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Abstract

This paper studies the impact of behavioral and non-behavioral information contained in consumer credit reports on the credit market. A repeated game of incomplete information between a borrower and a sequence of lenders is considered. Lenders' beliefs regarding the borrower's type depend on the borrower's historical loan performance and other informative signals. It is established that for low beliefs, strategic defaults occur in equilibrium as the borrower defaults on his loans despite having the ability to repay them. Further, as the ancillary signals become increasingly informative of the borrower's type, the range of beliefs over which strategic defaults occur expands. This increase in strategic defaults highlights an unintended negative effect of allowing informative signals to influence beliefs. On the upside, the informative signals allow the lenders to learn the borrower's type more quickly.

Keywords: Credit bureaus, Information, Signals, Repeated Games

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1 Introduction

Credit markets are plagued with information asymmetries that create incentives for a borrower to behave opportunistically. The risk of opportunism can lead to some undesirable market outcomes, such as an inefficient allocation of resources and complete market failure (Akerlof, 1970; Atakan and Ekmekci, 2014; Ausubel, 1991; De Meza and Webb, 1987; Jaffee and Russell, 1976; Mankiw, 1986; Stiglitz and Weiss, 1981). The borrower’s self-interest seeking behavior can be kept under check if his current actions have an effect on his future as well (Andreoni and Miller, 1993; Bó, 2005; Kreps et al., 1982).

It is easy to link the borrower’s current loan performance to the future if the same lender always meets his recurring need for funds. Although this was common in earlier times when credit markets were underdeveloped, it is unlikely to happen in today’s market where a typical American owns an average of 2.6 credit cards with different banks.¹ In such a fragmented market credit bureaus, such as Experian and TransUnion, play a critical role in ensuring that the present behavior of the borrower will affect his future. Lenders report their respective borrowers’ loan performances to these credit bureaus which, in turn, disseminate this information to other prospective lenders as a part of the borrower’s credit report.

In addition to containing information on the borrower’s current and past loan performances, the credit report also contains other information, such as past bankruptcies, public records, length of credit history, recent credit inquiries and lawsuits.² Arguably, all this information contained in the credit report is useful in predicting the borrower’s probability of default (Arya et al., 2013; Chatterjee et al., 2008). Furthermore, credit bureaus synthesize this information to calculate a credit score that is viewed as a measure of the borrower’s creditworthiness.³ Evidence indi-
cates that potential lenders place a great deal of importance on credit reports while scrutinizing loan applications and immediately reject applications with bad credit scores (Arya et al., 2013). This suggests that lenders base their decisions on information beyond the historical loan performance of the borrower. Motivated by this observation, this paper studies how various kinds of information utilized by lenders affect the credit market.

Consider a finitely repeated dynamic game of incomplete information between a borrower and a sequence of lenders. In every period of the game, the borrower seeks funding for an independent and risky project. The project’s probability of success is privately known to the borrower. Depending upon this probability, the borrower can be labeled as either a high type or a low type borrower. In comparison to the low type borrower, the high type borrower is more likely to succeed in his projects. A successful project yields a sufficiently high return to the borrower and permits him to make strategic choices. An unsuccessful project forces the borrower to non-strategically default. Adverse selection arises due to the borrower’s private information regarding his type or the likelihood of the project’s success. Moral hazard arises as the borrower privately observes the project’s outcome and lenders cannot distinguish a strategic default from a non-strategic default. In every period of the game, a non-strategic rating agent provides the lender with the borrower’s historical loan performance and a history of imperfectly informative signals. The rating agent also provides a rating based on this information that coincides with the lender’s beliefs about the borrower’s type.

This paper finds that the borrower with a successful project adheres to the following strategy in equilibrium: despite having the ability to repay his loan, he strategically defaults when his rating falls below a certain threshold. Low ratings in the current period are associated with low ratings in the future as well. As interest rates are high for low ratings, the borrower, being a residual claimant, receives a low payoff. For low ratings, high interest payments in the current period and
smaller potential future gains from repayment motivate the borrower to default. As ratings increase, the interest rate decreases which translates to lower costs and increased benefits of repayment. Furthermore, due to a lower likelihood of success, the low type borrower has expected future benefits which are smaller than those of the high type borrower, and as a result the low type borrower chooses default over a wider range of ratings.

Next, this paper addresses the question of how exogenous informative signals affect the behavior of different market participants. It is found that the aforementioned repayment threshold increases with the degree of informativeness of signals. This increase happens because more informative signals weaken the impact of the behavioral history on lenders’ beliefs and on the interest rates which, in turn, lowers the borrower’s expected net gains from repayment. In response, the borrower becomes unwilling to bear very high interest costs and ends up defaulting on his loans over a wider range of ratings. If lenders’ beliefs are based on the history of either loan performance or informative signals, they eventually learn the borrower’s type. If lenders’ beliefs are based on both the history of loan performance and informative signals, it hastens their learning of the borrower’s type and allows them to weed out the low type borrower more quickly.

Unsurprisingly, the rise in strategic defaults associated with informative signals translates to either a higher interest rate or no lending over certain ranges of ratings. Both a higher interest rate and credit rationing hurt the borrower. This combined with the faster dissipation of information asymmetry confirms that additional informative signals have negative welfare implications for the low type borrower. However, the rise in the number of strategic defaults by the low type borrower presents an additional opportunity for the high type borrower to separate himself from the low type and maximize his future returns. This opportunity comes at an expense of a higher interest payment in the current period. Depending upon the relative magnitudes of current cost and future gains, the high type
borrower may be better or worse off for these ratings. Hence, the welfare impact of ancillary signals on the high type borrower is ambiguous.

From the lenders’ perspective, although this ancillary information speeds up their learning about the borrower’s type, competition prevents them from earning any ex-ante surplus from their learning. While lenders may receive a higher interest rate ex-post, they also observe more defaults. Consequently, these ancillary signals may not always be beneficial to lenders either.

The key contribution of this paper lies in identifying an undesirable consequence of using more information in making decisions. In the setup presented in this paper, although more informative signals are beneficial for resolving the problem of adverse selection, they also increase the incidence of moral hazard as the strategic borrower responds to the precise nature of information used by lenders. This finding highlights the need for re-evaluating the marginal value of non-behavioral information used by creditors in their decision-making.

1.1 Related Literature

This paper is most closely related to the literature on the impact of repeated interactions on the players’ behavior. If the actions of a player have additional implications for his future, it incentivizes him to behave “nicely” (see e.g., Behr and Sonnekalb, 2012; Brown and Zehnder, 2007; Brown et al., 2009; Chatterjee et al., 2011; Cole et al., 1995; Diamond, 1989; Gehrig and Stenbacka, 2007; Jappelli and Pagano, 2006; Pagano and Jappelli, 1993; Vercammen, 1995). Similar disciplinary effects arise for the borrower in this paper as well.

Many times players have privately known characteristics or a type that affects their performance. Others may discover a player’s type by observing either his behavioral history (see e.g., Atakan and Ekmekci, 2012, 2013; Cripps et al., 2005; Fudenberg and Levine, 1989) or various exogenous informative signals (see e.g.,
Cripps et al., 2008; Ekmekci, 2011; Wiseman, 2009). This paper extends this stream of literature by allowing the discovery of the borrower’s type to depend on both his behavioral history and additional exogenous informative signals.

In this respect, this work is most closely related to Padilla and Pagano (2000). In their paper, a low type borrower always defaults, but a high type borrower can alter the risk of default by choosing a non-contractible costly effort, which affects the likelihood of his project’s success. Although lenders are ex-ante identical, a lender learns the borrower’s type after one interaction and gains an ex-post advantage. Padilla and Pagano (2000) compare the high type borrower’s effort levels under different information-sharing regimes. When lenders share loan performance only and a default is observed, competing uninformed lenders cannot distinguish between the two types of the borrower. Because of being pooled together with the low type borrower, the high type borrower who defaults is offered a higher interest rate by competing lenders which, in turn, allows an informed lender to quote a higher interest rate to the borrower as well. Anticipated higher interest payments in the future extort a higher effort from the high type borrower. When lenders share information on loan performance as well as the borrower’s type, all lenders become ex-post identical. Enhanced competition forces lenders to extend interest rates based on the borrower’s type only. As the borrower’s current loan performance ceases to have any impact on his future, he reduces his effort on the project.

Instead of focusing on information sharing, as Padilla and Pagano (2000) do, this paper focuses on the marginal impact of using non-behavioral information. In this paper’s model, an information intermediary ensures that in any period all lenders have identical information and previous interactions with the borrower are not a source of competitive advantage. It is found that in comparison to a case where lenders base their decisions on repayment history only, more strategic defaults occur when lenders utilize both the repayment history and exogenous signals. As a result, in an attempt to reduce adverse selection, lenders themselves
reduce the weights given to historical loan performance and end up weakening the borrower’s repayment incentives. Another somewhat minor distinction lies in this paper’s focus on *ex-post* default behavior, instead of *ex-ante* effort levels.

This paper also relates to the literature on marginal value of information in repeated games with imperfect monitoring where signals are informative about hidden actions. One stream of literature studies, under varying assumptions about signal structures, how improved observability of historical actions affects the set of possible equilibria and achievable payoffs (see e.g., Atakan and Ekmekci, 2015; Ekmekci et al., 2012; Fudenberg and Levine, 1994; Kamada and Kominers, 2010; Kandori, 1992; Kandori and Obara, 2006; Sugaya and Wolitzky, 2014). In contrast, the signals in this paper are informative not of actions but of hidden characteristics. In this respect, the framework of this paper relates closely to that of Wiseman (2009) who, however, characterizes the bounds on equilibrium payoffs when actions can be perfectly observed and signals are informative of a player’s hidden characteristics. Some of the recent papers in this area study the impact of restricting the uninformed players’ access to the informed player’s history of observable actions by limiting record-keeping capacities (Liu and Skrzypacz, 2014), costly information acquisition (Liu, 2011), or information censoring by only providing a summary statistic for history (Dellarocas, 2011; Doraszelski and Escobar, 2012; Ekmekci, 2011). These papers find that limiting short-run players’ access to the long-run player’s behavioral history elevates the significance of his recent performance on his future and provides continual disciplinary incentives. Instead of focusing on limiting the history, this paper focuses on the nature of information provided in the history. The results are driven by the fact that access to ancillary information reduces the signaling value of behavioral history, thereby weakening the borrower’s repayment incentives. Reliance on the history eliminates adverse selection in Cripps et al. (2004, 2007), Vercammen (1995), and Wiseman (2008) as well. The distinguishing characteristic of this paper is simultaneous inclusion of
both behavioral history and exogenous information, which more quickly reveals the borrower’s type to lenders.

Lastly, this paper also contributes to the vast literature on the drivers of strategic defaults, whereby a borrower chooses to default on his loans despite having the ability to repay them. The theoretical literature on this topic is fairly limited and identifies exogenous shocks (see e.g., Campbell and Cocco, 2011; Deng et al., 2000) and variations in external environmental factors (see e.g., Athreya et al., 2012; Guiso et al., 2013; Karlan, 2005) as the key determinants of strategic defaults. In contrast, this paper identifies the widely popular organizational practice of acquiring and using an increasing amount of information about the borrower as a determinant of strategic defaults.

The remainder of this paper is organized as follows. Section 2 describes the model, and the equilibrium is characterized in Section 3. Section 4 analyzes the impact of information on the market. Section 5 offers a conclusion. All the proofs are provided in the Appendix.

2 Model

Consider a finitely repeated game where a given borrower sequentially interacts with T lenders.\(^4\) In every period \(t \leq T\), the borrower has access to a one-period project requiring an investment of $1. He has no monetary endowment and relies exclusively on external sources for funding.\(^5\) A funded project yields a privately observed payoff of \(Y_P \in \{M, 0\}\). If \(Y_P = M\), the project is deemed a success. If \(Y_P = 0\), the project is a failure.\(^6\) The probability for the project’s success is \(\theta_j \in \{\theta_H, \theta_L\}\)

\(^4\)This is similar to reputation games studied in Diamond (1989), Vercammen (1995), and their successors.

\(^5\)The output from the project cannot be stored and transferred inter-temporally; hence the borrower is dependent on the unsecured credit market for all T periods.

\(^6\)Though it is assumed that a failed project yields nothing, it can equivalently be assumed that the failed projects yield a positive but sufficiently low payoff as long it forces the borrower to non-
where $\theta_{H(\text{igh})} > \theta_{L(\text{ow})}$. $\theta_j$ corresponds to the privately known type of the borrower which remains unchanged across periods.\(^7\) The common prior belief that $\theta_j = \theta_H$ is $q \in (0, 1)$. If $Y_P = M$, the borrower’s action set is $\{\bar{d}, d\}$ where $\bar{d}$ denotes repayment and $d$ denotes default. For $j \in \{H, L\}$, the $\theta_j$ borrower’s strategy is denoted by $r_j^t$ reflecting his probability of repayment in period $t$. Otherwise, if $Y_P = 0$, the borrower’s action set is $\{d\}$ and he non-strategically chooses default. Let $c^t$ denote the borrower’s observed choice in period $t$. Although $Y_P$ is privately observed, the borrower’s choice $c^t \in \{d\}$ is observable to both the current lender and a rating agent. In the event that the project goes unfunded, $c^t \equiv \emptyset$. The borrower is the residual claimant of the returns from the project.

The rating agent is a non-strategic player and his payoff is exogenously determined in every period.\(^8\) Although $c^t$ is observable to the rating agent, $Y_P$ is not. Accordingly, upon observing $c^t = d$, the rating agent cannot identify the underlying driver of the borrower’s choice. In addition to $c^t$, the rating agent observes an exogenous signal $s^t \in \{s_H, s_L\}$ that is imperfectly informative of the borrower’s type $\theta_j$. If $\theta_j = \theta_H$ (resp. $\theta_L$), then a signal $s^t = s_H$ (resp. $s_L$) is observed with probability $p_s \in [0.5, 1]$.\(^9\) A higher value of $p_s$ is interpreted as signals being more informative. Let $H^t = \bigcup_{t=1}^t \{s^t, c^{t-1}\}$ denote the history at period $t$ and $H^t$ denote the set of all possible histories at period $t$. In every period, the rating agent issues

\(^7\)\(\theta_j\) lends itself to multiple interpretations. It can reflect the riskiness of available projects, where the $\theta_H$ borrower has access to less risky projects. Alternatively, it can be the borrower’s discount factor, where the $\theta_H$ borrower is the more patient one. It can also be interpreted as a measure of cost of effort that the borrower can put into the project with the $\theta_H$ borrower incurring lower costs of expending effort and hence finding it optimal to exert more effort, thereby increasing the likelihood of a successful project.

\(^8\)This formulation is consistent with the fixed-fee model adopted by the U.S. credit bureaus where credit reports are provided in lieu of a predetermined fixed fee.

\(^9\)Signals are equivalent to the ancillary information contained in the credit reports such as public records, credit limits on various accounts and past requests for credit histories. If $p_s = 0.5$, then these signals are completely uninformative of the borrower’s type and will not have any informational value. This scenario will coincide with the one where no signals are observed. If $p_s = 1$, then signals fully reveal the borrower’s type. If $p_s \in [0, 0.5)$, then each signal $s_i$ suggests that the probability of the borrower being type $\theta_j$, $j \neq i$ is $\rho_i$, where $i, j \in \{H, L\}$.
a rating $R_t \in [0, 1]$ based on $H^t$ and provides it to the lender along with $H^t$.\textsuperscript{10} $R_t$ is interpreted as the likelihood that the borrower’s type is $\theta_H$ and coincides with the lender’s belief about the same.\textsuperscript{11,12}

Similar to Diamond (1989), lenders are modeled as anonymous creditors in a perfectly competitive market and not as a single specific creditor. Accordingly, each lender lives for only one period.\textsuperscript{13} In each period, the lender is endowed with $1$, which can either be invested for a risk-free return of $i^f \geq 0$ or lent to the borrower for a perfectly competitive return of $i^t$. The lender’s strategy is denoted by a binary variable $l^t$, which equals 1 if lending occurs in period $t$ and is otherwise 0. If $l^t = 1$ and $c^t = d$ occur, then the lender’s payoff is $Y_l = 1 + i^t$. If $l^t = 1$ and $c^t = d$ arise, then the lender’s expected payoff is $Y_l = \beta Y_P$. Although $Y_P$ is ex-ante privately observed by the borrower, upon viewing a default the lender can costlessly and privately use an enforcement mechanism which, with probability $\beta$, can confiscate and transfer $Y_P$ to the lender before the borrower can consume it. $\beta$ can be interpreted as the effectiveness of debt-enforcement remedies that are legally available to lenders when the borrower defaults on his loan.

In addition, the following simplifying assumptions are made:

\textsuperscript{10}$R_t$ is analogous to the credit scores issued by the U.S. credit bureaus. Similar to the credit bureaus, the rating agent provides both $H^t$ and $R_t$ to the lenders.

\textsuperscript{11}In the model, ratings are used for predicting the borrower’s type. Lenders utilize the rating for further predicting the likelihood of repayment. This is in slight variation from the U.S. credit markets where ratings (or credit scores) directly reflect the likelihood of repayment. The model can easily be adapted to capture this variant by redefining ratings as $R'_t \equiv R_t \theta_H E_r^{H,t} + (1 - R_t) \theta_L E_r^{L,t}$, where $E_r^{j,t}$ is the expected probability of repayment by the $\theta_j$ borrower for $j \in \{L, H\}$. The results of the model will continue to hold.

\textsuperscript{12}Instead of solely relying on information provided by the credit bureaus, lending institutions sometimes conduct their own surveys, audits, etc. to collect more information and calculate scores using their own credit-scoring models. In that case, although lenders’ beliefs would be different from the credit scores provided by the credit bureaus, all the results would continue to hold for given beliefs.

\textsuperscript{13}The assumption of short-lived lenders explicitly rules out any long-term relationships between borrowers and lenders, thereby preventing any one lender from gaining informational advantages over his competitors due to its previous interactions with the borrower. It is a reasonable assumption for a credit market (such as the U.S. credit market) where borrowers can conveniently switch lenders and information intermediaries, such as credit bureaus, streamline the flow of information, making it difficult for any one lender to earn informational rents.
Assumption 1(a): $\theta_L M > 1$.

Assumption 1(b): $\theta_L (M - 1 - i^f) > 1 - \beta$.

Together, assumptions 1(a) and 1(b) are sufficient conditions to ensure that in every period the investment decision of the borrower is independent of his privately known type and the equilibrium strategies for both types of the borrower. If these assumptions hold, then both types of the borrower will always find it optimal to invest in their projects.\(^1\) Assumption 1(a) imposes a lower bound on $\theta_L$ and also implies that it is socially optimal to invest. Assumption 1(b) imposes an upper bound on permissible interest rates. Such an exogenous bound on interest rates can be understood as usury laws put in place to guard the borrower by making it illegal for lenders to charge extremely high interest rates.

Assumption 2(a): $\theta_H \beta M > 1 + i^f$.

Assumption 2(b): $1 + i^f > \theta_L \beta M$.

Together, assumptions 2(a) and 2(b) state that if the borrower will choose default, lenders will find it optimal to lend when $\theta_j = \theta_H$, but not when $\theta_j = \theta_L$. Assumption 2(a) imposes a lower bound on $\beta$. That is, it requires that $\beta > \beta \equiv \frac{1 + i^f}{\theta_H M} > 0$ and the borrower faces a non-negligible threat of punishment in every period. This assumption is necessary to prevent the game from unraveling. Assumption 2(b) imposes an upper bound on $\theta_L$. This implies that the two types of the borrower are sufficiently different from each other so that lenders have an interest in distinguishing between them.

The timing and payoffs of the stage game are presented in Figure 1. At the beginning of each period, the rating agent observes an imperfectly informative

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\(^{14}\)The analysis of this paper can easily be modified to endogenize investment decisions and make them dependent on the borrower’s type and equilibrium strategies for both types of the borrower. Allowing for this possibility would result in higher interest rates and less frequent lending in equilibrium. The main result of this paper will continue to hold.
signal \( s^t \in \{ s_H, s_L \} \). Based on \( H^t = \bigcup_{\tau=1}^{t} \{ s^\tau, c^{\tau-1} \} \), the rating agent issues a rating \( R_t \) and communicates it to the lender along with \( H^t \). Upon observing \( H^t \) and \( R_t \), the lender decides whether to lend at a competitive interest rate \( i^t \). If \( l^t = 0 \), then the lender receives \( Y_l = 1 + i^t \), \( c^t \equiv \emptyset \), the borrower receives 0 and the period ends. If lending occurs, \( Y_P \) is privately learned by the borrower, who then chooses \( c^t \in \{ d, \overline{d} \} \), which is observed by the current lender as well as the rating agent. If \( c^t = \overline{d} \), then the lender receives \( Y_l = 1 + i^t \), the borrower receives \( M - (1 + i^t) \) and the period ends. If \( c^t = d \), the lender’s expected payoff is \( Y_l = \beta Y_P \), the borrower’s expected payoff is \( (1 - \beta) Y_P \) and the period ends. The above stage game is repeated for \( T \) periods.

The analysis in this paper focuses on the Markov Perfect Bayesian Equilibrium for this dynamic game of incomplete information. The payoff relevant state variable is the lender’s beliefs \( R_t \).

**Definition 1** A strategy profile \( \{ r^t_H (R_t), r^t_L (R_t), l^t (R_t) \}_{t=1}^{T} \) and associated beliefs \( R_t \) constitute a Markov Perfect Bayesian Equilibrium if, for all \( t \):
• The players’ strategies \{r^t_H(R_t), r^t_L(R_t), l^t(R_t)\} are sequentially rational.

• For all realized histories \(H^t = \cup_{t=1}^T \{s^t, c^{t-1}\}\) and prior \(q\), beliefs \(R_t\) are determined using Bayes’ Rule in the following manner:

\[
R_t = \frac{q \prod_{\tau=1}^t \Pr(s^\tau, c^{\tau-1}|\theta_j = \theta_H)}{q \prod_{\tau=1}^t \Pr(s^\tau, c^{\tau-1}|\theta_j = \theta_H) + (1 - q) \prod_{\tau=1}^t \Pr(s^\tau, c^{\tau-1}|\theta_j = \theta_L)}
\]

Or equivalently,

\[
R_t = \frac{R_{t-1} \Pr(s^t, c^{t-1}|\theta_j = \theta_H)}{R_{t-1} \Pr(s^t, c^{t-1}|\theta_j = \theta_H) + (1 - R_{t-1}) \Pr(s^t, c^{t-1}|\theta_j = \theta_L)}
\]  

(1)

If (1) above is indeterminate, then any beliefs \(R_t \in [0, 1]\) are admissible.

• For any two different histories \(H^t, \tilde{H}^t \in \mathcal{H}^t\), if beliefs \(R_t = \tilde{R}_t\), then players’ strategies \(r^t_H(R_t) = \tilde{r}^t_H(\tilde{R}_t)\), \(r^t_L(R_t) = \tilde{r}^t_L(\tilde{R}_t)\) and \(l^t(R_t) = \tilde{l}^t(\tilde{R}_t)\) as well.

Markov Perfect Bayesian Equilibrium always exists for the above finitely repeated dynamic game (Fudenberg and Tirole, 1991). Although \(R_t\) is a function of \(H^t\) and players’ equilibrium strategies, its arguments are dropped whenever it refers to a rating that has already been announced by the rating agent.

Sequential rationality requires that in every period \(t\), both the borrower and the lender choose their strategies such that their individual payoffs are maximized for any given beliefs. Given \(R_t\), the lender’s expected payoff from lending is:

\[
\begin{align*}
(R_t \theta_H r^t_H(R_t) + (1 - R_t) \theta_L r^t_L(R_t)) (1 + i^t) + \\
(R_t \theta_H (1 - r^t_H(R_t)) + (1 - R_t) \theta_L (1 - r^t_L(R_t))) \beta M
\end{align*}
\]  

(2)

The borrower makes a strategic decision only if \(Y_P = M\). Given \(R_t\), the expected likelihood of \(Y_P = M\) and \(c^t = \tilde{d}\) is \(R_t \theta_H r^t_H(R_t) + (1 - R_t) \theta_L r^t_L(R_t)\), in which case the lender receives a precontracted amount of \(1 + i^t\). The probability that
\[ Y_P = M \text{ and } c^t = d \] is \( R_t \theta_H (1 - r^t_H (R_t)) + (1 - R_t) \theta_L (1 - r^t_L (R_t)) \) in which case the lender’s expected payoff is \( \beta M \). If \( Y_P = 0 \), then the lender receives nothing.

The competitive credit market equalizes the lender’s expected return from lending with the risk-free return \( i^f \). Further accounting for interest rates permissible under Assumption 1(b) gives the interest rate function:

\[
1 + i (R_t) = \min \left\{ \beta M + \frac{(1 + i^f) - [R_t \theta_H + (1 - R_t) \theta_L] \beta M}{[R_t \theta_H r^t_H (R_t) + (1 - R_t) \theta_L r^t_L (R_t)]}, M - \frac{1 - \beta}{\theta_L} \right\} \tag{3}
\]

If \( Y_P = M \), then the borrower seeks to maximize:

\[
V_j^P (R_t) = \left[ r^t_j (R_t) \left( M - (1 + i (R_t)) + V_j^E (R_{t+1} (\bar{d}, \cdot)) \right) \right] + \left( 1 - r^t_j (R_t) \right) \left( (1 - \beta) M + V_j^E (R_{t+1} (d, \cdot)) \right)
\]

where \( R_{t+1} (c^t, \cdot) \) denotes the realized rating in period \( t + 1 \) if choice \( c^t \in \{d, \bar{d}\} \) is observed at the end of period \( t \). \( V_j^E (\cdot) \) denotes the expected value of future continuation payoffs, which depends on the borrower’s type \( \theta_j \) and the rating \( R_{t+1} (\cdot) \). If \( c^t = \bar{d} \), which happens with probability \( r^t_j (R_t) \), then the borrower receives \( M - (1 + i (R_t)) \) in the current period and \( V_j^E (R_{t+1} (\bar{d}, \cdot)) \) in the future periods. If \( c^t = d \), then the borrower receives a payoff of \( (1 - \beta) M + V_j^E (R_{t+1} (d, \cdot)) \).

Note that \( V_j^P (\cdot) \) and \( V_j^E (\cdot) \) differ from each other as the former reflects the borrower’s payoff ex-post an observed success in the project while the latter reflects his expected payoff ex-ante of the realization of the project’s outcome. Then

\[
V_j^E (R_{t+1}) \equiv \left\{ \begin{array}{ll} 1 & (l^{t+1} = 1) \left[ \theta_j V_j^P (R_{t+1}) + (1 - \theta_j) V_j^E (R_{t+2} (d, \cdot)) \right] \\ & + \mathbb{1} (l^{t+1} = 0) V_j^E (R_{t+2} (\emptyset, \cdot)) \end{array} \right\}
\]

Positive gains occur in any period only if lending happens and the project succeeds thereafter. Post lending, if the project succeeds (which happens with proba-
bility \( \theta_j \)), then the borrower’s payoff is the same as in equation (4). Otherwise, if either the project fails or lending does not occur, then the borrower only receives an expected continuation payoff of \( V^E_j (R_{t+2} (c_{t+1}^+)) \) for \( c_{t+1}^+ \in \{d, \emptyset\} \). In the last period \( T \), the expected value of continuation payoffs is \( V^E_j (R_{T+1}) = 0 \). Then, sequential rationality requires that given \( R_t \)

\[
r^I_j (R_t) \in \arg \max_{r^I_j \in [0, 1]} V^P_j (R_t) \text{ s.t. } V^E_j (R_{T+1} (c^T, \cdot)) = 0
\]

for all \( c^T \in \{d, \overline{d}, \emptyset\} \) and \( l (R_t) = 1 \) if and only if the lender’s expected return from lending given in equation (2) is at least as high as \( 1 + i^f \) for all \( t \).

3 Equilibrium Characterization

First, the equilibrium is characterized in a market without a rating agent. Doing so provides a benchmark case against which the benefits of having a rating agent can be evaluated.

3.1 Benchmark : Without A Rating Agent

Consider the game specified in Section 2, with the slight modification that there are only two players in any period – the borrower and the lender. If there is no rating agent, the lender in period \( t \) does not have access to the history \( H^l = \bigcup_{\tau=1}^t \{s^\tau, c^{\tau-1}\} \) and his beliefs regarding the borrower’s type coincide with the prior \( q \) for every \( t \leq T \). For notational consistency, the lender’s beliefs about the borrower being type \( \theta_H \) are denoted by \( R_t \). Equilibrium for this game is as follows:

**Lemma 1 (Equilibrium with no rating agent)** Given \( R_t \in (0, 1) \) and \( R^b \equiv \prod_{\tau=1}^T \frac{1+\alpha^f}{\beta^M - \theta_L} \), there exists a unique pooling equilibrium such that
1. For $R_t < R^b$, both types of the borrower choose default ($d$), and the lender, in turn, finds it optimal not to lend. That is, $r^t_H(R_t) = r^t_L(R_t) = l^t(R_t) = 0$ for $R_t < R^b$.

2. For $R_t \geq R^b$, both types of the borrower choose repayment ($\bar{d}$), and the lender, in turn, finds it optimal to lend at an interest rate given by equation (3). That is, $r^t_H(R_t) = r^t_L(R_t) = l^t(R_t) = 1$ with $1 + i(R_t) = \frac{1 + r_f}{\beta M - \theta_L} \frac{1}{\theta_H + (1 - \beta) \theta_L}$ for $R_t \geq R^b$.

Lemma 1 states that in the absence of the rating agent, a pooling equilibrium arises where both types of the borrower choose the same action. If there is no rating agent, then the ex-ante expected continuation payoffs from repayment ($\bar{d}$) and default ($d$) are identical for both types of the borrower; that is, $V^E_j(R_{t+1}(\bar{d}, \cdot)) = V^E_j(R_{t+1}(d, \cdot)) = V^E_j(q)$ as information on historical $c^t$ is not available to the lenders in future periods. As the borrower’s continuation payoffs no longer depend on his choice, he makes his decisions by comparing his current period payoffs between repayment ($\bar{d}$) and default ($d$). Ex-ante heterogeneous types of the borrower display ex-post identical behavior and choose repayment if and only if $M - (1 + i(R_t))$ is higher than $(1 - \beta) M$.\(^{15}\) Plugging in the optimal responses of both types of the borrower in equation (3), it can be seen that $1 + i(R_t) \leq \beta M$ if and only if $R_t \geq R^b = \left[\frac{1 + r_f}{\beta M - \theta_L} \frac{1}{\theta_H + (1 - \beta) \theta_L}\right] < 1$. Given the borrower’s equilibrium response and competitive interest rate, lending is optimal only for $R_t \geq R^b$.

Note that the above equilibrium will also arise if the borrower and the lender interact only once; that is, if $T = 1$. This equilibrium will also arise in the last

\(^{15}\)Note that the ex-post identical behavior stems from the assumption that both types of the borrower receive identical expected payoffs from choosing default. Empirical evidence suggests that often a borrower’s expected payoff from default is influenced by other factors such as moral values, social norms, etc. (see e.g., Fehr and Zehnder, 2009; Guiso et al., 2013; Karlan, 2005). Often, individuals differ from each other in this dimension as well, which may induce them to behave differently under identical financial environments. Such a possibility can be captured by allowing $\beta$ to be a function of the borrower’s type and assuming that $\beta(\theta_H) \neq \beta(\theta_L)$. In this case, the qualitative nature of the equilibrium stays the same; that is, both types of the borrower continue to follow a threshold strategy where they strategically default for lower ratings and choose repayment for higher ratings. However, these thresholds would now be different for different types of the borrower.
Figure 2: Equilibrium strategies in the absence of a rating agent

period $t = T$ of the repeated game laid out in Section 2. A graphic representation of the results in Lemma 1 is provided in Figure 2 with the help of following example.

Example 1 Consider an interaction between a borrower and a sequence of lenders, where lenders do not have access to the borrower’s repayment history. Let risk-free return be $i^r = 0.4$, output from a successful project be $M = 10$ and probability of punishment in case of default be $\beta = 0.3$. The probability that the project succeeds is 0.8 if $\theta_j = \theta_H$ and 0.12 if $\theta_j = \theta_L$. For these values of parameters, $R^b \approx 0.51$.

Figures 2(a) and 2(b) show that the equilibrium strategies are identical for both types of the borrower: default ($d$) for $R_t < R^b = 0.51$ and repayment ($\bar{d}$) for $R_t \geq 0.51$. 

0.51. Figure 2(c) shows that the lender finds it suboptimal to lend for $R_t < R^b$. For $R_t \geq R^b$, lending occurs at an interest rate reflected in Figure 2(d). Notice that the interest rate is non-increasing in $R_t$. A higher $R_t$ corresponds to a stronger belief that the borrower is $\theta_H$ type. From the lender’s perspective, an increase in $R_t$ increases his expected return from lending, thereby making lending a more attractive option. Unsurprisingly, competition posed by other creditors leads to a lower interest rate.

3.2 With A Rating Agent

The remainder of this paper utilizes the original model specified in Section 2, where a rating agent provides the lender with the realized history $H^t = \cup_{t=1}^{t} \{s^T, c^{T-1}\}$ and a rating $R_t$. As a result, the lender’s beliefs depend on $H^t$. In equilibrium, lender’s beliefs are determined using Bayes’ rule and are given by equation (1). The equilibrium strategies for two types of the borrower in this game are summarized in the following proposition.

**Proposition 1 (Borrower’s equilibrium behavior)** Given $R_t \in (0, 1)$, $p_s \geq \frac{1}{2}$, for $j \in \{H, L\}$ there exists

$$R_{tj}(p_s) \equiv \inf \left\{ \begin{array}{l} R_t | \left\{ V^p_j (R_t) \text{ when } r^j_t = 0 \right\} \leq \left\{ \max r^j_t V^p_j (R_t) \text{ when } r^j_t \in (0, 1] \right\} \end{array} \right\}$$

(5)

such that

1. For $R_t < R_{tH}(p_s)$, a pooling equilibrium arises with $r^H_t (R_t) = r^L_t (R_t) = 0$.

2. For $R_{tH}(p_s) \leq R_t < R_{tL}(p_s) \leq R^b$, a semi-separating equilibrium arises with $r^H_t (R_t) = 1$ and $r^L_t (R_t) \in (0, 1]$. 

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3. For \( R_t \geq R_{tL}(p_s) \), a pooling equilibrium arises with \( r^L_{ti}(R_t) = r^H_{ti}(R_t) = 1 \).\(^{16}\)

Proposition 1 states that given \( p_s \), in equilibrium, the \( \theta_j \) borrower follows a threshold strategy where he chooses default for all ratings below \( R_{tj}(p_s) \). In making his decision, the borrower compares the costs of repayment against its benefits. In the current period, repayment costs \( 1 + i(R_t) \) but saves the expected payment of \( \beta M \) to be made in the event of default. Additionally, repayment results in a gain in continuation payoffs \( V^E_j(R_{t+1}(d, \cdot)) \). Recall that ex-ante expected continuation payoff \( V^E_j(\cdot) \) depends on whether lending takes place in future periods. For a very low \( R_t \), when the competitive interest rate \( i(R_t) \) is very high, the expected \( R_{t+1}(\bar{d}, \cdot) \) is too low to support any future lending. Consequently, for low ratings, the borrower finds it optimal to default despite having the ability to repay his loans. As \( R_t \) increases, the corresponding interest rate \( i(R_t) \) becomes lower.

For the intermediate ratings, while \( R_{t+1}(\bar{d}, \cdot) \) secures future lending, \( R_{t+1}(d, \cdot) \) does not, thereby strengthening the incentives for repayment. Although \( i(R_t) \) is somewhat high in this region, repayment is motivated by the large positive gains in the ex-ante expected continuation payoffs \( V^E_j(R_{t+1}(\bar{d}, \cdot)) - V^E_j(R_{t+1}(d, \cdot)) \). For higher ratings, future lending occurs despite default, thereby reducing the gains in the ex-ante expected continuation payoffs. For these ratings, it is the low interest rates that prompt the borrower to choose repayment.

In comparison to the low type borrower, the ex-ante expected continuation payoffs \( V^E_j(\cdot) \) are higher for the high type borrower. This is because, ceteris paribus, the

\(^{16}\)For \( R_b < R_t < \frac{1+r_f - \theta_l}{\theta_l \beta M - \theta_H} \), there may exist another equilibrium such that \( r^L_{ti}(R_t) = 0 \) and \( r^H_{ti}(R_t) = 1 \). However, such an equilibrium would result in \( R_{t+1}(\bar{d}, \cdot) = 0 < R_{t+1}(d, \cdot) \). In line with Kreps and Wilson (1982), such beliefs are considered implausible as they imply that credit bureaus issue a lower rating upon observing repayment \( \bar{d} \) instead of default \( d \); that is, despite repayment lenders believe that the borrower is less likely to be \( \theta_H \) type. Hence, only equilibria with \( r^L_{ti}(R_t) = r^H_{ti}(R_t) \) are identified in this paper. Additionally, for all \( R_t \leq R_b \), there also exists a trivial pooling equilibrium with \( r^L_{ti}(R_t) = r^H_{ti}(R_t) = 0 \) and lenders find it in-optimal to lend. Equilibria where both types of the borrower pool on default is supported by the off-equilibrium belief of \( R_{t+1}(\bar{d}, r^H_{ti} = 0, \cdot) = 0 \).
\( \theta_H \) borrower is not only more likely to observe high signals but also more likely
to succeed in his projects. Consequently, the \( \theta_H \) borrower is willing to pay a much
higher interest rate in order to receive the higher continuation payoffs. This leads
to \( R_{tH}(p_s) \leq R_{tL}(p_s) \) with equality being met only for \( t = T \).

Proposition 1 further states that the high type borrower always finds it optimal
to choose a pure strategy. In order to understand this result, notice that the \( \theta_H \)
borrower’s expected continuation payoff from choosing repayment \( V^E_{tH}(R_{t+1}(\tilde{d}, r'_{tH}, \cdot)) \)
increases in \( r'_{tH} \), and his expected continuation payoff from choosing default \( V^E_{tH}(R_{t+1}(d, r'_{tH}, \cdot)) \)
decreases in \( r'_{tH} \), causing \( V^E_{H}(R_{t+1}(\tilde{d}, r'_{tH}, \cdot)) - V^E_{H}(R_{t+1}(d, r'_{tH}, \cdot)) \) to increase in \( r'_{tH} \). Accordingly, the \( \theta_H \) borrower will maximize his continuation payoffs
by choosing \( r'_{tH}(R_t) \in \{0, 1\} \) for any given interest rate. In contrast, for intermediate levels of ratings, the low type borrower would find it optimal to choose a mixed strategy and not pool with the \( \theta_H \) borrower. This is because his expected continuation payoff from choosing repayment \( V^E_{tL}(R_{t+1}(\tilde{d}, r'_{tL}, \cdot)) \)
decreases in \( r'_{tL} \) and his expected continuation payoff from choosing default \( V^E_{tL}(R_{t+1}(d, r'_{tL}, \cdot)) \)
increases in \( r'_{tL} \), causing \( V^E_{L}(R_{t+1}(\tilde{d}, r'_{tL}, \cdot)) - V^E_{L}(R_{t+1}(d, r'_{tL}, \cdot)) \) to decrease in \( r'_{tL} \). As a result, the \( \theta_L \) borrower may maximize \( V^P_{L}(R_t) \) for an intermediate value of \( r'_{tL} \in (0, 1) \).

Given Proposition 1, the lender’s equilibrium behavior can be easily characterized as follows:

**Lemma 2 (Lender’s equilibrium behavior)** Given \( R_t \in [0, 1] \) and \( p_s \geq 0.5 \), there exists a cutoff \( R^l_t(p_s) \geq R_{tH}(p_s) \) such that \( l(R_t) = 0 \) for all \( R_t < R^l_t(p_s) \) and \( l(R_t) = 1 \) otherwise with the interest rate given by equation (3).

Lemma 2 states that, similar to the borrower, the lender also finds it optimal to
follow a threshold strategy whereby he avoids lending for ratings below a certain
cutoff. For ratings \( R_t < R_{tH}(p_s) \), default is observed irrespective of the project’s
outcome and borrower’s type. The lender’s expected payoff from lending is given
by \( [R_t \theta_H + (1 - R_t) \theta_L] \beta M \), which is lower than \( 1 + ivf \) in this range of ratings.
For \( R_t \geq R_{tH}(p_s) \), at least the \( \theta_H \) borrower finds it optimal to repay his loan. This change in the \( \theta_H \) borrower’s behavior discontinuously increases the lender’s expected payoff from lending. Because equation (3) decreases in \( R_t \), there exists a \( R^l_t(p_s) \geq R_{tH}(p_s) \) such that \( i(R^l_t(p_s)) \) is below \( M - \frac{1-\beta}{\bar{y}_L} \) for all \( R_t \geq R^l_t(p_s) \), thereby making it optimal for the lender to lend.

Figure 3 combines and summarizes the results of Proposition 1 and Lemma 2.

The findings of this section are reiterated in Figure 4 with the help of a numerical example.

**Example 2** Consider a game where a borrower interacts with a sequence of lenders for \( T = 5 \) periods. The informativeness of the signal is \( p_s = 0.8 \). The values for the remaining parameters are the same as given in Example 1 above.

Figures 4(a) and 4(b) show the equilibrium strategies for both types of the borrower at time \( t = 1 \) for given \( p_s \).

\[ \text{Figures 4(a) and 4(b) show the equilibrium strategies for both types of the borrower at time } t = 1 \text{ for given } p_s . \]

As can be seen, \( R_{tH}(p_s) \) is always at least as high as \( R_{tL}(p_s) \). When compared with Figures 2(a) and 2(b), Figures 4(a) and 4(b) immediately highlight the undisputed usefulness of the rating agent who links a borrower’s present with his future. The presence of the rating agent alleviates the moral hazard experienced by the borrower and results in fewer strategic defaults. Figure 4(c) shows that \( R_{tH}(p_s) \) and \( R_{tL}(p_s) \) are increasing in \( t \). This suggests that the disciplinary impact of a rating agent tends to weaken towards the end of the game. This is because, as \( t \to T \), the expected gains in ex-ante continuation payoffs

\[ \text{\textsuperscript{17}The equilibrium strategies for times } t = 2, 3, \text{ and } 4 \text{ are qualitatively similar. The equilibrium strategies for } t = 5 \text{ are the same as those in Figures 2(a) and 2(b).} \]
become smaller and the borrower is unwilling to pay very high interest rates in order to receive these gains.

4 Impact of Information

Similar to the U.S. credit bureaus, the rating agent in our model equips the lender with two kinds of information pertaining to the borrower: (1) the history of loan performance, and (2) the history of informative signals. This paper seeks to disentangle the effects of these two kinds of information. Recall that if $p_s = 0.5$, then the signals are completely uninformative, or equivalently, the lender has access only
to the borrower’s repayment history \( H^t = \bigcup_{\tau=1}^{t} \{ c^{\tau-1} \} \). Analyzing the impact of increasing \( p_s \) on the borrower’s repayment incentives highlights the marginal impact of the information contained in \( \bigcup_{\tau=1}^{t} \{ s^\tau \} \) on the market and that brings us to the central result of this paper.

**Proposition 2 (Discouraging impact of additional information)** The repayment threshold \( R_{tj}(p_s) \) is increasing in \( p_s \). Further, \( R_{tj}(p_s) \) converges to \( R^b \) as \( p_s \to 1 \).

Proposition 2 states that as signals becomes more informative, the repayment threshold for both types of the borrower become higher as well. This happens because informative signals reduce the impact of current choices on the continuation payoffs. As \( p_s \) increases, the difference between expected future ratings \( R_{t+1}(\bar{d}, \cdot) - R_{t+1}(d, \cdot) \) and ultimately, ex-ante expected continuation payoffs \( V^E_j(R_{t+1}(\bar{d}, \cdot)) - V^E_j(R_{t+1}(d, \cdot)) \) starts converging to 0. In response, the borrower is willing to pay an increasingly lower interest rate in order to receive the reduced future gains. As a result \( R_{tj}(p_s) \), as given by equation (5), is increasing in \( p_s \). This finding highlights that informative signals increase the incidence of moral hazard experienced by the borrower. Even though the borrower has the ability to repay his loans, as these signals get increasingly informative he strategically chooses default over a wider range of ratings.

Additionally, as \( V^E_j(R_{t+1}(\bar{d}, \cdot)) - V^E_j(R_{t+1}(d, \cdot)) \) converges towards 0, the borrower’s multi-period optimization problem given in equation (4) starts resembling his one-period optimization problem, and consequently \( R_{tj}(p_s) \) converges to \( R^b \). For the limiting case of \( p_s = 1 \), \( V^E_j(R_{t+1}(\bar{d}, \cdot)) - V^E_j(R_{t+1}(d, \cdot)) = 0 \). As a result, the borrower’s optimization problem becomes identical to the one in Section 3.1 leading to \( R_{tj}(p_s) = R^b \).

The findings of Proposition 2 are reiterated in Figure 5. Figures 5(a) and 5(b) show the values of \( R_{tH}(p_s) \) and \( R_{tL}(p_s) \) respectively for \( t = 1 \) to 5 and for varying values of \( p_s = 0.5, 0.8, \) and 1. The values for the remaining parameters are the
same as given in Example 2 above.

(a) Changes in $R_{tH}$ with respect to $p_s$  
(b) Changes in $R_{tL}$ with respect to $p_s$

Figure 5: $R_{tj}(p_s)$ for different values of $p_s$

In summary, informative signals have an undesirable impact on the borrower’s equilibrium behavior as they tend to weaken the disciplinary impact of providing the borrower’s repayment history to the lenders, thereby incentivizing the borrower to default more often. The impact of increased signal informativeness on the lender’s behavior is fairly straightforward to analyze and is given by the following lemma.

**Lemma 3 (Frequent market failure)** The lending threshold $R^l_{tj}(p_s)$ is increasing in $p_s$.

The above result follows directly by combining Lemma 2 with Proposition 2. Frequent strategic defaults in the presence of informative signals translate to infrequent lending, which results in the inefficient outcome of investments in the risk-free assets by lenders. Credit rationing highlighted by Lemma 3 implies that more information has a welfare-reducing impact on the market, as it prevents some efficient investments from taking place. However, moral hazard is only one aspect of information asymmetry in the market. It is equally important to explore the impact of this ancillary information on adverse selection in the market. This can be done by studying the asymptotic behavior of rating for various values of $p_s$. 

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The following lemma characterizes the asymptotic behavior of ratings in the presence of the rating agent as the number of periods increases to $\infty$ for a given $p_s$.

**Lemma 4 (Convergence of ratings)** As $t \to \infty$, if $\theta_j = \theta_H$, the equilibrium belief $R_t$ converges to 1. Otherwise, if $\theta_j = \theta_L$, $R_t$ converges to 0.

Lemma 4 states that in the long run, if lenders base their beliefs on history $H^t$, then the borrower’s type is eventually revealed to them. That is, ratings for the $\theta_H$ borrower converge to 1 and ratings for the $\theta_L$ borrower converge to 0. Post-convergence, observed $c^t$ has no impact on the borrower’s continuation payoffs, and equilibrium will be the same as in Lemma 1. This temporary nature of disciplinary incentives generated by linking the history with the future has previously been established by Cripps et al. (2004). In the context of this paper, the interest is in investigating how informative signals affect the rate of convergence of beliefs. The rate of convergence is calculated as the second derivative of ratings with respect to $p_s$ as $t$ goes to $\infty$. Proposition 3 presents this result.

**Proposition 3 (Faster convergence)** The rate of convergence of the equilibrium beliefs $R_t$ is increasing in $p_s$.

Proposition 3 states that the informative signals hasten lenders’ learning of the borrower’s type and hence dissipate the problem of adverse selection at a faster rate. In the extreme case where $p_s = 1$, lenders learn the borrower’s type in one period only.

One may be tempted to conclude that although the $\theta_L$ borrower is negatively affected when informative signals reveal his type sooner, the $\theta_H$ borrower is certainly better off but this may not necessarily be true. For ratings $R_t \in [R_{tH}(0.5), R_{tH}(p_s))$,\(^{18}\) both types of the borrower are worse off in the presence of informative signals because no lending occurs in this region. Further, for ratings $R_t \in [R_{tL}(0.5), R_{tL}(p_s))$,

\(^{18}\)Recall that $p_s = 0.5$ corresponds to the case where the signals are completely uninformative, or equivalently, the lender has access only to the borrower’s repayment history $H^t = \cup_{t=1}^{\infty} \{c^{t-1}\}$.
the $\theta_H$ borrower receives an additional opportunity to separate himself from the $\theta_L$ borrower. However, lenders charge higher interest rates in this range because the low type borrower no longer finds it optimal to choose repayment. Depending upon whether the gain in the continuation payoffs from separation exceeds its costs, the $\theta_H$ borrower may be better off or worse off in this range of ratings. For all other ratings $R_t \in [R_{tH}(p_s), R_{tL}(0.5)) \cup [R_{tL}(p_s), 1]$ for which the behavior of both types of the borrower remains unaltered, the $\theta_H$ borrower is unambiguously better off.

5 Conclusion

In addition to the borrower’s historical repayment records, consumer credit reports contain other information: credit mix, credit inquiries, public records, etc. These reports are widely used by lenders in deciding the terms for the loans extended to him. This paper studies the impact of various kinds of information contained in credit reports and finds that incentives for repayment are maximized when only information pertaining to the borrower’s repayment history is used to determine the terms of loans. Considering the fact that other ancillary information will discount the impact of his repayment behavior on future loans, the borrower might strategically default on his loans. As this additional information becomes increasingly indicative of the hidden characteristics of the borrower, repayment incentives become even weaker. This additional information, however, allows lenders to screen out undesirable borrowers sooner than before.

As discussed below, the assumptions made in this paper are not very restrictive. The findings will continue to hold under various alternative modeling assumptions as well. The assumption that the borrower is a long-lived player is necessary in order to capture the inter-temporal dynamics studied in this paper. Although one might argue that often lenders are big organizations that outlive
borrowers, evidence suggests that information sharing and enhanced competition among lenders has shortened the length of relationships between banks and their customers, with borrowers often switching lenders (Boot, 2000; Frame et al., 2001; Sutherland, 2015), which justifies the lender’s perceived short life in the model. The model is also applicable to loans which last over multiple time periods. These loans often come with various legal provisions that permit the lender to change the terms and conditions of an existing loan at any time. Additionally, upon observing a default, the lender can charge a higher interest rate and reserves the right to immediately rescind the entire amount. Also, the borrower retains the option to refinance a loan at any time. These features of long-term loans allow them to be viewed as a series of short-term loans. The results of this paper also extend to repeated interactions with the same lender as long as the lender operates in a Bertrand competitive credit market with no switching costs. Although the model in this paper focuses on unsecured lending, the insights also apply to secured loans with the value of collateral being $\beta M$.

Future works in this area can be carried out in a number of directions. Although the assumption of a non-strategic rating agent seems plausible in consumer credit markets, ample evidence indicates that rating agents in capital markets are often strategic agents motivated by their own reputational and competitive concerns. Accordingly, it would be interesting to see how the presence of a strategic rating agent that endogenizes the nature and precision of this ancillary information influences the market. Once the borrower’s type is revealed, the rating agent ceases to play any role in this paper’s model. It will be interesting to identify the marginal value of information under an alternative rule to determine lenders’ beliefs which prohibit this phenomenon by preventing the ratings from converging.
Appendix

The appendix contains proofs of all results presented in the text.

Proof of Lemma 1. In the absence of a rating agent $R_t = R_{t+1} (d, \cdot) = R_{t+1} (\tilde{d}, \cdot) = q$ for every $t$. The borrower’s future continuation payoffs become independent of his equilibrium strategy and the history of observed choices. The borrower chooses $r_t^j (R_t)$ so as to maximize:

$$V_j^P (R_t) = r_t^j (R_t) \left[ M - (1 + i (R_t)) \right] + \left( 1 - r_t^j (R_t) \right) \left[ (1 - \beta) M \right] + V_j (R_t)$$

Hence, $r_t^H (R_t) = r_t^L (R_t) = 1$ if $1 + i (R_t) \leq \beta M$ and $r_t^H (R_t) = r_t^L (R_t) = 0$ otherwise. Plugging in the equilibrium strategies for both types of the borrower in equation (3) gives the interest rate as:

$$1 + i (R_t) = \min \left\{ \beta M + \frac{(1 + i^{\tau_f}) - [R_t \theta_H + (1 - R_t) \theta_L] \beta M}{R_t \theta_H + (1 - R_t) \theta_L}, M - \frac{1 - \beta}{\theta_L} \right\} \quad (A.1)$$

Then $1 + i (R_t) \leq \beta M$ if and only if $R_t \geq R^b \equiv \left[ \frac{1 + i^{\tau_f}}{\beta M} - \theta_L \right] \frac{1}{\theta_H - \theta_L}$. Otherwise, $1 + i (R_t) = M - \frac{1 - \beta}{\theta_L} > \beta M$. Substituting for $1 + i (R_t)$ in equation (2), it can be seen that for $R_t < R^b$, the lender’s expected payoff from lending is $[R_t \theta_H + (1 - R_t) \theta_L] \beta M$, which is less than the risk-free return of $1 + i^{\tau_f}$. Hence, $l (R_t) = 0$ for this range of ratings. For $R_t \geq R^b$, the lender is able to charge the competitive interest rate given by equation (A.1) and will find it optimal to lend. ■

Proof of Proposition 1. This paper identifies the equilibrium for which $r_t^H (R_t) \geq r_t^L (R_t)$. It can be shown that $R_{t+1} (d, \cdot) - R_{t+1} (d, \cdot) \triangleq \theta_H r_t^H - \theta_L r_t^L \geq 0$ in equilibrium. Given an interest rate $i^t$, the $\theta_j$ borrower will choose $r_t^j > 0$ if and only if

$$0 \leq \left( 1 - \beta \right) M + V_j^E (R_{t+1} (d, r_t^j = 0, \cdot)) \quad \leq M \left( 1 + i^t \right) + V_j^E (R_{t+1} (\tilde{d}, r_t^j \in (0, 1], \cdot)) \quad (A.2)$$
First, it is shown that \( r_H^t (R_t) \in \{0, 1\} \). Suppose not. Now \( r_H^t (R_t) \in (0, 1) \) if and only if \( (1 - \beta) M + V_H^E (R_{t+1} (d, r_j^t \in (0, 1], \cdot)) = M - (1 + i^t) + V_j^E (R_{t+1} (\bar{d}, r_H^t \in (0, 1], \cdot)) \). By combining this fact with the observation that \( V_H^E (R_{t+1} (\bar{d}, r_H^t, \cdot)) \) increasing in \( r_H^t \) and \( V_H^E (R_{t+1} (d, r_H^t, \cdot)) \) is decreasing in \( r_H^t \), it can be seen that \( r_H^t (R_t) = 1 \) strictly dominates \( r_H^t (R_t) \in (0, 1) \). Hence, the \( \theta_H \) borrower maximizes his payoff by choosing a pure strategy in equilibrium.

For \( R_t \in (0, 1) \), it can be seen that \( R_{t+1} (\bar{d}, r_j^t, \cdot) \) increases at a decreasing rate in \( R_t \) and \( R_{t+1} (d, r_j^t, \cdot) \) increases at an increasing rate in \( R_t \). This in turn implies that \( V_j^E (R_{t+1} (\cdot)) \) is increasing in \( R_t \) and \( V_j^E (R_{t+1} (\bar{d}, r_j^t, \cdot)) \geq V_j^E (R_{t+1} (d, r_j^t, \cdot)) \geq 0 \). However, positive gains occur only if lending occurs. For a very low \( R_t \), no future lending occurs and \( V_j^E (R_{t+1} (\cdot)) = 0 \). For this range of ratings, \( r_j^t (R_t) = 1 \) if \( 1 + i^t \leq \beta M \) and \( r_j^t (R_t) = 0 \) otherwise. For an intermediate \( R_t \) while \( V_j^E (R_{t+1} (\bar{d}, r_j^t, \cdot)) \) is positive and increasing in \( R_t \), \( V_j^E (R_{t+1} (d, r_j^t, \cdot)) \) is still zero. For higher \( R_t \), \( V_j^E (R_{t+1} (d, r_j^t, \cdot)) \) becomes positive as well and \( V_j^E (R_{t+1} (\bar{d}, r_j^t, \cdot)) - V_j^E (R_{t+1} (d, r_j^t, \cdot)) \) is decreasing in \( R_t \). The shape of \( V_j^E (R_{t+1} (\bar{d}, r_j^t, \cdot)) - V_j^E (R_{t+1} (d, r_j^t, \cdot)) \) can be summarized as follows: it is zero for a low \( R_t \), it is increasing in \( R_t \) for intermediate \( R_t \), and it is decreasing in \( R_t \) for higher \( R_t \). Hence, there will exist \( 0 \leq R_{ij} (p_s) \leq \bar{R}_{ij} (p_s) \leq 1 \) such that \( \max_{i^t \geq 0} V_j^E (R_{t+1} (\bar{d}, r_j^t, \cdot)) - V_j^E (R_{t+1} (d, 0, \cdot)) \geq 1 + i^t - \beta M \) for all \( R_t \in \left[ R_{ij} (p_s), \bar{R}_{ij} (p_s) \right] \). In this range of \( R_t \), given \( i^t \), the borrower will find it optimal to choose \( r_j^t > 0 \) such that \( r_j^t (R_t) \in \arg \max \left\{ V_j^P (R_t) \text{ when } r_j^t \in (0, 1] \right\} \). As \( i^t \) decreases, the range of ratings over which the borrower finds it optimal to choose \( r_j^t \in (0, 1] \) expands. This equilibrium behavior is supported by the off-equilibrium belief of \( R_{t+1} (\bar{d}, r_H^t = 0, \cdot) = 0 \). The lender uses this knowledge regarding the borrower’s optimal behavior in determining \( i (R_t) \) using equation (A.1). It can be seen that given the optimal behavior for both types of the borrower and the interest rate \( i (R_t) \), there exists \( R_{ij} (p_s) > 0 \) such that
Next, it is shown that \( r^i_j (R_t) > 0 \) for all \( R_t \geq R_{ij} (p_s) \). This result follows from the observations that \( i (R_t) \) is decreasing in \( R_t \), \( R_{ij} (p_s) \) is increasing in \( i (R_t) \) and \( \overline{R}_{ij} (p_s) \) is decreasing in \( i (R_t) \).

Notice that for \( R_t > R_{bi} \), the optimal interest rate \( 1 + i (R_t) < \beta M \). In this case \( R_{ij} (p_s) = 0 \), \( R_{ij} (p_s) = 1 \) and \( r^i_j (R_t) = 1 \). Hence, it follows that \( R_{ij} (p_s) \leq R_{bi} \).

Lastly, it can be shown using induction that the ex-ante expected value of continuation payoffs is increasing in \( \theta_j \). Hence, \( R_{iH} (p_s) \leq R_{iL} (p_s) \).

**Proof of Lemma 2.** Lemma 1 shows that \( r^i_H (R_t) = r^i_L (R_t) = 0 \) for \( R_t < R_{iH} (p_s) \). Then the lender’s payoff from lending is \([R_t \theta_H + (1 - R_t) \theta_L] \beta M \) is less than \( 1 + r^f \) – his return from not lending. As a result \( l (R_t) = 0 \) for this range of ratings. For \( R_t \geq R_{iH} (p_s) \), because \( r^i_H (R_t) = 1 \), \( \beta M + \frac{(1 + r^f) - [R_t \theta_H + (1 - R_t) \theta_L] \beta M}{[R_t \theta_H r^i_H (R_t) + (1 - R_t) \theta_L r^i_L]} \) becomes finite. This expression is continuous and decreasing in \( R_t \). Accordingly, there will exist \( R^i_t (p_s) \in [R_{iH} (p_s), 1] \) such that

\[
R^i_t (p_s) \equiv \inf \left\{ R_t | \beta M + \frac{(1 + r^f) - [R_t \theta_H + (1 - R_t) \theta_L] \beta M}{[R_t \theta_H r^i_H (R_t) + (1 - R_t) \theta_L r^i_L]} \leq M - \frac{1 - \beta}{\theta_L} \right\}
\]

and \( \beta M + \frac{(1 + r^f) - [R_t \theta_H + (1 - R_t) \theta_L] \beta M}{[R_t \theta_H r^i_H (R_t) + (1 - R_t) \theta_L r^i_L]} < M - \frac{1 - \beta}{\theta_L} \) for all \( R_t > R^i_t (p_s) \). Hence, \( l (R_t) = 1 \) for \( R_t \geq R^i_t (p_s) \).

**Proof of Proposition 2.** For any given \( R_t \), first consider the case where \( s^{t+1} = s_H \). Then, the difference between \( R_{t+1} (\bar{d}, s_H, \cdot) \) and \( R_{t+1} (d, s_H, \cdot) \) can be simplified to:

\[
\frac{R_t (1 - R_t) (\theta_H r^i_H - \theta_L r^i_L) p_s (1 - p_s)}{[R_t \theta_H r^i_H p_s + (1 - R_t) \theta_L r^i_L (1 - p_s)] [R_t (1 - \theta_H r^i_H) p_s + (1 - R_t) (1 - \theta_L r^i_L) (1 - p_s)]}
\]
Taking limits, it is immediate that the above expression converges to 0 as \( p_s \) converges to 1. Similar calculations for \( s^{t+1} = s_L \) reveal that \( R_{t+1}(d, s_L, \cdot) - R_{t+1}(d, s_L, \cdot) \rightarrow 0 \) as \( p_s \rightarrow 1 \). An immediate implication of this result is that \( V_j^E(R_{t+1}(d, \cdot)) - V_j^E(R_{t+1}(d, \cdot)) \rightarrow 0 \) as \( p_s \rightarrow 1 \). It then follows immediately that \( R_{ij}(p_s) \) (as defined in Proof of Proposition 1) and \( R_{ij}(p_s) \) are increasing in \( p_s \). At the extreme, when either \( p_s = 1 \) or \( R_t = 0 \) or 1, the borrower’s optimization problem starts coincides with his one-period optimization problem (similar to the one in Section 3.1) and hence, \( R_{ij}(p_s) = R_b^b \). ■

**Proof of Lemma 3.** The result follows by combining Lemma 2 and Proposition 2. ■

**Proof of Lemma 4.** The rating \( R_t \) in stage \( t \) is given by:

\[
R_t = \frac{q \prod_{\tau=1}^{t} \Pr(s^{\tau}, c^{\tau} \mid \theta_j = \theta_H)}{q \prod_{\tau=1}^{t} \Pr(s^{\tau}, c^{\tau} \mid \theta_j = \theta_H) + (1 - q) \prod_{\tau=1}^{t} \Pr(s^{\tau}, c^{\tau} \mid \theta_j = \theta_L)}
\]

As \( s^t \) and \( c^{t-1} \) are independent of each other, \( \Pr(s^t, c^{t-1} \mid \theta_j) = \Pr(s^t \mid \theta_j) \Pr(c^{t-1} \mid \theta_j) \). Then \( R_t \) can be rewritten as

\[
R_t = \frac{1}{1 + \frac{1 - q}{q} \prod_{\tau=1}^{t} \Pr(s^t \mid \theta_j = \theta_L) \prod_{\tau=1}^{t} \Pr(c^{t-1} \mid \theta_j = \theta_L) / \Pr(s^t \mid \theta_j = \theta_H) \prod_{\tau=1}^{t} \Pr(c^{t-1} \mid \theta_j = \theta_H)}
\]

Define \( \rho_{1t} \equiv \prod_{\tau=1}^{t} \frac{\Pr(s^t \mid \theta_j = \theta_L)}{\Pr(s^t \mid \theta_j = \theta_H)} \) and \( \rho_{2t} \equiv \prod_{\tau=1}^{t} \frac{\Pr(c^{t-1} \mid \theta_j = \theta_L)}{\Pr(c^{t-1} \mid \theta_j = \theta_H)} \)

1. Let \( \eta_{1t} \) denote the number of times the observed signal is \( s^\tau = s_H \) for all \( \tau \leq t \). Then signal \( s^t = s_L \) is observed in \( t - \eta_{1t} \) periods. Because the observed signals \( s^t \) are independently and identically distributed across time, any history of signals can be summarized by the number of times a given
signal is observed in that history. Thus, \( \rho_{1t} \) can be rewritten as:

\[
\rho_{1t} = \left[ \frac{p_s^{1-\eta_1}}{p_s^{\eta_1} (1-p_s)^{1-\eta_1}} \right]^t = \left[ \frac{p_s^{1-2\eta_1}}{p_s^{\eta_1} (1-p_s)^{1-2\eta_1}} \right]^t
\]

Notice that \( \frac{p_s}{1-p_s} > 1 \). A direct application of the law of large numbers indicates that as \( t \to \infty \), \( \frac{\eta_1}{t} \to \theta_H \) and \( 1 - 2\frac{\eta_1}{t} \to 0 \). This, in turn, implies that \( \rho_{1t} \to \left\{ \begin{array}{ll} 0 & \text{if } \theta_j = \theta_H \\ \infty & \text{if } \theta_j = \theta_L \end{array} \right. \).

2. Let \( \eta_{2t} \) denote the number of times \( c^T = \tilde{a} \) until period \( t \). Given the law of large numbers and equilibrium strategies for both types of the borrower, it can be said that if \( \theta_j = \theta_H \) then \( \eta_{2t} \to \theta_H \). Otherwise, it is bounded above by \( \theta_L \). Hence, a sufficiently large history of choices would perfectly reveal the borrower’s type through the number of times repayment is observed. This implies that for any given sequence of observed choices \( \cup_{t=1}^{T} \{c^T\} \), \( \prod_{t=1}^{T} \Pr (c^T | \theta_j = \theta_L) = 0 \) if \( \theta_j = \theta_H \) and \( \prod_{t=1}^{T} \Pr (c^T | \theta_j = \theta_H) = \infty \) if \( \theta_j = \theta_L \).

Combining the behavior of \( \rho_{1t} \) and \( \rho_{2t} \), it is easy to see that \( R_t \to \left\{ \begin{array}{ll} 1 & \text{if } \theta_j = \theta_H \\ 0 & \text{if } \theta_j = \theta_L \end{array} \right. \).

**Proof of Proposition 3.** The derivatives of \( R_t \) with respect to \( p_s \) is \( \frac{dR_t}{dp_s} = -R_t^2 \cdot \frac{1-q}{q} \rho_{2t} \frac{dp_{1t}}{dp_s} \) where \( \rho_{1t} \) and \( \rho_{2t} \) are as defined in the proof of Lemma 4. The derivative of \( \rho_{1t} \) with respect to \( p_s \) reveals that \( \frac{dp_{1t}}{dp_s} = t \frac{1-2\eta_1}{p_s^{\eta_1} (1-p_s)^{1-2\eta_1}} \). Applying the law of large numbers reveals that \( \frac{dp_{1t}}{dp_s} \to \left\{ \begin{array}{ll} < 0 & \text{if } \theta_j = \theta_H \\ > 0 & \text{if } \theta_j = \theta_L \end{array} \right. \). Hence,
as \( t \to \infty \), \( R_t \) is increasing in \( p_s \) if \( \theta_j = \theta_H \) and is decreasing in \( p_s \) if \( \theta_j = \theta_L \).

Differentiating \( R_t \) with respect to \( p_s \) yields

\[
\frac{d^2 R_t}{dp_s^2} = - \frac{1 - q}{q} \rho_{2t} \left[ R_t^2 \frac{d^2 \rho_{1t}}{dp_s^2} + 2R_t \frac{d \rho_{1t}}{dp_s} \frac{dR_t}{dp_s} \right] = - \frac{1 - q}{q} \rho_{2t} \frac{d \rho_{1t}}{dp_s} \left[ R_t^2 \frac{2p_s - 1}{p_s (1 - p_s)} + 2R_t \frac{dR_t}{dp_s} \right]
\]

If \( \theta_j = \theta_H \), \( \frac{dR_t}{dp_s} > 0 \) as \( t \to \infty \) and hence, \( \frac{d^2 R_t}{dp_s^2} \) is positive. If \( \theta_j = \theta_L \), then \( R_t^2 \frac{2p_s - 1}{p_s (1 - p_s)} + 2R_t \frac{dR_t}{dp_s} \geq \frac{1}{2} > 0 \) in limit. Hence, \( \frac{d^2 R_t}{dp_s^2} \to \begin{cases} > 0 & \text{if } \theta_j = \theta_H \\ < 0 & \text{if } \theta_j = \theta_L \end{cases} \)

That is, convergence happens at a faster rate as signals become more informative.

\begin{center} \textbf{References} \end{center}


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