ROTATIONS AND TRANSLATIONS OF GRAPHS IN THE \( xy \)-PLANE

THEOREM: The polar graph of \( G ( r, \theta - \alpha ) = 0 \) in the \( xy \)-plane is that of the equation \( G ( r, \theta ) = 0 \) only rotated about \( O \) thru the angle \( \alpha \).

In the figure, the points \( P, Q, R, S \) on \( G ( r, \theta ) = 0 \) have the polar coordinates:

\[
\begin{align*}
P &= [ r_0, \frac{\pi}{2} ] \implies P' = [ r_0, \frac{\pi}{2} + \alpha ] \\
Q &= [ r_0, \theta_Q ] \implies Q' = [ r_0, \theta_Q + \alpha ] \\
R &= [ r_0, \theta_R ] \implies R' = [ r_0, \theta_R + \alpha ] \\
S &= [ r_1, \frac{3\pi}{2} ] \implies S' = [ r_1, \frac{3\pi}{2} + \alpha ]
\end{align*}
\]

THEOREM: The rectangular graph of \( F ( x - h, y - k ) = 0 \) in the \( xy \)-plane is that of \( F ( x, y ) = 0 \) only translated \( h \) units horizontally and \( k \) units vertically.

In the figure, the points \( P, Q, R, S \) have the rectangular coordinates:

\[
\begin{align*}
P &= ( 0, d ) \implies P' = ( h, d + k ) \\
Q &= ( -a, b ) \implies Q' = ( -a + h, b + k ) \\
R &= ( a, b ) \implies R' = ( a + h, b + k ) \\
S &= ( 0, c ) \implies S' = ( h, c + k )
\end{align*}
\]
I. Rotation of the parabola: \( y^2 = 4px \) by 60° followed by a translation \( h \) units rightward and \( k \) units upward.

II. Rotation of the lemniscate: \( y^2 = x^2 - x^4 \) by 60° followed by a translation 3 units rightward and 2 units upward.