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Brief Research Summary
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Mostly I study graph theory: sometimes more as mathematics (structural, extremal) and sometimes more as computer science (algorithms, complexity), although one often helps with the other, and I have interests (graph drawing, tree-width) that fall squarely in the middle of of the two fields. As for topics, I have not particularly specialized or restricted the types of problems I work on. Rather, as I look back it seems that for the most part, I selected projects based simply on *niceness*: of a problem, the people involved, and whether there is an application.

Unfortunately my style of working makes it harder to quickly summarize my work, even assuming that the reader knows is familiar with basic graph theory concepts (terminology will agree with [**84**] or [**27**] whenever possible.) However, this is what I must do next.

Unless specified otherwise, all graphs are assumed to be simple, and G is a graph on n vertices.

Graph Drawing

The *crossing number* $\mathrm{cr}(G)$ is the minimum number of edge crossings in a (nondegenerate) drawing of G in the plane. The *odd-crossing number* $\mathrm{ocr}(G)$ is the minimum number of pairs of edges that cross each other an odd number of times (called *odd pairs*, other pairs are *even*). Clearly $\mathrm{ocr}(G) \leq \mathrm{cr}(G)$. A theorem of Hanani [**25**] and Tutte [**80**] (weak version) asserts that if a graph can be drawn such that no two edges cross each other an odd number of times, then the graph is planar; in other words, $\mathrm{ocr}(G) = 0$ implies $\mathrm{cr}(G) = 0$.

For this and other reasons, people thought that perhaps $\mathrm{ocr}(G) = \mathrm{cr}(G)$ for any graph G (see [**9,62,85**]). Pach [**61**] called it "perhaps the most exciting open problem in the area". Schaefer, Štefankovič, and I [**69**] disproved this, providing graphs for which $\mathrm{ocr}(G) \leq (\sqrt{3}/2 + o(1))\,\mathrm{cr}(G)$. Moreover, the graphs satisfy the bound $\mathrm{ocr}(G) \leq (\sqrt{3}/2 + o(1))\,\mathrm{pcr}(G)$, where $\mathrm{pcr}(G)$ is the *pair-crossing number*, defined by Pach and Toth in 1998. (G. Toth [**79**] has recently improved the separation factor.) On the other hand, we have shown that $\mathrm{ocr}(G) = \mathrm{pcr}(G) = \mathrm{cr}(G)$ whenever $\mathrm{ocr}(G) \leq 3$ [**70**], and Pach and Toth [**62**] showed that $\mathrm{cr}(G) \leq 2\,\mathrm{ocr}(G)^2$ for every graph G. An important open question is whether there exists a constant c such that $\mathrm{cr}(G) \leq c \cdot \mathrm{ocr}(G)$ for all graphs G.

Subsequently we have continued to investigate the relationship between different notions of crossing numbers. Highlights include: An intuitive, algorithmic proof of the Hanani-Tutte Theorem [**70**] (which already seems to have made it into the graph theory curriculum, see [**32**] and [**78**]); strengthenings of results of Cairns & Nikoleyevsky [**19,20**] about crossing numbers on surfaces other than the plane [**71**], again with nice proofs; extending the Pach-Toth theorem to other surfaces [**71**]; theorems for eliminating crossings between even pairs in the presence of odd edges [**70,71**] (strengthening a theorem of Pach and Toth [**62**]); a simpler proof of Hliněný's result [**40**] that computing the crossing number of a cubic graph is NP-complete [**67**], and a proof that odd crossing number is fixed-parameter tractable [**68**] (following Grohe's proof that crossing number is FPT [**37**]).

A graph is *confluent* if can be represented in the plane by vertices and differentiable curves such that two vertices are adjacent if there is a smooth route between them (a curve with no sharp turns) with no self-intersections. Confluent graphs capture the connection properties of train tracks, offering a very natural generalization of planar graphs, giving us an important tool in graph visualization. (See page 12 of [**29**] for an example.) The definition is due to Dickerson, Eppstein, Goodrich, and Meng [**26**], who identified several graphs classes which are confluent (including interval graphs and cographs), gave examples of graphs that are not confluent, and gave a heuristic recognition algorithm. Hui, Schaefer, Štefankovič, and I [**42**] showed that a natural strengthening of confluency can be recognized in NP. We also defined confluency analogues of two subclasses of planar graphs: trees and outerplanar graphs. It turns out that *tree-confluent graphs* are equivalent to the $(6,2)$-chordal bipartite graphs, and bipartite *outerconfluent graphs* are equivalent to the bipartite permutation graphs (see [**16**] for definitions). Eppstein, Goodrich, and Meng subsequently defined Δ-confluency [**28**], which generalizes tree-confluency, and they asked [**29**] about further natural generalizations of tree-confluency. Together with Schaefer and a student, we have recently investigated this [**72**], yielding a hierarchy of natural definitions with recognition algorithms.

Reliable Single-Message Transmission

End-to-end communication considers problems of sending messages from a sender s to a receiver r through an asynchronous, unreliable network, such as the internet. The following model was introduced by Fich in [**33**]; she explored it further with several coauthors in [**1**] and [**2**]: Consider the problem of transmitting a single message from s to r through a network in which edges may fail and cannot recover. We can assume that there is at least one s,r-path without failed edges, but we do not know which path it is. A protocol is *k-robust* if it ensures that a message sent by s will be received by r when at most k edges fail, and it will never generate an infinite number of messages.

Fich, Kündgen, Ramamurthi and I [**34**] gave a forbidden rooted-minor characterization of the family of networks with a protocol that is k-robust for all k, and we described the protocol. We have also shown that there is a forbidden rooted-minor characterization for the case when we can attach a fixed-length header containing routing information to the message. We also discuss the algorithmic consequences of these characterizations.

For networks that do not have a k-robust protocol for all k, the *robustness* is the maximum k for which there is a k-robust protocol. Kündgen, Ramamurthi and I [**50**] provided general lower bounds for robustness by improving a natural algorithm obtained from Menger's Theorem. We determined robustness for several examples, such as complete graphs, meshes, and hypercubes.

In [**34**], the proofs bound the *tree-width* of graphs for which a good protocol exists. For the characterizations, we use a *tree decomposition* of small width to investigate the structure of such a graph. (A nice survey of the recent history of these concepts is given by Bodlaender [**12**].)

Coloring

Graph coloring has applications in resource allocation, and the possibility of a restricted list of resources for each user motivates the notion of *list coloring* (for a survey, see Tuza [**81**]). Equitable coloring is another very active area of research (see Lih [**51**]). Kostochka, West, and I [**48**] have combined these notions: An n-vertex graph is *equitably k-choosable* if every assigment of k-element lists to its vertices permits a list coloring such that each color class has size at most $\lceil n/k \rceil$. We seek the least k such that G is equitably j-choosable for all $j \geq k$. (Eroh [**30**] proved that for any hereditary class of graphs, this "threshold" for equitable choosability is the same as the minimum k such that every graph in the class is equitably k-choosable.)

We have determined the least such k in terms of maximum degree for trees (within 1), graphs with maximum degree at least $n/2$, interval graphs, and graphs with maximum degree 3 [**48,64**]. (The last was obtained independently by Lih and Wang [**52**].) We also have upper bounds in terms of the maximum degree for outerplanar graphs, graphs of bounded tree-width, chordal graphs, and graphs in general [**63,64,65**].

Alon, Grytczuk, Hałuszczak, and Riordan [**6**] introduced the following graph coloring variation. A sequence of the form $s_1 s_2 \dots s_k s_1 s_2 \dots s_k$ is called a *repetition*. A vertex-coloring of a graph is called *nonrepetitive* if none of its paths is repetitively colored. (See Grytczuk [**38**] for a survey of this and related topics.) The original inspiration for this is Thue's 1906 discovery [**77**] that for an arbitrarily long path, 3 colors suffice. Kündgen and I [**49**] answered a question of Grytczuk by proving that every outerplanar graph has a nonrepetitive 12-coloring. (This result was obtained independently by Barát and Varjú. [**11**]) We also showed that graphs of treewidth t

have nonrepetitive 4^t-colorings.

The *upper chromatic number* of a hypergraph $H = (V, E)$ is the maximum number of colors that can be assigned to vertices such that on every edge there is a repeated color. (For a graph, this is simply the number of components.) Skokan and I [**73**] are studying this for random k-uniform hypergraphs, where each k-set is an edge with probability $p = p(n)$. Our work also has consequences for random *mixed hypergraphs*, which were introduced by Voloshin over 20 years ago (see [**83**] for a survey).

Berry and Modiano [**10**] studied the benefits of using electronic ports in WDM/TDM Optical Networks, and in certain cases the problem reduces to the following: Given a multigraph $G = (V, E)$ and an integer g, color the edges by colors $1, \ldots, g$ so that $\sum_{v \in V} d(v, i)$ is minimized, where $d(v, i)$ is the number of of edges colored i that are incident to vertex v. The problem is NP-Complete; it is even APX-Hard. Berry and Modiano provide a conceptually simple 2-approximation. Călinsecu and I [**21**] improve this to a 3/2-approximation. An interesting lemma is that if the maximum degree is at most g then we can find an edge coloring that is proper in at least half of the vertices. We show how to carefully implement and analyze the algorithm to obtain a good expected running time.

Induced subgraphs

A *feedback vertex set* in a graph G is a vertex subset S such that $G - S$ has no cycles. For a given graph G and integer k, the *feedback vertex set problem* (FVS) is to decide if G has a feedback vertex set of size k. FVS has applications in fields like circuit testing, deadlock resolution, and analyzing manufacturing processes. Since FVS is NP-complete, there has been a lot of work on approximation algorithms and also in approaching the problem from the point of view of parameterized complexity (see [**45**] for details). The general idea of parameterized complexity is that a hard problem may be solved quickly as long as some parameter (like tree-width) is bounded by a constant. In particular, FVS is said to be *fixed-parameter tractable* because it can be solved in $f(k)n^{O(1)}$-time, where f is a function that depends only on the size of the minimum feedback vertex set. Kanj, Schaefer and I [**45**] gave an algorithm that was faster than the previous best by a factor of roughly 2^k. (Since then, other groups have made further improvements.)

Since a graph is acyclic if and only if it is a forest, finding a feedback vertex set of minimum size is equivalent to finding a maximum subgraph that is a forest. Albertson and Berman [**3**] conjectured that every planar graph has an induced subgraph that is a forest and has at least half of the vertices. (This would directly imply that a planar graph contains an independent set with at least a quarter of the vertices, without using the Four-Color Theorem.) Akiyama and Watanabe [**5**] gave examples showing that this would be best possible. Borodin [**14**] proved the best known lower bound of 2/5 on this ratio by proving that the vertex set of a planar graph may be partitioned into five independent sets, any two of which together induce a forest.

There are similar conjectures, examples, and results for induced forests in bipartite planar graphs [**5**], induced forests in outerplanar graphs [**41**], induced linear forests in planar graphs [**17,75**] (a linear forest is a graph for which every component is a path), and induced linear forests in outerplanar graphs [**4,17,23,56**] (for details, see [**66**]). In particular, Chappell conjectured that every outerplanar graph has an induced linear forest with more than 4/7 of the vertices, and he found examples to show that 4/7 is the best possible ratio.

I proved that every n-vertex outerplanar graph has an induced linear forest with at least $\lceil \frac{4n+2}{7} \rceil$ vertices [**66**]. Furthermore, this bound is sharp. Chappell and I [**24**] generalized the proof technique to investigate the analogous problem about induced forests of degree at most d in graphs of fixed bounded tree-width w; we obtained sharp results for all $d \geq 2$ and $w \geq 2$. (Bose, Dujmović, and Wood [**13**] recently investigated a similar-sounding problem: they sought large induced forests in graphs of bounded tree-width such that each vertex in the forest has degree at most d in the original graph.)

The induced matching problem is to compute, for a given graph G and an integer k, whether G has an induced matching of size at least k. (The nature of this problem differs from the case of induced forests and linear forests is a key sense: induced matchings are not *hereditary*; that is, a subgraph of an induced matching is not necessarily an induced matching itself.) The complexity and approximability of this problem has been studied extensively for special classes of graphs, and for graphs in general. For planar graphs it is NP-complete, and Moser and Sidkar [**57**] recently showed that this is fixed-parameter tractable, but without specifying the constants involved in their algorithm. Kanj, Schaefer, Xia, and I [**43**] considered this problem more carefully and obtained an $O(91^k + n)$ time solution. For the proof, we study graphs that are *twinless* (distinct vertices have distinct neighborhoods) and we obtain a purely extremal theorem: A planar twinless graph has an induced matching of size at least $n/40$. We also give examples showing that this bound cannot be improved beyond $(n + 10)/27$.

Other topics

Matheson and Tarjan [**54**] considered the problem of asymptotically minimizing the size of dominating set in triangulated planar graphs. They showed that for any such graph there is a dominating set of size at most $n/3$, and conjectured that $n/4$ is possible if n is sufficiently large. Intuitively, perhaps, the presence of large degree vertices should make this easier. King and I [**47**] recently proved the conjecture for graphs of maximum degree 6 (which includes the fullerene graphs).

A graph is *chordal* if it has no induced cycle of length at least 4; such graphs have been studied extensively since their introduction almost 50 years ago. Farber [**31**] introduced a subclass; a graph is *strongly chordal* if it is chordal and every even cycle of length at least 6 has a *strong chord*, meaning a chord joining vertices whose distance along the cycle is odd. Farber's motivation was a polynomial-time algorithm for the weighted dominating set problem on strongly chordal graphs; the problem is NP-hard for chordal graphs in general. Farber and others explored graph, hypergraph, and matrix characterizations of strongly chordal graphs (for example, see [**8,15,18,31,82**]).

Tokaz, West, and I [**74**] give new proofs of well-known characterizations of strongly chordal graphs and chordal bipartite graphs. The key ingredient is a better understanding of the dual hypertree structure for totally balanced hypergraphs. (For definitions and background, see [**16**].)

Given a set of clients on a network that are to be serviced by a facility (e.g., a fire station, a supply depot, a highway), one wants find a location for the facility that optimizes certain criteria. Mulder, Reid, and I [**59**] consider certain classes of problems on a tree network that generalize common notions (the *center*, *median*, *branchweight center*, and Slater's path analogues [**76**]) in several ways, with an overall framework that allows a unified approach to finding such central structures quickly. I have worked with two undergraduate students on related questions.

We also give a new, short proof [**58**] of a results of McMorris, Roberts, and Wang [**55**]: that four certain axiomatic properties uniquely define the center of a tree.

Given a set of objects V (for example, boxes in R^d), the associated *intersection graph* $G = (V, E)$ has $uv \in E$ if u intersects v. The *transversal number*, denoted $\tau(V)$, is the minimum size of a set of points that intersects every object in V. The *independence number* of V, denoted $\alpha(V)$ (or of G, denoted $\alpha(G)$), is the maximum size of a subset of objects in V that are pairwise disjoint; clearly $\alpha(G) \le \tau(V)$. Gyárfás and Lehel [**39**] found upper bounds for $\tau(G)$ in terms of $\alpha(G)$ when V is a set of boxes in R^d; Károlyi [**44**] and Fon-Der-Flaass and Kostochka [**35**] give improvements. Kim, Nakprasit, Skokan, and I [**46**] obtain similar bounds for a more general class of objects: translates of a centrally-symmetric convex sets in R^d. We have better bounds for the case $d = 2$, and we also study the case when we drop the centrally-symmetric condition.

Cao and I [**22**] proved that every toroidal graph with connectivity 3 and girth 6 is bipartite. This implies that its toughness is at most 1, which answers a question in Goddard, Plummer, and Swart [**36**] in which it was shown that such a graph has toughness at least 1. In fact our result follows as a corollary of results in [**7**] and [**60**], but we give a direct proof.

Luttamaguzi, Shen, Yang, and I [**53**] gave integer programming formulations of some graph problems. We also solve some small cases using IP solvers.

Future Plans

A few projects that I mentioned are not in their final form. While all have publishable results, some still require a nontrivial amount of work before they can be submitted—this of course is an immediate priority. There are also a few items connected with these projects that I still intend to investigate, including:

- I believe that we can show that there is a dominating set of size at most $n/4$ (if n is sufficiently large) for a much wider class of triangulations than those of maximum degree 6, using many of the proof techniques developed for that case.
- Conjecture [**38**]: There is a constant c such that every planar graph has a nonrepetitive c-coloring.

In general, however, I plan to cut down the number of different projects and instead research fewer topics more extensively, in order to increase the rate and depth of my output. In the near future, I will mostly concentrate on research in graph drawing. There are many interesting open questions in both crossing numbers and confluency, and we have submitted grant proposals that detail our plans for continuing this work. I will only mention a few of the larger questions here.

- (Mentioned already:) Does there exists a constant c such that $\mathrm{cr}(G) \le c \cdot \mathrm{ocr}(G)$ for all graphs G?
- Let $\mathrm{iocr}(G)$ be the minimum number of *non-adjacent* pairs of edges that cross in a drawing of a graph G in the plane. The original theorem (not weak version) of Hanani and Tutte is this: If $\mathrm{iocr}(G) = 0$ then $\mathrm{cr}(G) = 0$. Thus, in a sense, intersections of adjacent edges do not matter. Tutte seemed to think this true in some more general sense [**80**]. We have two specific questions: Can this be generalized to surfaces other than the plane? Does $\mathrm{iocr}(G) = \mathrm{ocr}(G)$ for all graphs G?
- The pair-crossing number $\mathrm{pcr}(G)$ is the minimum number of pairs of edges that cross in a drawing of G in the plane. Clearly $\mathrm{pcr}(G) \le \mathrm{cr}(G)$, but is it in fact the case that $\mathrm{pcr}(G) = \mathrm{cr}(G)$ for all G [**61**]? What about on surfaces other than the plane?

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