | 6. $r = 3 + 11 \cos \theta, 0 \leq \theta \leq 2\pi$ |
| 7. $r = 2 \sin 2\theta, 0 \leq \theta \leq 2\pi$ | 8. $r = 3 \sin 3\theta, 0 \leq \theta \leq \pi$ | 9. $r = 3 \sin \theta, 0 \leq \theta \leq \pi$ |
| 10. $r = \frac{3}{\sin \theta}, 0 < \theta < \pi$ | 11. $r^2 = 9 \sin 2\theta, 0 \leq \theta \leq \frac{\pi}{2}$ | 12. $r^2 = 16 \cos 2\theta, -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$ |

..... **Exam 2 – Wednesday, November 11**

Assignment 8

Due date : Friday, November 20

Section 20.1, Page 564

#40, 43, 50, 51, 52, 53.

Section 20.2, Page 568

#3, 4, 7 – 10, 25, 27, 28, 47.

Additional Exercises

#1 – 7 (below).

- Find the slopes of the lines through the two pairs of points; then determine whether the lines are parallel, coincident, perpendicular or none of these.
(a) $(1, -2), (-2, -11); (2, 8), (0, 2)$ (b) $(1, 5), (-2, -7); (7, -1), (3, 0)$
(c) $(1, 1), (4, -1); (-2, 3), (7, -3)$ (d) $(1, -1), (5, 2); (9, 5), (-3, -4)$.
- Find the slope of the line bisecting the angle from l_1 to l_2 with slopes m_1 and m_2 respectively.
(a) $m_1 = -3$ and $m_2 = 5$ (b) $m_1 = \frac{3}{4}$ and m_2 does not exist.
- Find the equations of the three sides of the triangle with vertices
 $A = (1, 5), B = (3, -1)$, and $C = (-1, -1)$.
- Use slopes to find the measures of each interior angle in the triangle from Problem 3.
- Find the equations of the medians of the triangle from Problem 3.
- Find the equations of the altitudes of the triangle from Problem 3.
- Find the equation of the perpendicular bisector of the segment joining $(4, 2)$ and $(-2, 6)$.

Assignment 9

Due date : Monday, November 30

Section 20.3, Page 577

#1,4,7,10,17,19,25,26,27.

Additional Exercises

#1 – 8 (below).

- Find the distance from the point to the given line by using the distance theorem
<http://mypages.iit.edu/~maslanka/PointtoLine.pdf>
(a) $2x - 4y + 2 = 0; (1, 3)$ (b) $4x + 5y - 3 = 0; (-2, 4)$
(c) $3x + 4 = 0; (2, 4)$ (d) $y = 3; (1, 5)$
- Find the distance between the parallel lines:
 $L_1: 2x - 5y + 3 = 0; L_2: 2x - 5y + 7 = 0.$
(Hint: It suffices to compute the distance from any particular point P_1 on L_1 to L_2 .)
- Use the *Distance formula from Point to Line* to find the lengths of the altitudes of $\triangle ABC$ with the vertices $A = (1, 2), B = (5, 5), C = (-1, 7)$.
- Find the area of $\triangle ABC$ from problem 3.
- Find the equation of the line L bisecting the angle from L_1 to L_2 in each case.
(a) $L_1: 3x - 4y - 2 = 0; L_2: 4x - 3y + 4 = 0.$
(b) $L_1: 12x + 35y - 4 = 0; L_2: 15y - 8x + 3 = 0.$
(Hint: Note that if $P = (x, y)$ is any point on the bisector L then its coordinates satisfy
the condition: $\frac{|A_1x + B_1y + C_1|}{\sqrt{A_1^2 + B_1^2}} = \frac{|A_2x + B_2y + C_2|}{\sqrt{A_2^2 + B_2^2}}$ (*)
where $L_1: A_1x + B_1y + C_1 = 0$ and $L_2: A_2x + B_2y + C_2 = 0.$
The solutions to (*) yield the equations of both lines bisecting the angles between L_1 and L_2 .)
- Find the center C of the circle inscribed in the triangle with vertices:
 $(0, 0), (4, 0)$ and $(0, 3)$.
(Hint: First note that $C = (a, a)$. Why?)
- Find the two points of intersection of the line and circle:
 $4x - y - 9 = 0; x^2 + y^2 - x - 3y - 6 = 0.$
- Find the two points of intersection of the circles:
 $x^2 + y^2 + 5x + y - 26 = 0; x^2 + y^2 + 2x - y - 15 = 0.$

Assignment 10

Due date : Monday, December 9

Section 20.4, Page 584 #1,2,5,6,7,14,15,17.

Additional Exercises #1 – 4 (below).

1. Find the equation of the parabola with vertex: $V = (0, 0)$ and passing thru the points: $(2, 3)$ and $(-2, 3)$.
2. Find the equation of the parabola \mathcal{P} with focus $F = (1, 1)$ and the directrix $\mathcal{D}: x + y = 0$ in two ways:
 - (a) By using the fact that : $Q = (x, y)$ on $\mathcal{P} \Leftrightarrow d(Q, F) = d(Q, \mathcal{D})$.
 - (b) By rotating the parabola $y^2 = 2\sqrt{2}x$ by 45° about its axis and translating its graph $+\frac{1}{2}$ units horizontally and $+\frac{1}{2}$ units vertically.
3. Find the equation of the tangent line to $x^2 = -5y$ at $(5, -5)$ by using only the techniques of analytic geometry.
4. Find the equation of the tangent line to $y^2 = -16x$ that is parallel to the line $x + y = 1$.

..... **Final Exam – Wednesday, December 9 at: 8 – 10 AM (Section 119-01)
2 – 4 PM (Section 119-03).**