Vertical-axis wind turbines (VAWTs) offer an inherently simpler design than horizontal-axis machines, while their lower blade speed mitigates safety and noise concerns. While VAWTs do offer significant operational advantages, development has been hampered by the difficulty of modeling the aerodynamics involved, further complicated by their rotating geometry. This proposal presents results from a simulation of a baseline VAWT computed using Star-CCM+, a commercial finite volume (FVM) code. VAWT aerodynamics are shown to be dominated at low tip-speed ratios by dynamic stall phenomena and at high tip-speed ratios by wake-blade interactions. Additional completed research includes recent work on the design of a blade-pitch control system and VAWT performance trade studies with preliminary results showing improvements in power output greater than 127%.

The proposed research will include completing and optimizing the control system design and implementation for the single-bladed VAWT, eliminating dynamic stall and expanding the feedback control to the rear half-cycle. Once the control system has been fully implemented for that case, the turbine configuration may then be changed to include two and three blades. Additional case studies will be completed including trade studies in order to find the optimal configuration for maximal efficiency. Lastly, the control system will need to be proven while operating in real-world conditions, including time-variant tip-speed ratios and turbulent flow fields.
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NOMENCLATURE

$c$  Blade chord length
$D$  Drag
$H_{\text{Domain}}$  Physical extent of the flow in the $y$-direction behind the turbine
$l_o$  Characteristic length of a boundary element of an overset grid
$l_u$  Characteristic length of a boundary element of an underset grid
$L$  Lift
$L_{\text{Domain}}$  Physical extent of the flow in the $x$-direction behind the turbine
$n$  Number of turbine blades
$P$  Power
$p$  Pressure
$R$  Turbine rotor radius
$t$  Blade thickness
$T$  Torque
$TSR$  Tip speed ratio
$u_{\tau}$  Friction velocity at the wall
$U_{\infty}$  Free stream velocity
$Y_{\text{wall}}$  Non-dimensional wall distance

$\alpha$  Blade angle of attack
$\beta$  Blade pitch
$\theta$  Azimuthal angle
$\sigma$  Solidity
1 Introduction

Global energy consumption has expanded since 1980 and remains closely related to quality of life issues such as clean water access, longevity, and personal income levels [52]. As a result, new energy sources must be developed in order to meet demand for a growing world population. Energy production in the United States has traditionally come from a variety of sources including petroleum, coal, natural gas, nuclear fission reactors, and hydroelectric dams. For most of the last decade, coal fired power plants have supplied over 70% of the US's electricity, but recent increases in the cost of coal as well as concerns about carbon emissions have caused that contribution to drop to 57% in 2012 [12]. Likewise, uncertainty over the supply and cost of petroleum has discouraged consumption. New extraction methods, including horizontal drilling and hydraulic fracturing, have resulted in a significant decrease in the price of natural gas, bringing up its relative contribution to the US electricity supply. Nevertheless, concerns over carbon emissions and the exhaustion of easily exploitable gas reserves will eventually contribute to future rises in the price of natural gas [41]. Over the next few decades, the proportional contribution from hydroelectric sources is expected to decrease simply because most profitable sites in the US have already been exploited. Growth in nuclear power plants is uncertain due to concerns regarding waste disposal, safety, and high capital costs. These recent developments in the last 20 years have made wind energy sources comparatively more attractive than in previous years.

The profitability of wind energy is highly dependent on the cost and availability of fossil fuel derived sources. In the 1970s and 1980s there was a boom in wind turbine development due to energy shortages at the time. However, as the price of fossil fuels dropped in the 1990s, commercial wind turbines fell out of use. This resulted in the failure of all but a

Figure 1.1: The contributions of all electricity sources in the United States during the 2012 year [70]. Due to recent growth, wind energy is now responsible for 3.5% of US electricity production, more than any other non-hydroelectric renewable source.
handful of wind energy developers throughout the 1990s [64]. Rapidly growing global energy demand, political instability in energy rich nations, and environmental concerns have lowered the relative cost of wind energy, as evidenced by exponential increases in wind energy investments over the last decade. Over the last five years, wind power generation has risen steadily by about 30% every year [50]. This growth is not expected to slow in the near future with wind providing an estimated 14% [12] to 20% [41] of US electricity by the year 2030.

Wind turbines may be categorized by their axis of rotation into vertical-axis wind turbines (VAWTs) and horizontal-axis wind turbines (HAWTs), as well as by the dominant aerodynamic forces used to provide a turbine's power including lift-based and drag-based turbines. HAWTs are lift-based machines that operate similarly to an airplane propeller in reverse. The set of VAWTs currently undergoing commercial development is much more diverse. These include drag-based Savonius turbines which use scoops to capture the wind and generate torque. Other VAWTs represent lift-based machines such as the curved-blade Darrieus turbines which are optimized for the centrifugal loads on the blades and the straight bladed H-rotor[48]. H-rotors are unique in that they combine the benefits of simple manufacturing along with the ability to use individual pitch control of the blade geometry. Except for small applications, lift-based turbines are exclusively used due to their enhanced efficiency[64].

Figure 1.2: Histogram showing exponential growth of wind energy’s contribution to the US electricity production [69].
1.1 VAWT GEOMETRY

VAWTs are differentiated from HAWTs by the axis of rotation, which is perpendicular to the free-stream velocity. This has several operational advantages, but also results in more complicated aerodynamics. Major features of VAWT aerodynamics include periodic blade angles of attack, dynamic stall, and wake dynamics, all of which make modeling and optimization of these turbines challenging. There are two ways of looking at the dominant two-dimensional aerodynamics of VAWTs: (1) examination of the individual blade aerodynamics and (2) observation of the kinetic energy loss of the airflow as it passes through the turbine. Turbine performance is also affected by three-dimensional flows near the ends of the turbine that are analogous to the wingtip vortices produced by aircraft. These effects cause an induced drag on the turbine blades, which cannot be predicted in a two-dimensional analysis. This, however, is best considered only after the dominant aerodynamic features have been dealt with.

Figure 1.4 shows the two-dimensional cross section of a Darrieus turbine or H-rotor, an azimuthal angle is determined by the orientation of the blades. The angle of attack of each turbine blade is determined by two main velocity components. One velocity component is the free-stream wind whose average flow is constant in magnitude and direction. The other velocity component is induced by the blade’s rotation around the central pivot point, resulting in a vector which is always anti-parallel to the blade’s velocity vector and whose magnitude is equal to the turbine radius multiplied with the angular velocity. This velocity component is defined as the blade’s induced velocity,

$$U_{\text{ind}} = \dot{\theta} R .$$

(1.1)

If the blade pitch angle is zero, the induced velocity vector alone cannot provide the blade with an angle of attack. Therefore, a blade’s geometric angle of attack must rely on its position with respect to the free-stream wind vector and the ratio of that vector’s projection normal to the blade’s tangent with respect to the induced velocity. This ratio is defined as
the tip-speed ratio,

$$\text{TSR} = \frac{U_{\text{ind}}}{U_\infty}. \quad (1.2)$$

The changing direction of the turbine blades will result in a variable angle of attack oscillating between positive and negative maxima at the front end (the left side of the turbine in the figure) and back end, respectively. At the front and back end of the turbine, the induced velocity and free-stream vectors are perpendicular, so that the maximum angle of attack is equal to the arctangent of the tip-speed ratio's inverse,

$$\alpha_{\text{max}} = \tan^{-1}\left(\frac{1}{\text{TSR}}\right). \quad (1.3)$$

The tip-speed ratio and azimuthal angle also have an effect on the magnitude and direction of the lift vector. When the free-stream wind vector and the induced velocity combine to produce the relative velocity the blade sees with respect to the blade-centered frame of motion, this causes the lift vector to be pushed forward producing an induced thrust, known as the Katzmayr effect[33]. It is therefore this lift, projected onto the rotor tangent which is responsible for the usable torque of the turbine.

Another important way of understanding basic wind turbine performance is to look at the kinetic energy of the airflow as it passes through the turbine, since the power that the turbine produces stems from the kinetic energy it extracts from the free-stream. It is best to start by assuming that the dominant behavior of the air is one-dimensional, flowing from left to right. Since energy is conserved, power produced by the turbine must result in a slowing of the airflow. As shown in Figure 1.5, the velocity remains nearly that of the free-stream until the flow reaches the front end of the blade path. Once the flow enters the
Figure 1.5: 2-D variation in the time averaged U-velocity for a turbine operating at TSR = 3.0. The figure shows that kinetic energy is lost as the flow passes through the turbine, with the greatest drops occurring as the flow passes through the blade path. There is also a variation in the speed in the vertical direction. At the top of the turbine, the blade pushes against the flow and slowing it down. At the bottom of the figure, the blade pulls the flow faster.

turbine, the airflow slows and contains less energy that may be extracted by the back end. It should also be noted that kinetic energy of the flow can be lost even if the turbine is not producing useful power. Flow energy can be lost due to viscous effects such as viscous drag in the boundary layer of the turbine blades, as well as spurious energy in the turbine wake. Sources of this energy are vortices and shear layers associated with the blade wakes. Aerodynamic stall, in particular, gives rise to substantial amounts of kinetic energy remaining in the turbine wake.

1.2 RELATED RESEARCH
EXPERIMENTS AND MODELING METHODS

There are several criteria for determining the appropriateness of a model or experiment: material cost, investment in time, and accuracy. All methods require some degree of compromise and emphasize or neglect particular aspects of the flow physics. For example, flow visualization can be extremely time consuming and difficult when conducting wind tunnel experiments; whereas in a finite volume simulation it is a rather straightforward process
that only requires already known information to be displayed. However, experiments allow an investigator to see the actual physics being studied whereas a simulation must rely on a model. Therefore, it is essential that simulations be validated by comparison with experimental data. Some of the investigational methods used to understand the aerodynamic behavior of VAWTs include particle image velocimetry [18], pitot ports [61], and direct measurement of forces/moments or power output [6].

Experimental studies of VAWT aerodynamics have been done in the past by placing scaled-down models in wind tunnels or by building full-scale turbines in the field. In a recent study, Castelli et al. [6] conducted experiments in the Politecnico di Milano wind tunnel in order to validate computational models. While the wind tunnel data was qualitatively similar to the computational results, the wind tunnel data could only serve as a rough estimate since tunnel blockage effects for VAWT experiments are distinct and not well understood. These tunnel effects were studied further by Battisti et al. [2] who concluded that the effects of tunnel blockage could be mitigated by utilizing open-section wind tunnels, which better facilitate bypass flow around the model. Battisti concurs with Castelli that the standard tunnel blockage corrections are not applicable for VAWT wind tunnel studies and attributes this to unexpected entrainment of the flow along the axis of rotation. As mentioned earlier

Table 1.1: Advantages / Disadvantages of VAWT Analysis Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Pros</th>
<th>Cons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind Tunnel</td>
<td>requires no modeling, necessary for validation of computational models</td>
<td>requires large scale wind tunnels, flow visualization is difficult, tunnel blockage effects are not well understood</td>
</tr>
<tr>
<td>Full Scale Prototype</td>
<td>demonstrates genuine performance in real world conditions, useful for determining feasibility for large scale implementation</td>
<td>extremely expensive in capital investment, unsuitable for parametric studies</td>
</tr>
<tr>
<td>BEM Model</td>
<td>computationally inexpensive, useful for optimization, easiest model to implement</td>
<td>limited to moderate TSRs, neglects viscous effects, highly dependent on stall models</td>
</tr>
<tr>
<td>Vortex Model</td>
<td>computationally inexpensive, extends to 3-d easily, approximates flow velocity field</td>
<td>approximates only high Re cases, neglects viscous effects, dependent on stall models</td>
</tr>
<tr>
<td>Grid Centered Models</td>
<td>moderately computational expensive, model most faithful to the physics, can provide insights regarding dynamic stall and wake effects, suitable for parametric studies</td>
<td>accuracy is highly dependent on grid and turbulence model, challenging to implement for complex or moving geometries</td>
</tr>
</tbody>
</table>
in the chapter, a large scale study was conducted in cooperation with Sandia National Laboratories by Kadlec [32] and Sheldahl et al. [61] in which over 600 Darrieus turbines were built. This massive undertaking involved building turbines as large as 200kW in order to collect basic performance data and to determine the feasibility of VAWTs for large scale energy production.

Due to the high cost and difficulty of experimental investigations, various aerodynamic models have been developed as alternatives. These models can be separated into three main types: blade element methods, vortex methods, and Navier-Stokes/grid methods. Blade element methods (BEM) have been the most popular approach to studying VAWT aerodynamics because of their usefulness for giving rough but reasonably accurate predictions of VAWT performance, while remaining relatively easy to implement. Blade element methods model the momentum loss of the free-stream flow as it passes through the turbine [58, 68]. BEM models make use of actuator disk theory which was first applied to wind turbines by Glauert [20]. Lift and drag forces are estimated from the angle of attack and empirical data, which is then used to compute the momentum lost by the airflow. The interference factor, the velocity divided by the free-stream velocity, is then used to compute the angle of attack and the aerodynamic forces on the turbine blades downstream. These methods tend to be relatively cheap in terms of computational power and provide good accuracy for moderate tip-speed ratios. The downside is that BEM models are heavily reliant on empirical airfoil data which neglects unsteady aerodynamics, dynamic stall behavior, and wake dynamics - major features of VAWT aerodynamics [50]. The first dynamic stall models applied to BEM models for analysis of VAWTs were by Gormont [21] and later modified by Strickland [66], Paraschivoiu [50], and Masse & Berg [3]. Claessens [11] made extensive use of BEM models to study the effects of airfoil shape on the dynamic stall behavior of VAWT blades. McCoy et al. [47] optimized a two dimensional turbine for power coefficient, solidity, and their corresponding interference coefficients. Combining BEM models with grid-centered methods, Castelli et al. [8] provided detailed performance characteristics of a low solidity VAWT at various tip-speed ratios. Periodic passage of turbo-machinery is sometime referred to as a cascade [27]. Similar to boundary element models, cascade models make use of the Bernoulli equation and empirical data to approximate the flow velocity in the near wake of the turbine blades. While this method is the simplest and cheapest method of accurately predicting VAWT performance, is heavily dependent on empirical data [31].

Vortex methods are more computationally expensive than BEM, but provide more information about the two and three dimensional behavior of the flow through a VAWT. These methods assume an ideal flow and use local circulation around the turbine blades by substituting the blade elements with bound vortex line. Due to Kelvin's theorem which states that the total circulation within the flow must remain constant, there must be a blade tip vortex which is equal and opposite to the circulation that is produced by the bound vortex lines. Furthermore, additional span-wise vortex lines must be shed whenever the strength of the bound vortex changes and whose magnitude is equal to the change in circulation. These vortex lines may then be used to approximate the surrounding velocity potential field [31]. As a result of the inviscid assumption, the overall flow approximates high Reynolds number cases while wake dynamics and viscous drag are neglected [22].
The first researcher to use vortex methods to model VAWT aerodynamics was Larsen [39]. Others have continued to make improvements to vortex methods, including Strickland et al. [67], who added dynamic stall models and fully extended the model to include three-dimensional effects as well as Masse [43] who approximated wake effects in the turbine operation, which compared favorably to experimental observations.

Grid-centered methods include finite difference (FDM) [57], finite element (FEM) [55], and finite volume methods (FVM) [26]. These methods work by discretizing the Navier-Stokes equations as well as the physical domain and solving the resultant system of equations. They are more computationally expensive than BEM or vortex methods, but they can approximate the physics more faithfully with minimal modeling and empirical data. While grid-based methods can be relatively straightforward for simple problems, solution accuracy becomes highly dependent on proper discretization of the physical domain and choice of turbulence model, once complex phenomena such as turbulence and separation are introduced. As of 2013, the most popular method utilized by computational fluid dynamics software is the finite volume method. Usage of FVM software is also likely to grow as computational resources become cheaper and its computational expense declines compared to other models. [71] One of the first researchers to use a grid-centered approach to investigate VAWT aerodynamics was Rajagopalan et al. [57], who used a finite difference model. Ponta et al. [55], combined a local circulation model with a finite element model in order to better approximate the instantaneous forces on the turbine blades as well as the near wakes. More recently, research has emphasized FVM as the primary tool for grid-based VAWT studies. Castelli et al. [6] developed a procedure for modeling a low-solidity VAWT at moderate tip-speed ratios. Hamada et al. [26], used FVM models to observe the effects of dynamic stall and wake dynamics for a set of turbines. Howell et al. [29] conducted case studies for various parameters such as tip-speed ratio, solidity, and blade surface finish. In his doctoral dissertation, Ferreira [17] investigated the two and three dimensional behavior of flows in the near wake of a VAWT, utilizing FVM models as well as other methods. At this point, the FVM approach seems to be the likely successor as the most predominant tool for VAWT aerodynamic research.

**VAWT Performance Studies**

Due to the complexity of VAWT aerodynamics and the sparsity of research in previous years, the number of parameters to be optimized for VAWTs remains immense. These parameters include tip-speed ratio, solidity, blade number, blade shape and camber, as well as constant or variable blade pitch offsets. While there have been several studies to investigate the effects of these parameters on overall turbine performance, there is still a tremendous amount of work to be done to optimize these parameters with respect to each other. As a result, comparisons should not be made between modern horizontal-axis turbines which have benefited from years of interest and vertical-axis ones whose performance characteristics often rely on 40-year old data [16].

Of the various parameters that have been studied, very little investigation has been done on the influence of the number of turbine blades on turbine performance, holding all other parameters constant. In the reference, *Wind Turbine Technology* [64], it is noted that blade
numbers of three or more minimize the effects of torque ripple. Castelli et al. [7] ran a case-
study with three, four, and five blade turbines which also demonstrated the dampening
effect of additional turbine blades on torque ripple. It was also shown that blade number
has little effect on overall power output at low tip-speed ratios, but the three-blade turbine
was shown to be superior at higher tip-speed ratios.

The effects of turbine solidity on VAWT performance characteristics are better known.
Paraschivoiu [50] has shown that solidity has the greatest influence at very high or very low
tip-speed ratios, as well as near the optimal tip-speed ratio. At high tip-speeds, solidity
has an unfavorable effect on turbine output due to blockage effects. However, higher so-
olidity does seem to dampen the effects of dynamic stall making turbines with large chord-
lengths relative to their diameters more attractive at very low tip-speed ratios. Since three-
dimensional effects such as induced drag are inversely proportional to the aspect ratio of
the turbine blades, larger chord-lengths (and higher solidity) can be expected to exacerbate
these phenomena. Howell et al. [29] show in their experimental observations that higher
solidity results in unfavorable changes in lift to drag ratios of the turbine blades and overall
turbine efficiency.

Studies have been conducted on the shape of turbine blades in order to minimize drag,
correct for virtual camber effects resulting from the curved blade trajectory, and to improve
self-starting characteristics of VAWTs. In their paper, Howell et al. [29] state that blade sur-
face roughness can help reduce drag at lower Reynolds numbers by encouraging transition
from laminar to turbulent flow within the boundary layer. Another parameter that was
demonstrated was the blade thickness, which was demonstrated to delay the onset of stall
at high blade angles of attack but decreases the lift-to-drag ratio overall. Claessens [11]
was able to increase the performance of a VAWT by changing the blades from a NACA 0018
shape to a DU 06-W-200 airfoil, resulting in a larger lift to drag ratio and discouraging the
formation of laminar separation bubbles at high blade angles of attack. The effect of cam-
ber on the overall performance of a VAWT was investigated by Danao et al. [13] who found
that the optimal camber line should follow the path line of the blade, since this eliminates
the virtual camber produced by the curved trajectory of the turbine blades. They also saw
interesting effects of blade camber on how energy is extracted from the airflow throughout
the turbine cycle. In particular, it was observed that a negative camber resulted in almost
all of the energy being extracted at the front half of the turbine cycle. Beri et al. [4] found
that VAWTs with cambered blades showed greater potential to self-start, though this also
results in reductions in peak efficiency.

For most cases, VAWT blades are mounted at 25 or 50% of their chordlengths along the
tangent line of the swept circle. By offsetting the blade's attitude away from the tangent
line, either in the "tow out" or "tow in" direction, the geometric angle of attack may be ad-
justed down or up, respectively (see Figure 1.10). Altering the blade angle of attack affects
dynamic stall behavior, the balance of energy extraction between the front and back halves
of the turbine, and self starting potential. Using wind tunnel observations, South et al. [65]
reported increases up to 11 % in power coefficients for a low solidity VAWT with a pitch
offset of 4° (tow out), compared to the zero pitch case. Klimas et al. [34] demonstrated that
with a pitch offset of 2° (tow out), a VAWT produced a 3% higher power coefficient with
a lower optimal tip-speed ratio, when compared to the zero pitch case. Fiedler et al. [19]
similarly shows significant improvements in power coefficient for a VAWT as the pitch angle increases, but with diminishing returns for angles greater than about 4°. These studies consistently show improved performance for modest tow out blade pitches, though the optimal pitch offset and the flow behavior are highly dependent on other parameters such as solidity, blade number, and airfoil shape. They also show that a tow-in blade pitch is detrimental to turbine performance. This would suggest that turbine performance is improved by lowering the maximum angle of attack (magnitude) on the front turbine half-cycle in order to prevent separation and increasing the maximum angle of attack on the back half cycle, which serves to smooth torque ripple.

Individual blade pitch control is already used on some HAWTs and becomes more feasible as turbines increase in scale. VAWTs equipped with blade pitch control can utilize the self-starting capabilities of turbines with blade pitch offsets, optimize blade angle of attack in order to maintain attached flow, maximize average power output at low tip-speed ratios, and mitigate torque ripple effects. Therefore, blade pitch regimes must be developed based on an understanding of VAWT performance characteristics for fixed pitch cases as well as dynamic stall effects and reattachment phenomenon, including hysteresis delay. Two such studies were published in the early 1990’s by Kirke et al. [35] and Lazauskas [40], who investigated using preset blade pitch regimes and self-stabilizing blades to maintain attached flow throughout the turbine cycle. Other active flow control devices such as trailing-edge flaps [62] and co-flow jets [72] could potentially be used to maintain attached flow for VAWTs. Acoustic devices such as plasma actuators [24], zero mass-flux jets [23] have also been studied. Greenblatt et al. [25] and Amitay et al. [1] have found that the introducing periodic perturbations (with a reduced frequency, \( F^+ = \frac{f}{U_{\infty}} \), of about 1.0) are effective at increasing average lift of airfoils at high angles of attack.

1.3 Motivation

The primary advantages of vertical-axis wind turbines over horizontal-axis machines stem from their lower tip speed ratios and omni-directionality. A lower tip speed ratio results in higher magnitude angles of attack, less influence of parasitic drag, and less noise production. HAWTs usually operate at tip speed ratios between six and ten whereas VAWTs operate between one half and four [50]. For an average wind speed of 15 mph that equates
to VAWT blade speeds of only about 30 mph and HAWT tip speeds of over 120 mph! Therefore, the safety and noise characteristics of VAWTs make them much more suitable for installation near populated areas.

A vertical-axis of rotation also eliminates the oscillating gravity loads that are exerted on HAWT blades, which create bending moments in one direction on the up-swing and then the other direction on the downswing. This problem quickly grows with turbine size and is considered one of the main limits on the scale of large HAWTs [48, 53]. It is for this reason that VAWTs are being considered for very large scale applications (see Figure 1.7) where HAWTs would be impractical, including multi-megawatt turbines [42]. Additional benefits of VAWTS include ease and safety of maintenance, as well as lower structural loading due to weight, since the heaviest and most complex parts of the turbine are installed near to the ground.

While there are significant operational advantages to VAWTs, they have not yet received the same attention and optimization efforts from which traditional horizontal-axis machines have benefited. In particular, the complex unsteady aerodynamics including hysteresis, dynamic stall, and wake interactions has made computational studies difficult. In particular, the phenomenon of turbulent flow separation which dominates VAWT aerody-

Figure 1.7: Vertax Wind Ltd. proposed multi-megawatt turbines. These sea-based turbines would rely on fewer moving parts than horizontal-axis machines, allowing a longer lifespan and less maintenance [30].
namics at moderate-to-low tip-speed ratios, drastically raises the computational expense and difficulty of accurately simulating turbine performance. As a result of this, power output and efficiency of VAWTs have remained significantly lower than HAWTs. If vertical-axis turbines are to be made competitive with respect to horizontal-axis machines, accurate predictions of VAWT aerodynamics and performance are necessary. If unsteady flow phenomena can be identified along with their effects on turbine performance, optimizations procedures may then be developed to improve efficiency and power output.
2 Completed Work

The completed research described in this section originates from previously published research on FVM simulations and BEM models of VAWT performance [37], identification of unsteady flow phenomena [36], and early attempts to improve VAWT power output using blade-pitch adjustments [60]. Additional unpublished research includes recent work on the design of a blade-pitch control system and VAWT performance trade studies.

2.1 FVM Studies

Methodology

Finite volume method (FVM) simulations work by discretizing the Navier-Stokes equations as well as the physical domain and solving the resultant system of equations [26]. They are more computationally expensive than BEM models, but they can approximate the physics more faithfully with significantly fewer assumptions and empirical data. While

![Diagram of physical domain and overset domain for FVM simulation]

Figure 2.1: Layout of the physical domain of the FVM simulation with boundary conditions (a) and the overset grids surrounding the turbine blades (b).
grid-based methods can be relatively straightforward for simple problems, solution accuracy becomes highly dependent on proper discretization of the physical domain and choice of turbulence model, once complex phenomena such as turbulence and separation are introduced. Adoption of FVM software is also likely to accelerate as the computational expense declines compared to other methods [71].

The two-dimensional VAWT geometry was modeled by mirroring a NACA 0021 airfoil three times around the axis of rotation. This geometry is then placed in a larger physical domain and conditions at the boundaries were defined (see Figure 2.1). Deciding on appropriate assumptions used to simplify the physics of the problem required more consideration. Since the flow speed was well below Mach = 0.3 with minimal blockage effects, compressibility effects were neglected. In order to solve the finite volume problem, the physical domain must be split into many control volumes (i.e. grid cells) with the overall discretization referred to as a grid or mesh. In order to ensure accuracy of the simulation, size and shape of the cells must be dictated by the magnitude and direction of gradients in the flow. This often requires corrections to the grid once a preliminary solution is determined. The simulation must then be run until the flow has reached a periodic, quasi-steady state. At that point, data may then be extracted from the simulations and observations regarding the aerodynamics may be made.

The two greatest challenges of grid-centered computational fluid dynamics methods are: (1) adequate discretization of complex and moving geometries while minimizing computational expense and (2) accurately describing the effects of turbulence in moderate to high Reynolds number flows. While the ability to deal with the turbulence problem is seriously hampered by the intractability of the governing equations and limitations with respect to computational resources [56], there have been recent advances in the ability to deal with complex and moving geometries. The most recent advance has been in the form of overset or chimera grid methods, beginning to be implemented in commercial codes only in the last few years. Overset grid methods have already been used to study the aerodynamics of helicopter rotors, [5, 14] which share many geometric features with VAWTs such as their axis of rotation, relative to the bulk flow. Hoke et al. [28] found that overset grid methods compared favorably to other methods such as conformal meshes and grid deformation techniques.

<table>
<thead>
<tr>
<th>Table 2.1: VAWT Simulation Parameters</th>
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<tbody>
<tr>
<td>Reynolds number [-]</td>
</tr>
<tr>
<td>Number of Elements [-]</td>
</tr>
<tr>
<td>( l_o ) [mm]</td>
</tr>
<tr>
<td>( l_o / \mu ) [-]</td>
</tr>
<tr>
<td>( \Delta t ) [s]</td>
</tr>
<tr>
<td>Inlet velocity [m/s]</td>
</tr>
<tr>
<td>TSR [-]</td>
</tr>
<tr>
<td>Turbulence model</td>
</tr>
</tbody>
</table>
Fully structured grids were created using Gridpro [15], an elliptic grid generation software. Gridpro allows the user to ensure orthogonality of the grid near the boundaries and provides control over cell shape by adjusting the grid topology. One of the strict requirements of overset grid methods is that the cells on the overset boundary be nearly uniform in size and shape. It is for that reason that a circular shape was chosen for the outer boundary of the overset grids. The portion of the underset grid that was overlaid by the overset grid was uniform with the cells growing outside of that region. The cells near the boundary of the overset grid should be smaller than those of the underset grid and the ratio of the two must fall within.

The computational solution was obtained using Star-CCM+, a commercial finite volume code with overset capability [9]. Star-CCM+ relies on the SIMPLE algorithm, an implicit, 2nd-order solver in space and time. The most important simulation parameters are listed in Table 2.1. The simulations made use of Star-CCM+’s implicit solver which permitted much larger time steps than otherwise possible with an explicit solver at a relatively low computational expense. For validation purposes, as well as to get a complete picture of the turbine performance at low to moderate tip-speed ratios, the turbine was run at tip-speed ratios of TSR = 1.5, 2.0, 2.5, and 3.0. Since the blade Reynolds number was defined to be 300,000 and the simulation geometry was limited to two-dimensional flow, a turbulence model was required. This, however, was not an obvious choice. Some investigators had chosen the two-equation $k$-$\varepsilon$ model [38] due to its insensitivity with respect to grid dimension within the boundary layer [6] and relatively good approximation of pure shear flows, which is useful for modeling wake dynamics. [56] In contrast, Paraschivoiu [50] demonstrated that the one-equation Spalart-Allmaras turbulence model [63] performed better at reproducing Piziali’s [54] experimental data for rotating airfoils. Since there is little difference between the $k$-$\varepsilon$ model and Spalart-Allmaras in predicting the behavior of pure shear flow, the results of Paraschivoiu’s study suggest the Spalart-Allmaras model to be better at predicting dynamic stall behavior. Because of this and the marginally lower computational expense of one fewer equation, the Spalart-Allmaras model was chosen for the simulation.

After some initial runs and adjustments to the mesh, two sets of simulations were produced. One simulation that utilized a grid that resolved the boundary layer only to $Y_{wall} > 5$ and, as a result, relied on wall modeling to approximate the flow in the boundary layer. The other simulation had a more refined boundary layer with $Y_{wall} \approx 1$, that did not rely on wall modeling to predict the boundary layer flow. The comparison of these simulations with each other as well as other case studies is discussed in the next section.

At each grid-point and for each of the governing equations, a residual,

$$r = f - \mathcal{L}\phi,$$  \hspace{1cm} (2.1)

is calculated, where $f$ is the solution for the given equation at that point. Once the residuals have been calculated for each grid point, the values are then normalized,

$$r_{\text{normalized}} = \sqrt{\frac{\sum_{i=1}^{n\text{ cells}} r^2}{n\text{ cells}}},$$  \hspace{1cm} (2.2)

providing a measure of how well the computational solution satisfies the governing equations. During each time step, the code was set to iterate until the residuals of the continuity
Figure 2.2: Fully elliptic overset grid containing a VAWT blade surface (a) and close-up of the trailing edge (b).
and momentum equations fell below $10^{-4}$ and the residual of the turbulent viscosity transport equation fell below $10^{-3}$. The simulations in each $Y^+_{wall}$ range were also run with twice the number of grid elements in order to demonstrate grid convergence; differences in the average turbine power coefficient were found to be less than 5% for all cases. The flow was determined to have reached a quasi-steady solution when the average turbine power coefficient of a full cycle was within 5% of the previous cycle. This typically occurred after about five full turbine cycles or about 2500 hours of CPU time.

One other issue should be noted with respect to computational stability. Simulations that utilize overset grids typically benefit from increased flexibility and accuracy than other methods. However, one disadvantage became apparent very quickly, which was that the computational stability was very sensitive to the grid and simulation parameters. It became quite clear that the simulation would diverge if the time step exceeded a value that was inversely proportional to the velocity of the overset grid with respect to the underset grid, $V_{o/u}$. The maximum stable time step also decreased as $\frac{I_u}{I_o}$ increased. This suggested that when the boundary cells of the overset grid moved too quickly with respect to the boundary cells of the underset grid, a large amount of error resulted that could cause the simulation to diverge. Therefore, it became necessary for this author to create a new criterion that could be used to ensure that a simulation using overset grids behaves in a predictable way. This quantity was named the Overset Slip Condition Number,

$$C_{o/u} = \frac{I_u}{I_o^2} V_{o/u} \Delta t.$$ (2.3)

Largely through trial and error, $C_{o/u} \approx 1$ was found to provide the best stability characteristics for the VAWT simulations and, therefore, determined the time-step size. Since the

![Figure 2.3: Boundary between the overset and underset grids.](image-url)
time scales of aerodynamic phenomena associated with VAWTs are orders of magnitude larger compared to the limitation imposed by the overset grids, the time-step size has little affect on the accuracy of the FVM simulation.

**PERFORMANCE LIMITING PHENOMENA**

Flow visualization is a laborious process, but it is also an effective tool for understanding the aerodynamics of concern. Since the effective angle of attack is more useful for looking at dynamic stall rather than wake dynamics, the vorticity magnitude distribution of a flow can be helpful for tracking the path of wake structures and identifying where they interfere with turbine blades. Figures 2.5 and 2.6 show a sequence of vorticity magnitude distributions for tip-speed ratios of TSR = 3.0 and 2.0, respectively. The sequences can be viewed from state 1 through state 10, and show the flow characteristics which repeat every 1/3 of a turbine cycle. The first sequence clearly demonstrates that attached flow is maintained on the blades for the full cycle. However, due to the faster rotation rate of the turbine with respect to the free-stream velocity, the blade will intersect with almost twice as many wake structures than for the tip-speed ratio of TSR = 2.0 case. The wakes that are produced on the front end of the turbine for the faster rotation rate also contain more vorticity.

Since efficient operation of VAWTs at low tip-speed ratios is desirable for a number of reasons, dynamic stall becomes the driving aerodynamic characteristic that determines performance. For all tip-speed ratios below TSR = 4.0, the blade angle of attack rises above the static stall angle of $12^\circ$, the angle of attack where stall effects begin to appear. However, full stall does not necessarily occur due to hysteresis delay, allowing the turbine to operate efficiently at much lower tip-speed ratios. Leading-edge separation does take place for this

![Figure 2.4: Close up view of the vorticity magnitude distribution around a turbine blade at an azimuthal angle of 180°. (TSR = 2.0)](image)

Near the end of the half-cycle, the blade stalls and the separation bubble is swept away as the angle of attack changes from negative to positive.
Figure 2.5: The sequence shows the vorticity magnitude distribution as the turbine passes through one third of a cycle at a tip-speed ratio of TSR = 3.0. The turbine maintains fully attached flow throughout the entire cycle.
Figure 2.6: The sequence shows the vorticity magnitude distribution as the turbine passes through one third of a cycle at a tip-speed ratio of TSR = 2.0. Around the 180° point, the blades undergo separation and reattachment.
turbine at all tip-speed ratios below TSR = 3.0. The azimuthal angle at which separation begins is difficult to predict, since the phenomenon is highly dependent on many different variables. For a tip-speed ratio of 1.5, separation and loss of lift occurs at azimuthal angles of approximately 90°, 210°, and again at 300°. For 2.0, separation takes place at azimuthal angles of 140° and 180°. Separation only occurs once for the tip-speed ratio of TSR = 2.5 case, occurring at an azimuthal angle of 150°, with the angle of attack going from negative to positive as the flow reattaches to the blade.

While the specific dynamic stall characteristics are difficult to predict from only the tip-speed ratio, such as the azimuthal angle at which separation occurs, there are a few patterns that should be noted. One interesting behavior is that full leading-edge stall is limited to the front half-cycle of the turbine’s operation for most of the operating tip-speed ratio’s, with the exception of the 1.5 case at the low end of that range. It is then reasonable to conclude that stall mitigation is most important for azimuthal angles between 0° and 180°. The angle of attack can also be raised significantly on the back half-cycle without necessarily causing the blade to stall. Another obvious pattern demonstrates that the problem of dynamic stall becomes dramatically worse as the tip-speed ratio is reduced from TSR = 3.0, with the number of occurrences of separation increasing as well as the performance damaging effects.

Aerodynamic forces on the blade change greatly as the flow becomes separated; the lift drops and the drag increases suddenly. Consequently, the performance of the VAWT is altered not only by how often the blade stalls during a cycle, but also by the point in the turbine cycle where the blade loses lift. This is demonstrated by the TSR = 2.5 case, where the blade stalls near the end of the front half-cycle. Even though the flow over the blade becomes completely separated, this cannot be seen in the blade torque profile since the blade wouldn’t be positioned to produce any torque anyway. The turbine can then operate at a higher angle of attack at this tip-speed ratio without dynamic stall effects reducing the lift at critical points.

Another important aspect of dynamic stall is the process of flow separation from the suction side of the blade and reattachment. In order for this to occur, the blade must be subjected to a high angle of attack for a significant amount of time. This can be seen in Figure 2.4, where the process of full leading-edge separation and reattachment can be seen for a turbine blade. As the blade angle of attack passes the static stall angle, separation begins at the trailing-edge and progresses toward the leading-edge. Lift drops significantly when the point of separation nears the quarter chord point since the majority of lift is generated near the leading-edge of the blade. The shear layer that separates the recirculation zone near the blade surface and the free-stream forms what is called the leading-edge vortex (LEV). As the lift and the circulation decreases for the blade, a trailing-edge vortex (TEV) is formed that is equal in magnitude and opposite in direction to the LEV. This is a consequence of Kelvin’s law, which states that for flows that can be approximated as inviscid with conservative body forces, the total circulation present in the flow,

\[ \frac{D\Gamma}{Dt} = 0, \quad (2.4) \]

remains constant. If the blade remains at a high angle of attack or the angle of attack is
suddenly decreased to below zero, the recirculation zone will eventually separate from the blade, taking the TEV with it. This results in a sudden increase in circulation around the blade, resulting in a rapid rise in lift. This phenomenon helps to increase the turbine performance for the $\text{TSR} = 2.5$ case, where the angle of attack magnitude rises sharply following reattachment.

From these observations some conclusions can be drawn regarding the dynamic stall effects on turbine operation. The blades will continue to produce lift at angles of attack above the static stall angle, but stall mitigation is needed to prevent separation at tip-speed ratios below $\text{TSR} = 3.0$. Separation is limited to the front half-cycle for tip-speed ratios above $\text{TSR} = 1.5$. The exact point at which separation occurs is difficult to predict, but separation at azimuthal angles less than $150^\circ$ is harmful to the overall turbine performance. Lastly, the reattachment process may be used to boost the torque produced by a turbine blade but this will require a way to induce separation at a given azimuthal angle.

As a turbine blade passes through a flow, a wake is produced downstream that is characterized by a low average velocity relative to the blade's frame of reference as well as high concentrations of vorticity. Wakes are produced by the reaction forces exerted on the fluid by the body, which are equal and opposite to the sum of the lift and drag vectors. As a result, the drag on a body and wake production are directly linked. Therefore, it is necessary to examine the causes of drag on the turbine blades. For the two dimensional case, there are three main components of drag: (1) friction drag generated by shear stresses in the boundary layer, (2) pressure drag resulting from the presence of flow separation and regions of recirculating fluid, and (3) viscous induced drag produced by added mass effects due to the blades' accelerating frames.

The structure of the wake may be characterized in terms of the relative velocity and the vorticity distribution. These two methods of observation are compared in Figure 2.7 for a VAWT blade. Near the trailing-edge surface, the wake is made up by a region of nearly uniform low relative velocity flow. Further downstream, the wake begins to diffuse producing a relative velocity distribution that is parabolic. Vorticity of opposite directions is generated along the top and bottom surfaces of the blade which then combine downstream. Other features may be introduced into the wake, such as shed vortices caused by separated flow over a blade. This stream of slowly diffusing wake structures trails behind the blade and is pulled along by the free-stream.

Wake production is indirectly related to the overall turbine performance since it is proportional to the drag forces on the turbine blades, resulting in lower torque. However, there is also a direct effect on the turbine performance because the wake is swept along by the free-stream from the front half-cycle into the path of the blades on the back half-cycle. This affects the individual blade aerodynamics in several ways, as shown in Figure 2.8. The lower average velocity relative to the blades results in a loss of lift, hampering energy extraction from the flow on the back half-cycle. Likewise, when a blade intersects with a wake at an oblique angle, flow may be cut off to one side of the blade and cause the blade to partially stall. Evidence of this may be seen in the form of ripples in the blade torque curve for azimuthal angles greater than $180^\circ$. These effects grow as the tip-speed ratio increases and the blade paths intersect with more numerous and stronger wakes.

The most straightforward way to limit wake interactions is to decrease wake production.
Figure 2.7: Two views of the flow downstream of a VAWT blade at an azimuthal angle of 0° and for a tip-speed ratio of TSR = 2.0.
in the first place, which means limiting the drag forces on the blades during the front half-cycle. Because wake production is a necessary consequence of lift generation, it is necessary to identify and mitigate the other major variables that increase drag. Clearly, stall is undesirable because it both reduces lift and adds greatly to blade drag. Second only to separation, the driving factor responsible for wake production and drag is the induced velocity and, therefore, the tip-speed ratio. Total drag is proportional to the net drag coefficient, but is also proportional to the induced velocity squared,

\[ C_d \sim U_{\text{induced}}^2 \cdot \]  

This explains the sudden drop in performance for tip-speed ratios above TSR = 2.5, where the tip-speed ratio is high enough to prevent the harmful effects of stall while also minimizing the tip-speed ratio, and thus, the drag as well as wake interactions. Also, sudden changes in lift are also to be avoided since, due to Kelvin's Law, vorticity must be shed from the blade equivalent to the change in circulation around the blade. The shed vorticity robs the surrounding flow of kinetic energy and produces drag. Lastly, the curvature of the blades’ trajectory has an effect on the individual blade aerodynamics, since it does impart a small angle to the flow near the leading and trailing-edge. This phenomenon is referred to as the "induced camber" of the blade and is dependent on the turbine geometry.
The effect of induced camber increases with the chord length relative to the turbine diameter, resulting in lift and drag being produced at azimuthal angles of $0^\circ$ and $180^\circ$. While the losses from this are small, they are not negligible.

The effects of dynamic stall allow VAWTs to operate at lower tip-speed ratios that raise the maximum blade angle of attack above the static stall angle, the effects of parasitic drag and wake interactions serve to reduce performance at higher tip-speed ratios. Therefore, the best way to mitigate both dynamic stall and wake interactions is to lower the tip-speed ratio below $\text{TSR} = 2.5$ and then attempt to prevent flow separation caused by sustained, high angles of attack. In order to do so, the effective angle of attack for the blade must be adjusted.

### 2.2 BEM Model

**Methodology**

Due to the high cost and difficulty of experimental investigations, various aerodynamic models have been developed as alternatives. Blade element momentum method (BEM) or streamtube models have been the most popular approach to studying VAWT aerodynamics because of their usefulness for giving rough but reasonably accurate predictions of VAWT performance. Blade element models, also known as streamtube models consist of discretizing the physical domain into streamwise sections and applying the governing equations in their integral form to the boundary of sections or streamtubes. Lift and drag forces are estimated from the angle of attack and empirical data, which is then used to compute the momentum lost by the airflow. From this, the interference factor are recalculated and the aerodynamic forces are computed again in the next iteration. The streamtube model assumes one-dimensional fluid flow in the streamwise direction, preventing mass, momentum or energy exchange between adjacent streamtubes. This method is computationally inexpensive and straightforward to implement. However, it neglects several unsteady effects that provide important insight into the behavior of VAWTs. The most significant are dynamic stall effects, added mass effects, and blade-wake interactions. The expansion of the cross-section of the streamtube that would be required by the continuity equation is also neglected in this model.

There are several distinct boundary discretization techniques, with varying levels of sophistication. The double-multiple-streamtube (DMST) model is the most complete. DMST models describe the flow in each streamtube section of the VAWT as affected by two consecutive actuator disks assuming unidirectional flow throughout the entire streamtube. A detailed discussion of the DMST scheme implemented in the streamtube model used can be found in Paraschivoiu[50, 51].

**Stall Model**

Dynamic stall is the dominant unsteady phenomenon affecting overall VAWT performance. Therefore, a great deal of effort has been invested into developing modifications to the original DMST model so that it can take into account these effects. When unsteady
effects are taken into account a hysteresis delay appears during large changes in angle of attack. Dynamic stall models, applied to the DMST model consist of a series of semi-empirical procedures applied in the calculation of the lift and drag coefficients of the VAWT blade. Therefore, applying a dynamic stall model to the DMST method requires that additional steps must be interlaced with the main algorithm. The stall models applied to the BEM model for analysis of VAWTs were by devised by Gormont\[21\] and later modified by Masse & Berg\[3\]. The Gormont method consists of applying a hysteresis delay ($\delta \alpha$) to the angle of attack. After applying this delay, the resulting reference angle of attack, $\alpha_{ref}$, behaves according to the simplified model,

$$\alpha_{ref} = \alpha - K \delta \alpha . \quad (2.6)$$

The constant, $K$, enforces the asymmetric behavior of the hysteresis delay,

$$K = \begin{cases} 1 & : \dot{\alpha} \geq 0 \\ -0.5 & : \dot{\alpha} < 0 \end{cases} . \quad (2.7)$$

When the absolute value of the angle of attack is increasing, the hysteresis delay is twice of that observed when it is decreasing, all other variables being the same in magnitude. The term $\delta \alpha$ is proportional to the non-dimensional rate parameter,

$$S = \sqrt{\frac{c \dot{\alpha}}{2U_{relative}}} . \quad (2.8)$$

$S$ can be interpreted as the square root of the velocity of the blade's leading or trailing edge as it rotates around its half chord point with respect to the relative wind velocity. This serves to take into account the blade's movement to the relative wind when calculating the magnitude of the hysteresis delay. The slope of $\delta \alpha$ vs $S$ decreases both with thickness over chord ratio and Mach number of the flow relative to the blade growth. Therefore, as the blade thickness with respect to the chordlength increases or the Mach number decreases, the hysteresis delay will drop as well. The full derivation for $\delta \alpha$ can be found in Gormont [21] or Paraschivoiu [50]. Once the delayed angle of attack has been found ($\alpha_{ref}$), the modified lift and drag coefficients can be calculated. The lift coefficient is calculated using the potential equation with a modified slope,

$$C_{L}^{dyn} = C_{L,0}(\alpha_{0}) + m(\alpha - \alpha_{0}), \quad (2.9)$$

where $\alpha_{0}$ is the angle of attack at zero lift, $m$ is the minimum value of either the slope of the linear portion of the lift curve or the value of $(C_{L}(\alpha_{ref}) - C_{L}(\alpha_{0}))/($$\alpha_{ref} - \alpha_{0})$. The drag coefficient is obtained simply by using $\alpha_{ref}$ to extract it from the available static data,

$$C_{D}^{dyn} = C_{D}(\alpha_{ref}) . \quad (2.10)$$

This method, so far, is purely Gormont's, which was designed for use in modeling helicopter rotor performance. Berg's [3] modification takes into account the fact that helicopter blades operate at much lower angles of attack than that of a VAWT, which typically operates
at angles of attack far higher than the stall angle of attack, $\alpha_{ss}$. Therefore, Berg theorized that Gormont’s method was likely to over-predict the lift and drag for high angles of attack. As a result, the following modification for both lift and drag$^1$ is introduced, so that both coefficients tend to the static values for very high values of $\alpha$, deep into the non linear regime,

$$C_{L, mod}^L = \begin{cases} C_L + \left[ \frac{A_M \alpha_{ss} - \alpha}{A_M \alpha_{ss} - \alpha_{ss}} \right] \left( C_{dyn}^L - C_L \right) & : \alpha \leq A_M \alpha_{ss} \\ C_L & : \alpha > A_M \alpha_{ss} \end{cases} . \quad (2.11)$$

The correction parameter, $A_M$, is set by the user. According to Berg, the best value for the correction parameter is $A_M = 6$. Paraschiviou, however, suggests that some VAWTs show a better response for $A_M \to \infty$, a special case that reduces the stall model to Gormont.

**Wake Model**

The streamtube model takes into account losses felt by the blades in the downstream half-cycle due to energy being extracted by the front half-cycle. However, this fails to fully capture the effects of the blades intersecting with discrete wakes, instead treating the flow as fully diffused and one-dimensional. As can be seen in Figures 2.2, the actual phenomenon plays a more dramatic role in influencing VAWT performance, particularly at higher tip-speed ratios. For azimuthal angles between $180^\circ$ and $360^\circ$, the effective angle of attack predicted by the streamtube model is significantly higher than that of the FVM simulation. The deviation is at its greatest near the top and bottom portions of the rear half-cycle, where the blade-wake interactions are concentrated. These interactions are not entirely understood, though localized flow separation and effective angle of attack variations have been observed, resulting in loss of lift and an increase of drag.

This necessitated the development of a wake model that could be used to correct for the effects of blade-wake interactions. First, the geometry of the wakes and the frequency of intersections between the wakes and blade path had to be modeled. For simplicity, a uniform relative velocity profile within the wake was assumed to be zero, and the width of each wake was modeled to be half the blade chord length. From vorticity magnitude plots (see Figure 2.8), the number of wakes that intersect with the blade path were counted and a roughly linear relationship was found with respect to tip-speed ratio and blade number,

$$n_{wakes} \approx 0.75 \ n \ TSR . \quad (2.12)$$

From this, the wake-length,

$$L_{\text{wake}} = c \ n_{\text{wake}} , \quad (2.13)$$

which represents the total distance the blade must transverse while interacting with wakes, may then be calculated. Once the geometry and wake-length have been taken into account, the effect of passing through these wakes must be understood. Since the relative velocity of the flow within the wake can be treated as negligible, the angle of attack within the wake drops to zero. These effects can be distributed evenly throughout the rear half-cycle by

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$^1$Only the lift modification was implemented.
Figure 2.9: Comparison of the effective angle of attack from the BEM models and the FVM simulation.
Figure 2.10: Comparison of the effective angle of attack from the BEM models and the FVM simulation at TSR = 3.0 for various solidities.
Figure 2.11: Average power coefficient with respect to tip-speed ratio for the FVM simulation, BEM model, and BEM model with blade-wake correction. The accuracy gained by the BEM model with wake correction is demonstrated at high tip-speed ratios.

taking the ratio of the wake-length with respect to half the circumference of the blade path in order to correct the angle of attack,

\[ \alpha_{\text{wake}}(\theta) = \left(1 - \frac{L_{\text{wake}}}{\pi R}\right) \alpha_{\text{ref}}(\theta) ; \quad 180^\circ < \theta < 360^\circ, \quad (2.14) \]

from which a new power coefficient for the rear half-cycle may be calculated.

As can be seen in Figures 2.2 & 2.10, the added wake correction significantly improves the accuracy of the streamtube model for predicting effective angle of attack when compared to the FVM simulation, particularly at higher tip-speed ratios and solidities. The lower angle of attack on the rear half-cycle due to wake corrections has different effects depending on the tip-speed ratio. At lower tip-speed ratios, the wake effects result in a lower power output. This is due to the fact that less power has already been extracted by the front half-cycle but cannot be extracted by the rear half-cycle, due to the lower angle of attack. At higher tip-speed ratios, the lower angle of attack results in less drag for the rear half-cycle and mitigating some of the losses. From these results, it is clear that by implementing a simple wake correction parameter, devised from the basic aerodynamic concepts, the accuracy of the streamtube model has been significantly improved for solidities above \( \sigma = 0.125 \).

2.3 Validation

The finite volume simulation was validated by comparing the average power coefficient curves with respect to tip-speed ratio to data extracted from other simulations by Castelli et
Figure 2.12: Average power coefficient with respect to tip speed ratio for (a) FVM simulations with high $Y_{+}^{+}$ wall treatments at the wall compared to the multiple-streamtube model and (b) for low $Y_{+}^{+}$ wall treatment. Plot (b) demonstrates considerable agreement between the simulations that rely on low $Y_{+}^{+}$ treatment along with the streamtube model.
There are three different approaches taken in these studies. Castelli and Mehrpooya simulated the VAWT aerodynamics using finite volume simulations with high $Y_{wall}^+$ treatment at the wall, which rely on wall functions rather than the governing equations to resolve the flow within the boundary layer. Mehrpooya also ran a FVM simulation with low $Y_{wall}^+$ treatment, which utilizes the Spalart-Allmaras turbulence model all the way to the wall surface.

As shown in Figure 2.12, all simulations share the same basic performance characteristics. For all cases except the BEM model, the average power coefficient peaks at a tip-speed ratio of TSR = 2.5, decreasing sharply as the tip-speed ratio decreases and falling more gradually as the tip-speed ratio is increased. The simulations with high $Y_{wall}^+$ treatment at the wall generally predict a higher average power coefficient at lower tip-speed ratios. The high $Y_{wall}^+$ treatment FVM simulations also vary significantly from one another, while the two low $Y_{wall}^+$ treatment simulations are more consistent, with less than 10% difference at lower tip-speed ratios. The BEM model provides the most qualitatively different average power coefficient curve, predicting the lowest power between tip-speed ratios of TSR = 1.5 and 2.5, then predicting a much higher power output than the others at tip-speed ratios above TSR = 2.5.

When different models predict qualitatively different performance and there is a dearth of experimental data to settle the issue, it becomes necessary to consider the critical assumptions of each computational model and the consistency between the various models. As mentioned previously, FVM simulations that utilize a high $Y_{wall}^+$ treatment at the wall, must rely on wall functions that are based on empirical observations, since the grid near the wall is not refined enough to resolve the boundary layer using the governing equations. The problem is that the wall functions are developed from cases where the flow is attached and fully developed, which makes wall functions unsuitable for predicting flow separation at high angles of attack [10]. Since the wall functions will replicate the fully attached boundary layer flow from which they were developed, it is reasonable to conclude that implementing these wall functions at high angle of attacks will have the effect of delaying stall and increasing the average power coefficient of the turbine. Combined with the fact that there is greater consistency between the two low $Y_{wall}^+$ treatment simulations, which rely on different methods of discretization (Mehrpooya's simulation is run using an unstructured, deformable grid), this supports the low $Y_{wall}^+$ wall treatment as the more accurate method of resolving the turbulent boundary layer for VAWTs operating at low tip-speed ratios.

2.4 BLADE-PITCH CONTROL SYSTEM

Improving turbine efficiency requires mitigation of power losses caused by blade-wake interactions and dynamic stall. In order to minimize drag and wake effects, a relatively low range of tip-speed ratios from TSR = 1.5 to TSR = 3.5 was chosen for optimization using blade pitch. Since the flow over the suction side of the blades becomes fully separated during the turbine cycle, the blade pitch must be adjusted in order to reduce the magnitude of the maximum angle of attack on the front side of the VAWT, below the static stall angle. If it is assumed that there are no limitations on the pitch of the blades during turbine operation, there are then an infinite number of ways to do this. Therefore, a procedure must then
Figure 2.13: Optimal angle of attack with respect to (a) tip-speed ratio as obtained from BEM studies and (b) azimuthal angle for an example $\alpha_{eff} = 10^\circ$ case.
be developed to optimize the blade pitch and, consequently, the effective angle of attack. Stall takes place at azimuthal angles of 150° and 180°, which represents the greatest loss in energy extraction. The lower angle of attack on the back end also results in significantly less torque produced during the back half-cycle. Since the goal was to lower the magnitude of the effective angle of attack on the front end, adding positive pitch was the first logical step in increasing the average torque for the turbine cycle.

**Brute Force Method**

If one assumes that the optimal angle of attack with respect to azimuthal angle is known, the variable pitch curve can be formulated by first subtracting the effective angle of attack from the desired angle of attack,

\[
\beta(\theta)_{i+1} = \alpha_{\text{optimal}}(\theta) - \alpha_{\text{effective}}(\theta)_i; \quad i = 0, 1, 2, \ldots, t_{\text{max}}.
\]  

Once the variable pitch has been implemented and the turbine is run until the aerodynamics returns to quasi-steady state, the effective angle of attack is measured again and a new variable pitch is computed using Equation 2.15. This iterating is necessary because any change to the pitch results in variation of the lift and drag forces, thereby altering the flow field. This dampening effect requires several iterations before an optimal pitch regime may be obtained.

Previous BEM studies conducted by Rempfer [59] strongly suggested that a constant angle of attack magnitude was most efficient for energy extraction from the flow. Since the target angle of attack must switch signs and little torque is produced near the point where the front and back half-cycles meet, a smooth sign change is desirable. Figure 2.13b shows the basic shape of the optimal effective angle of attack curve that should be produced by the variable pitch scheme. In order to find the optimal angle of attack for a given tip-speed ratio, the brute-force procedure was first applied to the streamtube model for various con-
Figure 2.15: Effective angle of attack compared to an optimal angle curve of $\alpha_{\text{optimal}} = 10^\circ$ for brute force iterations of $i = 1, 3, \text{ and } 6$ (TSR = 2.0).
stant angles of attack at several tip speed ratios. From this, the optimal angle of attack that maximized power output was found (see Figure 2.13a).

After six iterations, the power output of the turbine for the TSR = 2.0 case had increased by 40% (see Figure 2.14). As shown in Figure 2.15, the effective angle of attack gradually adjusted to more closely fit the optimal pitch curve. By the completion of the sixth iteration, the effective angle of attack conformed within a degree to the optimal pitch between azimuthal angles of $\theta = 0^\circ$ and $\theta = 75^\circ$, after which the effective angle of attack jumps to $\alpha_{\text{effective}} = -14^\circ$ and the blade stalls. Within the rear half-cycle, the blade pitch evidently had less effect and only resulted in a gradual smoothing of the angle of attack, likely due to wake effects and less kinetic energy within the flow available for extraction.

The blade-pitch curve for the $i = 6$ case (see Figure 2.16) was then implemented for the full range of tip-speed ratios. Figure 2.17 compare the turbine power output with blade-pitch implemented to the zero pitch case simulated using Star-CCM+ aside the BEM models with and without optimal pitch. Of note is not just significant improvement in the power output and efficiency, but also the dramatic extension of the turbine’s operational range, allowing the turbine to produce power at lower tip speed ratios. Figure 2.17 also compares the VAWT performance to the power curve of a commercially available horizontal-axis turbine, demonstrating that variable blade-pitch may allow VAWTs to perform competitively when shown next to HAWTs currently on the market.

While using a brute force method allowed for favorable improvements to overall turbine performance and delaying flow separation, there is still work left before the angle of attack can be controlled with precision. The jump in angle of attack at $\theta = 75^\circ$ can be explained by Gormont’s stall model, which predicts in Equation 2.7 that there will be a change in the direction of the hysteresis delay as $\dot{\alpha}_{\text{effective}}$ switches signs. This suggests that a feedback controller more accurately regulate the effective angle of attack.
Figure 2.17: Average power coefficients for various VAWT configurations and modeling methods compared to a commercially-available Mod-5B HAWT [64]
CONTROL SYSTEM DESIGN

Designing the control system began by defining the system using a set of governing equations. The input for the system was chosen to be $\dot{\beta}$, the pitch rate and $\alpha_{\text{effective}}$, the effective angle of attack, was chosen as the output. Once again, Gormont’s stall model,

$$
\alpha_{\text{effective}} = \alpha_{\text{geo}} - K\gamma \sqrt{\frac{c}{2} \frac{\dot{\alpha}_{\text{geo}}}{U_{\text{relative}}}},
$$

(2.16)

may be used to construct the transfer function for the plant. The geometric angle of attack, $\alpha_{\text{geo}}$, for a single blade starting at the $\theta = 0^\circ$ point may be defined in terms of the time and pitch angle,

$$
\alpha_{\text{geo}} = \beta - \tan^{-1}\left(\frac{U_x}{R \dot{\theta}}\right) \sin(\dot{\theta} t),
$$

(2.17)

where $\dot{\theta}$ is the rotation rate and $U_x$ is the streamwise velocity. If we treat the streamwise velocity and rotation rate as roughly constant in time, we may differentiate Equation 2.18 with respect to time,

$$
\dot{\alpha}_{\text{geo}} = \dot{\beta} - \dot{\theta} \tan^{-1}\left(\frac{U_x}{R \dot{\theta}}\right) \cos(\dot{\theta} t),
$$

(2.18)

and substitute into Equation 2.16,

$$
\alpha_{\text{effective}} = \beta - \dot{\theta} \tan^{-1}\left(\frac{U_x}{R \dot{\theta}}\right) \cos(\dot{\theta} t),
$$

(2.19)

Equation 2.19 may now be be put in terms of the input, $u$, output $y$, and taking the Laplace transform of each term separately,

$$
Y = \mathcal{L}\{y\} = \mathcal{L}\{\alpha_{\text{effective}}\},
$$

(2.20a)

$$
\frac{1}{s} U = \mathcal{L}\left\{\int_0^t u \, dt\right\} = \mathcal{L}\{\dot{\beta}\},
$$

(2.20b)

$$
-\tan^{-1}\left(\frac{U_x}{R \dot{\theta}}\right) \frac{\dot{\theta}}{s^2 + \dot{\theta}^2} = \mathcal{L}\{-\tan^{-1}\left(\frac{U_x}{R \dot{\theta}}\right) \sin(\dot{\theta} t)\},
$$

(2.20c)

including the last term by linearizing as a Taylor series expansion,

$$
K\gamma \sqrt{\frac{c}{2 U_{\text{relative}}} \left(u_o - \dot{\theta} \tan^{-1}\left(\frac{U_x}{R \dot{\theta}}\right)\right)} \frac{1}{s} + \frac{\dot{\theta}^3 \tan^{-1}\left(\frac{U_x}{R \dot{\theta}}\right)}{2 \left(u_o - \dot{\theta} \tan^{-1}\left(\frac{U_x}{R \dot{\theta}}\right)\right)} \frac{1}{s^3} \approx \mathcal{L}\left\{K\gamma \sqrt{\frac{c}{2 U_{\text{relative}}} \left(\dot{\beta} - \dot{\theta} \tan^{-1}\left(\frac{U_x}{R \dot{\theta}}\right) \cos(\dot{\theta} t)\right)}\right\}.
$$

(2.20d)

In order to compute the Taylor series expansion, $u_o$ is defined as the initial input condition where

$$
u_o > \dot{\theta} \tan^{-1}\left(\frac{U_x}{R \dot{\theta}}\right),
$$

(2.21)
must be satisfied. Since all the terms on the right side of the equation are independent of $U$, except for Equation 2.20b, the remaining terms may be treated as disturbance and may be neglected. This leaves the transfer function of the plant simply as

$$\left(\frac{Y}{U}\right)_{\text{plant}} = \frac{1}{s},$$  \hspace{1cm} (2.22)

resulting in a neutrally-stable system. In order to improve the system stability, the control transfer function,

$$\left(\frac{U}{E}\right)_{\text{control}} = \frac{k_1}{s + k_2},$$ \hspace{1cm} (2.23)

can be used to create a pole on the left-hand size of the z-plane \cite{49} and improve the stability of the system, with the parameters $k_1$ and $k_2$ to be determined.

Using MATLAB’s Simulink application \cite{44, 45}, the parameters were tuned so as to achieve a settling period of approximately $\Delta \theta = 10^\circ$ and an overshoot of less than $\text{PO} = 5\%$ (see Figure 2.19). The control system gain, $k_1$, was given special attention so as to limit the maximum pitch rate to $u \leq 10\text{rad/s}$ (see Figure 2.20). Based on these considerations, the optimal parameters were found to be $k_1 = 500,000$ and $k_2 = 1,000$ for a complete rotation of $\Delta t = 0.1798\text{s}$ at TSR = 2.0.

A challenge presented itself in that Star-CCM+ has no built-in means to implement a control system. However, it proved relatively easy to convert the control system as designed into a discrete form using finite-difference approximation. Starting with the equation describing the Laplace transform of the input, $U$, as a function of the error, $E$,

$$U = \frac{k_1}{s + k_2} E,$$ \hspace{1cm} (2.24)

the denominator on the right-hand side may be multiplied and distributed on the left-hand side of the equation,

$$sU + k_2 U = k_1 E.$$ \hspace{1cm} (2.25)

The inverse Laplace transform is applied to Equation 2.25,

$$\dot{u} + k_2 u = k_1 e,$$ \hspace{1cm} (2.26)

resulting in an ordinary differential equation that must now be discretized. For the first time-step, only two points are available. Therefore, $\dot{u}$ must be approximated using the first-order accurate, two-point backward difference equations,

$$\dot{u}_1 = \frac{u_1 - u_o}{\Delta t}$$ \hspace{1cm} (2.27a)
Figure 2.19: Control system response for a step disturbance.
Figure 2.20: Control system response for a sinusoidal disturbance.
and
\[ u_1 = \frac{\Delta t \ k_1 \ e_1 + u_o}{\Delta t \ k_2 + 1} \] (2.27b)

The second-order accurate, three-point backward difference equations may be used for all other time-steps,
\[ \dot{u}_i = \frac{u_{i-2} - 4 \ u_{i-1} + 3 \ u_i}{2 \ \Delta t} \] (2.28a)
and
\[ u_i = \frac{2 \ \Delta t \ k_1 \ e_i - u_{i-2} + 4 \ u_{i-1}}{2 \ \Delta t \ k_2 + 3} \] (2.28b)

These equations were then programmed into Star-CCM+ via user-defined field functions, allowing the simulation to compute a new blade-pitch rotation rate for each time-step as a function of the error,
\[ e = \alpha_{\text{optimal}} - \alpha_{\text{effective}} \] (2.29)
and stored values for \( u \) from previous time-steps. Additionally, a maximum pitch rate magnitude was imposed (\( u \leq 10^\circ \)) so as to keep the value within realistic limits. There is also a real-world justification for this since server motors typically are speed-limited.

**Preliminary Results**

As a proof-of-concept, the blade-pitch control system was implemented for a single-blade VAWT operating at TSR = 2.0 and only for the front half-cycle. For azimuthal angles between \( \theta = 180^\circ \) and \( \theta = 360^\circ \), the pitch was programmed to return to zero. The effect of the feedback controller on the turbine aerodynamics responded quickly, settling into quasi-steady behavior within a single rotation. Figure 2.21 shows the moment coefficient compared to the azimuthal angle for the controlled blade-pitch and zero-pitch cases,

![Figure 2.21: Moment coefficient compared to azimuthal angle for two VAWTs, with and without blade-pitch control.](image-url)
Figure 2.22: Effective angle of attack curves compared to the optimal angle of attack for a full revolution.
Figure 2.23: Effective angle of attack curves compared to the optimal angle of attack for the front half-cycle only.
Figure 2.24: Control system input (a) and blade pitch (b) with respect to the azimuthal angle for the front half-cycle.
with the moment curve significantly smoothed and of higher average magnitude. The average power coefficient rose dramatically for the variable pitch case from $C_p = 0.093$ to $C_p = 0.213$, an improvement of over 127%.

Figures 2.22 and 2.23 compare the effective angle of attack for the zero-pitch case to that of the variable pitch case. As the blade passes through the front half-cycle, the effective angle of attack matched the optimal angle of attack curve to within $e \leq 0.25^\circ$ for the majority of the half-rotation from $\theta = 15^\circ$ to $\theta = 120^\circ$. The spans of higher error ($e \leq 4^\circ$) can be attributed to two different causes: (1) The high error in the $0^\circ \leq \theta \leq 15^\circ$ range can be attributed to transient behavior of the control system, since the pitch was fixed throughout the rear half-cycle. Therefore, the control system response required a settling time before the error could be reduced significantly. (2) The error in the $120^\circ \leq \theta \leq 180^\circ$ range was likely caused by the onset of dynamic stall due to partial flow separation on the suction-side of the turbine blade. While the optimal angle of attack was shown to be $\alpha_{\text{optimal}} = 13^\circ$ for the TSR = 2.0 case using the BEM model (see Figure 2.4), the Gormont stall model predicts 6 delay but not unsteady flow separation. Despite the fact that feedback control was implemented only on the front half-cycle, the effective angle of attack was smoothed throughout the rear half-cycle and benefited from less prominent wake structures convected downstream.

Lastly, the implementation of a control system within an FVM simulations requires certain considerations. While the only limit to the pitch rate, $u$, is the overset slip condition number ($C_o/u \leq 1$), the simulation became numerically unstable if higher-order derivatives of the input ($\dot{u}$, $\ddot{u}$, etc...) became too large. This behavior can be explained by the SIMPLE algorithm employed within the Star-CCM+ software to solve the governing equations. Since the SIMPLE algorithm is 2nd-order accurate in time, the gain for the feedback control must be restricted more than would be necessary for real-world applications.
3 Future Work

At this point in the research, the framework has been laid and many of the fundamental questions have been answered. Finite-volume simulations and streamtube models have been completed and validated, necessary for the accurate prediction of VAWT performance and optimization studies. Using these simulations and models, performance-limiting flow phenomena, such as dynamic stall, hysteresis delay, and blade-wake interactions have been identified and understood. This knowledge provided the basis and justification for several variable blade-pitch schemes that may be utilized to mitigate unfavorable aerodynamic effects and improve energy extraction. Two such schemes were investigated, including a brute force method and a dynamic feedback system for controlling blade-pitch, with each providing an additional 40% and 127% power increase, respectively.

From here, the next immediate tasks will be to complete and optimize the control system design and implementation for the single-bladed VAWT, eliminating dynamic stall and expanding the feedback control to the rear half-cycle. Once the control system has been fully implemented for that case, the turbine configuration may then be changed to include two and three blades. Additional case studies will be completed including trade studies in order to find the optimal configuration for maximal efficiency. Lastly, the control system will need to be proven while operating in real-world conditions, including time-variant tip-speed ratios and turbulent flow fields.

3.1 Control System Improvement

The first step toward refining the blade-pitch control system is to finish the optimization of the feedback control for the TSR = 2.0, single-bladed VAWT. The goal should be to ensure that the effective angle of attack conforms completely to the chosen optimal angle of attack curve. Most importantly, the trailing-edge flow separation that takes place at the end of the front half-cycle must be entirely eliminated. To accomplish this, the optimal angle of attack must be adjusted to a lower value than the current magnitude of $\alpha_{\text{optimal}} = 12^\circ$. This will bring the effective angle of attack well below the static stall angle, preventing any loss of lift and reducing the error values within azimuthal angles of $120^\circ \leq \theta \leq 180^\circ$. The transient that results in high error for azimuthal angles between $0^\circ \leq \theta \leq 15^\circ$ can be eliminated by implementing the pitch control system throughout the entire cycle.

Expanding the scope of the control system to function through the rear half-cycle is complicated by the insensitivity of the effective angle of attack to pitch adjustments, as demonstrated by the results of the brute force procedure (see Figure 2.15). The causes for this insensitivity are both the lower kinetic energy density within the airflow that passes through the rear half-cycle and the buffeting of the blade by wakes as they are convected downstream. While the previous BEM models that were used to justify a constant optimal angle of attack could accurately predict the former, they did not include a wake correction model to account for the latter. Equations 2.12, 2.13, and 2.14 from the wake correction model may then be used to calculate a new optimal angle of attack for the rear half-cycle that is a
Table 3.1: Optimal Angle of Attack

<table>
<thead>
<tr>
<th>TSR</th>
<th>Front Half-Cycle</th>
<th>Rear Half-Cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>11 10.34 9.68</td>
<td>9.02</td>
</tr>
<tr>
<td>2.0</td>
<td>11 10.12 9.24</td>
<td>8.36</td>
</tr>
<tr>
<td>2.5</td>
<td>11 9.90 8.80</td>
<td>7.71</td>
</tr>
<tr>
<td>3.0</td>
<td>11 9.68 8.36</td>
<td>7.05</td>
</tr>
<tr>
<td>3.5</td>
<td>8 6.88 5.76</td>
<td>4.65</td>
</tr>
</tbody>
</table>

function of the front end optimal angle, the tip-speed ratio, and number of blades,

$$\alpha_{\text{optimal/wake}}(\theta) = \left(1 - \frac{L_{\text{wake}}}{\pi R}\right) \alpha_{\text{optimal}}(\theta) ; \quad 180^\circ < \theta < 360^\circ \quad (3.1)$$

The effect of this correction will be to reduce the optimal angle of attack as the buffeting of the blades on the rear half-cycle increases, since pitch adjustments are unlikely to mitigate this behavior. Table 3.1 shows the new optimal angles for the front half-cycle as well as the rear half-cycle optimal angles for various tip-speed ratios and blade numbers.

### 3.2 Additional Control Cases

Once the control system has been fully implemented and refined for the baseline configuration, a series of trade studies may be conducted, running FVM simulations in order to investigate the system response to various configurations and optimize the turbine’s power output with respect to several important parameters. The most important of these is the tip-speed ratio, of which there are five different values ($n_{\text{TSR}} = 5$). Another parameter is the number of blades, which may include one, two, and three-bladed VAWTs ($n_{\text{blades}} = 3$). If those were the only parameters of importance, then the total number of simulations that would be needed in order to test every configuration would be,

$$n_{\text{TSR}} * n_{\text{blades}} = 15 \text{ simulations} \quad (3.2)$$

However, each of these configurations will likely require several runs to test various optimal angle of attack values ($n_{\alpha_{\text{max}}}$) which will further raise the total number of simulations.

Table 3.2: Number of Test Values

<table>
<thead>
<tr>
<th>$n_{\text{TSR}}$</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_{\text{blades}}$</td>
<td>3</td>
</tr>
<tr>
<td>$n_{\alpha_{\text{max}}}$</td>
<td>$\approx 5$</td>
</tr>
<tr>
<td>$n_{u_{\text{max}}}$</td>
<td>$\approx 5$</td>
</tr>
<tr>
<td>$n_{\text{unsteady}}$</td>
<td>$\approx 20$</td>
</tr>
</tbody>
</table>
needed to be run. If additional runs are considered to investigate the effects of limiting the input value below the maximum required values \( (n_{\text{max}}) \) along with simulations where the VAWT is subjected to unsteady disturbances, this brings the total number of simulations \( (n_{\text{unsteady}}) \) to,
\[
 n_{\text{TSR}} \times n_{\text{blades}} \times n_{\text{max}} + n_{\text{max}} + n_{\text{unsteady}} \approx 100 \text{ simulations} .
\] (3.3)

Table 3.2 give the number of the different variations of test cases. Some of these values are approximate estimates and marked accordingly.

Conducting a full case study by varying the number of blades, solidity, and the tip-speed ratio allows the limits of operation of the control system and the overall performance of the VAWT to be measured. The last set of cases will be to evaluate the VAWT’s performance and the control system response to be evaluated under real-world conditions, where the wind speed may vary with time. Since the angle of attack is dependent on the free-stream velocity and there will be a time-delay required for the disturbance to be convected downstream from the inlet, the wind speed will be averaged between two probes roughly 1.5 diameters from the turbine’s center in the span-wise direction. Two different methods will be used to produce the free-stream disturbance: (1) A simple sinusoidal disturbance can be used to model periodic gusting behavior and (2) a random disturbance that can be approximated as a Fourier series,
\[
 \frac{U}{U_o} = \sum_{i=0}^{8} \left( a_i \cos(i \omega \theta) + b_i \sin(i \omega \theta) \right) ; \quad \omega = 1.20 ,
\] (3.4)

where the coefficients, \( a_i \) and \( b_i \), are given in Table 3.3. Figure 3.1 shows what such a disturbance might look like. In order to ensure that the VAWT remains within its normal operating range, the disturbance will produce a tip-speed ratio varying within a range of 2.0 \( \leq \text{TSR} \leq 3.0 \).

<table>
<thead>
<tr>
<th>( i )</th>
<th>( a_i )</th>
<th>( b_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.84458</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>0.03769</td>
<td>0.01378</td>
</tr>
<tr>
<td>2</td>
<td>0.00836</td>
<td>0.00620</td>
</tr>
<tr>
<td>3</td>
<td>0.00272</td>
<td>-0.00291</td>
</tr>
<tr>
<td>4</td>
<td>-0.00088</td>
<td>-0.01758</td>
</tr>
<tr>
<td>5</td>
<td>-0.03301</td>
<td>0.01535</td>
</tr>
<tr>
<td>6</td>
<td>-0.00557</td>
<td>-0.08566</td>
</tr>
<tr>
<td>7</td>
<td>-0.04537</td>
<td>0.01634</td>
</tr>
<tr>
<td>8</td>
<td>-0.00496</td>
<td>0.00161</td>
</tr>
</tbody>
</table>
Figure 3.1: Proposed tip-speed ratio variations to study the control system's response to random disturbances.
3.3 COMPUTATIONAL RESOURCES AND SIMULATION RUN-TIME

Two workstations and part of a high-performance computing cluster will be dedicated to running FVM simulations as well as post-processing of data. The workstations each make use of a 8MB Memory Cache Quad Core Intel i7-4790 hyper-threaded processor (3.60GHz clock-speed) and 32GB of DDR3 1600MHz (4x8GB) RAM. "Karlin", the computer cluster that will be available for use, has one master node with two 8MB Cache Quad Core Xeon processors with 12GB Memory (6x2GB) and 16 nodes with two 8MB Cache Quad Core Xeon processors (2.4GHz clock-speed) with 24GB RAM (12x2GB). From previous experience, each simulation can be estimated to require approximately 100 CPU hours (using the workstations as a benchmark) to complete. Assuming that only the two workstations (or the equivalent) are available at any time for computation, the amount of time required to run the total number of simulations calculated in Equation 3.3 is calculated to be

\[
\frac{10,000 \text{ CPU hours}}{2 \text{ workstations} \times 8 \text{ threads}} = 625 \text{ hours} = 26 \text{ days}.
\]  

(3.5)

This however is a conservative estimate, since additional resources including "Karlin" will likely be available for at least some portion of the ongoing research.
REFERENCES


