

Some suggestions and resources

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There are many books devoted to functional equations. By far the simplest introduction (and accessible to beginners) is an old slim volume entitled "On Applications and Theory of Functional Equations" written by the master János Aczél. It's in our library (QA 431). You may wish to look at nearby books as well.

If you Google "Functional Equations", you can get to some good (and some not so good) sites. Wikipedia is by far the best — the references are excellent — and the treatment of particular equations (e.g., Cauchy's) is useful. I strongly urge visits to eqworld and planetmath. The Wolfram site is not bad, as are several others farther down the list.

A wonderful two-page article on Cauchy's FE, written by G. Young, appeared in the American Mathematical Monthly, Vol. 65 (1958), pp. 37-38. Additional print resources are available upon request.

A suitable project might include (1) a detailed report on solution methods for several of the most important FE's and on modeling applications found in the literature, or (even better) (2) solution of the FE's given below.

A. "Warm-ups". Using elementary (but not necessarily simple) methods, find the general solution, i.e., all solutions with no assumptions on the unknown function.

1. $f(xy) = y^2 f(x)$, for all x and y
2. $f(1-x) = 1 + f(x)$, for all x
3. $f(xy) = f(x/y)$, for all $x > 0$ and $y > 0$
4. $f(xy) = f(x/y)$, for all x and all $y \neq 0$
5. $xg(x) + g(1-x) = 2x - x^2$, for all x
6. $f(x)f(y) - f(xy) = x + y$, for all x and y
7. $2f(x+y) = f(x+2y) + x$, for all x and y
8. $2g(x+y) = g(x+2y) + x^2$, for all x and y
9. $F(x, y) + F(y, z) = F(x, z)$, for all x, y and z

(over)