RECIPIENT PROBLEM SET 1

1. Let \( f(x) = x + \cos x \) for \( -\frac{\pi}{2} \leq x \leq \frac{\pi}{2} \).
   (a) Show that \( f \) is always increasing on its given domain.
   (b) Determine the range of \( f \).
   (c) Find the value of \( f^{-1}(1) \).
   (d) Find the value of \( (f^{-1})'(1) \).

2. Let \( f(x) = \frac{1 - \sqrt{x}}{1 + \sqrt{x}} \).
   (a) Find a formula for \( f^{-1}(x) \).
   (b) Find \( (f^{-1})'(x) \) by differentiating the formula from part (a).
   (c) Find \( f'(x) \) using the Quotient Rule.
   (d) Find \( (f^{-1})'(x) \) using Theorem 7:

   \[
   (f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}
   \]

3. Use the graph of \( f \) shown at the right to:
   (a) sketch the graphs of
       \( g(x) = f^{-1}(x) \)
       \( h(x) = \frac{1}{f(x)} \)
       \( h^{-1}(x) \)
   (b) state the domain of each of the functions: \( f, g, h \) and \( h^{-1} \).

4. Use the Laws of Logarithms to expand the quantity:
   (a) \( \ln \sqrt{a (b^2 + c^2)} \)
   (b) \( \ln \left[ \frac{3x^2}{(x+1)^5} \right] \)
   (c) \( \ln \left( \frac{x y^3}{z^6} \right) \).

5. Differentiate the function:
   (a) \( y = \frac{1 + \ln t}{1 - \ln t} \)
   (b) \( y = \ln (x^4 \sin^2 x) \).

6. Differentiate and state the domain of \( f \):
   (a) \( f(x) = \sqrt{1 - \ln x} \)
   (b) \( y = \ln (\ln x) \)
7. Use logarithmic differentiation to find \( \frac{dy}{dx} \):

(a) \( y = \frac{(x+1)^3(5-2x)^5}{x^2 + 4} \)  
(b) \( y = (x^2 + 1)(x^2 + 2)(x^2 + 3)(x^2 + 4) \)

8. Integrate the function:

(a) \( \int \frac{1}{x \ln x} \, dx \)  
(b) \( \int \frac{1}{(\ln x)^2} \, dx \)  
(c) \( \int \frac{\cos x}{2 + \sin x} \, dx \).

9. Let \( y = e^{-x^3} \) for \( 0 \leq x \leq 1 \).

Use the Disc Method to find the volume obtained by rotating the region under the graph of the curve \( y = e^{-x^3} \) about the \( y \)-axis.

10. Solve the equation for \( x \).

(a) \( e^{(2x+3)} - 7 = 0 \)  
(b) \( \ln(5 - 2x) = -3 \)  
(c) \( \ln(\ln x) = 1 \)  
(d) \( 7e^x - e^{2x} = 12 \).

11. Differentiate:

(a) \( y = \frac{e^x}{1 + x} \)  
(b) \( y = e^x \ln x \)  
(c) \( y = \cos(e^{7x}) \)  
(d) \( y = \sqrt{1 + xe^{-2x}} \).

12. Evaluate the integral:

(a) \( \int \frac{e^{(1/x)}}{x^2} \, dx \)  
(b) \( \int e^x \sin(e^x) \, dx \)  
(c) \( \int \frac{e^x}{1 + e^x} \, dx \)  
(d) \( \int \frac{e^{2x}}{1 + e^x} \, dx \).