B. Use Cauchy's FE, $f(x+y) = f(x) + f(y)$, or another FE on this page to obtain all <u>continuous</u> solutions.

1. $g(x+y) = g(x)\,g(y)$, for all x and y
2. $h(xy) = h(x)\,h(y)$, for all $x > 0$ and $y > 0$
3. $g\left(\frac{x+y}{2}\right) = \frac{g(x)+g(y)}{2}$, for all x and y.
   (Hint: let $f(x) = g(x) - g(0)$.) This is <u>Jensen's</u> FE. Interpret the FE and its solutions geometrically.
4. $h(x+y) = h(x) + h(y) + h(x)\,h(y)$, for all x and y
5. $f(xy) = x\,f(y) + y\,f(x)$, for all $x > 0$ and $y > 0$
   (Hint: let $g(x) = f(x)/x$.)
6. $f(x) + g(y) = g(x+y)$, for all x and y
7. $f(x+y) + f(x-y) = 2f(x)$, for all x and y
   (Hint: $x = \frac{1}{2}(x+y+x-y)$.)
8. $\varphi(u+v) = \varphi(u^2) + \varphi(v^2)$, for all u and v
9. $g(\sqrt{s^2+t^2}) = g(s)\,g(t)$, for all s and t

C. Find families of non-constant solutions.

1. $f(x+y) + f(x-y) = 2f(x)\,f(y)$, for all x and y
2. $q(x+y) + q(x-y) = 2q(x) + 2q(y)$, for all x and y