Math 497 – Special Problems: Advanced Problem Solving

Course Description: 1 credit. Strategies and tactics for solving hard, contest-style problems. General strategies and methods. Symmetry, extremality, pigeonhole principle, invariants. Selected topics from graph theory, complex numbers, generating functions, algebra, enumeration, number theory, geometry, calculus.

Enrollment: Elective for AM and other majors.

Textbook(s): *The Art and Craft of Problem Solving, 2nd* Edition, by Paul Zeitz, John Wiley & Sons, Inc. (2007).

Other required material: None

Prerequisites: an A or B in MATH 230 or consent of the instructor

Objectives: Students will:

1. participate in every class meeting (at most one unexcused absence), where students will collaboratively attempt to solve hard problems.
2. write clear, concise proofs, for a small number of problems: revised several times or not at all, as needed.
3. read the book ahead of class. Lectures will be short and will not cover the reading. Reading ahead is necessary for class participation (which is required).
4. learn how to approach a problem when it is not clear how to begin.
5. participate in Virginia Tech Regional Math Contest (Saturday morning in October) and the Putnam Competitions (Saturday in December).
6. be able to recognize and write valid proofs. Proof techniques include bijective/combinatorial proofs, induction, and the pigeonhole principle.
7. be able to discuss mathematics, including: presenting solutions at the board, generating examples for illustration as appropriate, seeking and finding holes in proposed proofs.
8. be able to apply the strategies, tactics, and specific methods covered.

Lectures: one 50 minute class per week.

Course Outline:	Hours
1. Strategies and Methods	4
2. Symmetry, Extremality, Pigeonhole, Invariants	4
3. Graph Theory, Complex Numbers, Generating Functions	3
4. Other topics	4

Assessment:		
	Class Participation	**Required**
	Contest Participation	**Required**
	Written, revised problems	70%
	Final Exam	30%

Syllabus prepared by: Michael Pelsmajer and John Erickson
Date: April 29, 2013

Players 1,2,3,…,n are seated around a table and each has a single penny. Player 1 passes a penny to Player 2, who then passes two pennies to Player 3. Player 3 then passes one penny to Player 4, who passes two pennies to Player 5, and so on, players alternately passing one penny or two to the next player who still has some pennies. A player who runs out of pennies drops out of the game and leaves the table. Find an infinite set of numbers n for which some player ends up with all n pennies.

(from http://math.scu.edu/putnam)

Fall 2013
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in which

students will study strategies, tactics, and mathematical techniques to attack mystifying problems such as these.

Forty-one rooks are placed on a 10×10 chessboard. Prove that there must be five rooks, none of which attacks each other.

Let P_1, P_2, …, P_{2013} be distinct points in the plane. Connect the points with line segments P_1P_2, P_2P_3, …, $P_{2012}P_{2013}$, $P_{2013}P_1$. Can one draw a line through the interior of every one of those segments?

Solve $x^4 + x^3 + x^2 + x + 1 = 0$.

Let a_1, a_2, …, a_n be a sequence of positive numbers. Show that

$$(a_1 + a_2 + \ldots + a_n)(1/a_1 + 1/a_2 + \ldots + 1/a_n) \geq n^2,$$

with equality holding if and only if the a_i are all equal.

Requirements: Class meetings will have short lectures and time for working in groups. Students must participate actively during class, read outside of class, write up nice solutions to a few problems, and participate in two math contests (VTRMC and Putnam).

Prerequisites: an A or B in Math 230, or permission from the instructor.

Time and Location: TBA.