Entry and Litigation under Sequential Innovation

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Abstract

We study a patent holding incumbent’s incentives to litigate versus accommodate a new market entrant with a quality improving product. Patent protection is uncertain and is characterized by a patentability standard and patent breadth determined during litigation. While litigation has the benefit of blocking an infringing product, it carries a risk of patent invalidation. As a result, the incumbent accommodates large improvements, which are both less likely to be infringing and more differentiated from the incumbent’s product. Litigation occurs for the improvements which are less differentiated from the incumbent’s product and are more likely to be deemed infringing. Improving the incumbent’s market position by increasing the quality of his product or the strength of his patent strengthens his litigation incentives, but it can also benefit the entrant since a stronger incumbent mitigates market competition from inferior non-patented product variants. As a result, we find that entrant’s innovation incentives may be higher if the current leading product on the market is protected by a patent.

Keywords: sequential innovation, patentability standard, infringement standard, litigation, accommodation, settlement.

JEL Classification: K4, L1, O3

PRELIMINARY AND INCOMPLETE DRAFT.
1 Introduction

Innovations are sequential in nature where each subsequent innovation often utilizes the know-how acquired for previous innovations and/or an improvement over them. This sequential nature of innovation makes it a natural breeding ground for intellectual property disputes. While patents grant some monopoly rights to innovators, the protection granted by them is far from perfect. This is so because while the term of most patents is set at 20 years, the range of innovations which are covered by them is often unclear thereby making it harder for an innovator to exercise his right of exclusion. In 2014, over 7000 patent lawsuits were filed in US courts suggesting the widespread use of courts for enforcing property rights. According to a report issued by PwC, over the last 25 years the number of patent related filings in court have registered an annual growth rate of about 8%. Patent disputes which go to trial stage in court are surrounded by a great deal of uncertainty due to ambiguity regarding patent’s scope. The alleged infringer often questions the validity of the patent for his defense, thereby exposing the patent to the risk of invalidity.

Given this environment, a question arises of what type of innovations tend to be litigated and how the structure of the patent system and the strength of patent protection affect the incumbent’s choice of litigation versus accommodation. Apart from understanding the proclivity for litigation, studying patent protection in the context of cumulative innovation is also important in understanding its impact on innovation incentives and thus the pace of technological progress.

This paper contributes to the theoretical literature on cumulative innovation by studying an infinite horizon game, in which innovators build on each other by introducing quality improving innovations. In this respect, an entrant is not simply an imitator, but an innovator who adds quality variety to the market. This is in stark contrast to some of the existing literature on patent infringement that models new entrants as pure imitators. Allowing for quality improving entrants has important implications on the possible market outcomes.

In each period, a new firm is endowed with an innovation capacity and chooses how much to invest in R&D to realize a higher quality product. Upon successful innovation and entry into market, the innovating firm faces a threat of litigation by a current patent holding incumbent. Patent protection in our model is characterized by a patentability standard, patent breadth and patent length. To account for uncertainty in court’s proceedings, we model the patentability standard as a stochastic variable that determines the minimum improvement that a new innovation needs to attain in order to be patentable. Thus, the larger the incumbent’s improvement, the more likely it is that his patent will withstand the court’s scrutiny. Similarly, patent breadth is modeled as a stochastic standard that determines whether a new improvement is infringing on a current patented technology. A larger improvement is thus associated with a lower likelihood of being found infringing.

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1Source: www.palexia.com
2Challenging the validity of the patent in question is one of the most common defenses employed by the alleged infringers in court. See any textbook in patent law, e.g., Bouchoux (2012) pp. 424-425
Finally, the patent length determines the time duration for which an innovation enjoys patent protection, which is normalized to two periods in our model. This allows each innovator to play the role of an entrant in one period and an incumbent in the next.

Our model gives rise to the prediction that the incumbent accommodates large improvements, which are both less likely to be infringing and less profit eroding for the incumbent. The lower profit erosion effect of larger improvements is driven by the consumer’s heterogeneous willingness to pay for quality, which introduces product differentiation in our model. In contrast, litigation occurs for incremental improvements since the innovator is stochastically more likely to win in court and such innovations are in closer competition with the incumbent’s product.

As expected, improving the incumbent’s market position by increasing the quality of his product or the strength of his patent increases the range of improvements that are being litigated and reduces the accommodation region. While strengthening the incumbent’s position increases the likelihood of litigation, we show that this is not necessarily detrimental to the entrant’s profits. This is so because a stronger incumbent may also mitigate market competition coming from inferior non-patented technologies. In particular, improving the quality of the incumbent’s product may benefit the entrant as it mitigates market competition by increasing quality differentiation. As a result, the entrant’s profit may increase in the incumbent’s quality improvement as the positive product differentiation effect dominates the negative litigation effect. Similarly, we show that strengthening the incumbent’s patent validity claim may increase the entrant’s profit. Intuitively, a weaker patentability standard affects the entrant through his current profit as an entrant and his expected continuation profit as an incumbent. Increasing the likelihood of patent validity above zero improves the entrant’s continuation payoff as it enables him to pursue litigation and settlement against incremental future innovations. At the same time, a moderate increase in the likelihood of patent validity is likely to have little effect on the current entrant’s profits if his innovation is fairly significant as it provides weak incentives for litigation for the current incumbent. As a result, we show that the impact of stronger validity claim has higher impact on the entrant’s continuation payoff than the current payoff when the patentability standard is sufficiently weak. This points to the desirability of some degree of patent protection by future innovators.

An important implication of the entrant’s benefit from facing a stronger incumbent is that the entrant’s innovation incentives may be higher if the current leading product on the market is protected by a patent. This is due to the fact that patents are instrumental in reducing existing market competition, which makes entry more profitable. This observation contrasts some of the existing literature on cumulative and overlapping innovations (see e.g. Bessen and Maskin (2009); Hunt (2006); Scotchmer (1991)), which argues that patent protection discourages cumulative innovations due to the threat of litigation. While this effect is present in our model as well, we show that the market competition effect may dominate the litigation effect when the incumbent is sufficiently small.

This paper is closely related to the theoretical literature that studies patent litigation and the role of patents in discouraging profit eroding imitation. Meurer (1989), Choi (1998), Aoki et al. (1999), Duchêne and Serfes (2012) study litigation incentives when the entrant
is a pure imitator, which contrast with our setting in which the entrants is an innovator himself. Notable exceptions are Crampes and Langinier (2002) and Llobet (2003) who allow for entry with differentiated products. Crampes and Langinier take a reduced form approach to competition and endogenize the monitoring effort of the incumbent that facilitates imitation detection. Our model complements theirs by considering how the degree of quality improvement affects the incumbent’s litigation incentives. In this respect, our model is closer to Llobet’s who also considers a quality improving entrant. However, while his model conjectures homogeneous consumers, resulting in only the best quality product being active on the market, we allow for heterogeneity in quality valuation among consumers. This has important implication on the market outcome. While in Llobet’s model accommodation is never an attractive option for the incumbent, who retains zero market share under such strategy, the incumbent in our model may find accommodation attractive since he retains the consumers with lower willingness to pay and thus realizes positive profits under accommodation.

Our work is also related to a growing literature that studies the role of patents in incentivizing innovation. Gilbert and Shapiro (1990), Klemperer (1990), Gallini (1992) study the socially optimal mix of patent breadth and length in awarding the innovator a compensation of a fixed size. While imitation in these models is allowed to the extent that patent breadth is narrow enough to allow inferior products (Klemperer, 1990) or costly invention around the patent (Gallini, 1992), the patent holder’s rights are assumed to be perfectly enforceable and thus never violated in equilibrium. In contrast, our model accounts for the uncertainty in the patent system, which results in litigation and possible patent infringement judgement by the court. In this respect, our paper is closest to the literature that studies the role of weak patents. Krasteva (2014) points out that an innovator with cost reducing innovation may benefit from imitation, which arises as a result of (stochastically) weak patent rights, since it encourages efficient imitation by competitors with inferior technology and some of the efficiency gain is transferred to the innovator via court’s appointed damages or settlement in the shadow of litigation. Since stronger protection discourages imitation, she finds that innovation incentives are maximized for intermediate level of protection. In contrast, our model considers cumulative and quality improving innovations, in which imitation is never desirable from the innovator’s point of view. Similar to the existing literature on cumulative innovation (e.g. Bessen and Maskin 2009; Hunt 2006; Scotchmer 1991), our model features a discouraging effect of stronger protection on future innovations due to the threat of litigation. However, this effect is countered by the positive effect of patents on mitigating existing competition on the market, which makes new entry more profitable. In this respect, we highlight an important aspect of patent protection not considered by the existing literature –namely its role in increasing not only the incumbent’s profits, but also the potential entrant’s profits via a less competitive market.

The rest of the paper is organized as follows. Section 2 sets up the base model, followed by the equilibrium analysis of litigation and accommodation outcome in Section 3. Section 4 discusses the impact of the incumbent’s size and patent policy on the equilibrium incentives to accommodate versus litigate, as well as on the equilibrium payoffs. Given the
equilibrium market outcome and profits, Section 5 studies the optimal investment in R&D and how it is impacted by the patent policy. The proofs of formal results are relegated to an appendix.

2 Base model

The environment consists of an infinite horizon discrete-time game, in which each period provides an opportunity for a quality improving innovation. In each period $t$, the market consists of at least one incumbent firm ($I$) and a potential entrant ($E$) firm. Product market is defined by the collection of patented and non-patented products of various qualities.

Consumer preferences: There is a continuum of consumers with heterogeneous willingness to pay for quality. If $q$ denotes product quality and $\theta_j$ consumer $j$’s willingness to pay for quality, $j$’s utility from consuming a product of quality $q$ and price $p$ is

$$u_j(q, p, \theta_j) = q\theta_j - p.$$ 

Based on their quality preference, $\theta_j$, consumers are uniformly distributed between 0 and 1. They base their consumption choice both on the available products’ prices as well as on their individual quality preferences.

Market competition: The firms in the market compete by setting their prices. There is free entry in the market and thus in absence of patent protection, the profit from new innovations would be instantaneously dissipated as firms freely appropriate the new technologies. However, a quality improving innovation enjoys patent protection for a limited time. We describe the patent environment below.

Patent protection: The patent policy is characterized by three policy variables: patentability standard, patent breadth and patent length.

Patentability: The patent law requires that a new invention is novel, non-obvious and useful in order to enjoy patent protection. While USPTO makes an initial determination of the patentability of a particular innovation, patent validity is often challenged in court when a patent holder initiates a patent infringement lawsuit. The patentability standard applied by the court determines the likelihood of a given patent being invalidated. Since patent validity is usually surrounded by a great deal of uncertainty, we model the patentability standard as a random variable $\Delta_v$ drawn from some distribution $F(\cdot | \tau_v)$ where $\Delta_v$ denotes the minimum improvement that an innovation needs to attain relative to prior art in order to be considered patentable. The parameter $\tau_v$ captures the stringency of the patentability standard, with higher $\tau_v$ denoting a more patentee friendly policy (i.e. $F_v(\cdot | \tau_v) > 0$). Suppose that in period $t$ a new innovator enters the market with a quality product $q_t$, which improves upon the existing prior art $q_{t-1}$ by $\Delta_t = q_t - q_{t-1}$. Then, the likelihood that the inventor $t$’s patent will be upheld by the court if challenged is $\gamma(\Delta_t, \tau_v) = F(\Delta_t | \tau_v)$ where $\gamma(0, \tau_v) = \gamma(\Delta_v, 0) = 0$ and $\gamma(\Delta_{\text{max}}, \tau_v) = \gamma(\Delta_v, \infty) = 1$.

Note that for simplicity we treat the USPTO office as a deterministic agency that automatically grants patents to new innovators and instead focus on the court as the main

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3See any textbook in patent law, e.g., [Bouchoux, 2012] pp 335-336
source behind the uncertainty in patent protection. This interpretation is for expositional ease and can be easily relaxed without affecting our qualitative results.

**Patent breadth:** Patent breadth determines how far reaching the patent is, i.e. the minimum size of improvement that another inventor must realize in order to be deemed non-infringing. A wider patent breadth provides stronger protection from new market entrants, but the actual breadth of a particular patent is highly uncertain and subject to the court’s determination during a patent infringement lawsuit. Similar to the patentability standard, patent breadth is modeled as a random variable $\Delta_b$, which is drawn from some distribution $G(\cdot|\tau_b)$, where $\Delta_b$ is the minimum inventive step that a new entrant needs to achieve in order to be deemed non-infringing. Higher $\tau_b$ denotes stochastically wider patent breadth (i.e. $G(\cdot|\tau_b) < 0$). Then, given a new entrants innovation size $\Delta_t$, the innovation is deemed infringing with probability $\beta(\Delta_t, \tau_b) = 1 - G(\Delta_t|\tau_b)$ where $\beta(0, \tau_b) = \beta(\Delta_t, \infty) = 1$ and $\beta(\Delta_t, 0) = 0$.

**Patent length:** The patent length is simply the length of time during which an innovator enjoys patent protection. In our base model, we consider a patent length of $t = 2$. This allows for each firm that enters the market to play the role of an entrant in one periods, risking a patent infringement lawsuit, and an incumbent in the subsequent period, having to defend its market share from a new market entry.

Given the above patentability environment, the complete timing of the game is described as follows. In each period $t$, a new firm has an opportunity to invest in R&D and draw a new innovation. The firm chooses its investment amount $R_t$ and realizes a new innovation with probability $p(R_t)$ where R&D has increasing and diminishing returns, i.e. $p'(R_t) > 0$ and $p''(R_t) \leq 0$. The quality of the new innovation $\Delta_t$ is a random draw from a stationary distribution $H(\Delta)$, where $\Delta$ denotes the quality improvement of the new innovation. Investment $R_t$ is associated with an increasing and convex cost function $c(R_t)$. Upon realizing an innovation of size $\Delta_t$, the firm obtains a patent with strength $\gamma(\Delta_t, \tau_v)$ and $\beta(\Delta_t, \tau_b)$, making the innovation public knowledge, and becomes an entrant in period $t$. If in period $t$ there is an incumbent with a valid patent, the incumbent chooses whether to litigate or accommodate the new entrant. Before proceeding to the equilibrium characterization for this game, we provide a more detailed description of the market competition in light of the patent environment laid out previously.

### 2.1 Market Competition in Period $t$

In each period $t$, there may be a number of non-patented technologies whose patents have expired. For simplicity, we assume that each new innovation enjoys 0 marginal production cost, which implies that only the best non-patented technology is present in the market at a particular period. The market price of the non-patented technology is $p^* = 0$ due to free entry.

In period $t$, there may also be a patent holding incumbent (I), who has entered in period...
$t - 1$ and has a product with quality improvement over the best non-patented technology of $\Delta_{t-1} = q_{t-1} - q_{t-2}$. If no new entry occurs in period $t$, the marginal consumer who is indifferent between the incumbent’s patented product and the non-patented product is given by

$$\theta_t q_{t-2} = \theta_t q_{t-1} - p \implies \theta_t = \frac{p}{\Delta_{t-1}}.$$  

Therefore, $I$ earns a market share of $1 - \frac{p}{\Delta_{t-1}}$ and a corresponding profit of

$$\pi(\Delta_{t-1}, p) = \left(1 - \frac{p}{\Delta_{t-1}}\right) p$$

Setting his price optimally at $p^* = \frac{\Delta_{t-1}}{2}$, the incumbent’s corresponding equilibrium profit in period $t$ is given by

$$\Pi(\Delta_{t-1}) = \frac{\Delta_{t-1}}{4}.$$  

Suppose instead that a new entry occurs in period $t$ with quality improvement $\Delta_t$ over $I$’s technology. There are three possible ways in which the incumbent may respond to new entry. First, he can choose to accommodate ($A$) the entrant, in which case the two firms compete on the product market by simultaneously setting their prices. Second, he can choose to litigate ($L$) the entrant, in which case the court has to determine whether the incumbent’s patent is valid and infringed upon. Below we describe the corresponding payoffs in period $t$ for each outcome $\sigma_t \in \{A, L\}$.

**Accommodation:** If the incumbent chooses to accommodate the entrant, the two firms compete by simultaneously setting their prices. The marginal consumer who is indifferent between the incumbent’s and entrant’s product is given by

$$\theta_E q_{t-1} - p^I = \theta_E q_t - p^E \implies \theta_E = \frac{p^E - p^I}{\Delta_t}.$$  

Therefore, the incumbent’s and the entrant’s corresponding profit functions are given by:

$$\pi^I(\Delta_{t-1}, \Delta_t, p^I, p^E) = \left[\frac{p^E - p^I}{\Delta_t} - \frac{p^I}{\Delta_{t-1}}\right] p^I$$

$$\pi^E(\Delta_{t-1}, \Delta_t, p^I, p^E) = \left[1 - \frac{p^E - p^I}{\Delta_t}\right] p^E$$

Solving for the Nash equilibrium prices of this simultaneous move game results in the following equilibrium profits for the incumbent and the entrant in period $t$:

$$\Pi^I(\Delta_{t-1}, \Delta_t, A) = \frac{(\Delta_t + \Delta_{t-1}) \Delta_t \Delta_{t-1}}{(4\Delta_t + 3\Delta_{t-1})^2}$$

$$\Pi^E(\Delta_{t-1}, \Delta_t, A) = \frac{4 (\Delta_t + \Delta_{t-1})^2 \Delta_t}{(4\Delta_t + 3\Delta_{t-1})^2}$$
Litigation: Litigation carries the risk of invalidating the incumbent’s patent with probability $1 - \gamma(\Delta_{t-1}, \tau_v)$, but also has the benefit of obtaining injunction against the entrant’s product with probability $\beta(\Delta_t, \tau_b)$. An injunction prevents the entrant from producing until the incumbent’s patent expires. Therefore, the expected profits from litigation in period $t$ for the incumbent and the entrant are given by:

$$\Pi^I(\Delta_{t-1}, \Delta_t, L) = \gamma(\Delta_{t-1}, \tau_v) \left[ \beta(\Delta_t, \tau_b) \Pi(\Delta_{t-1}) + (1 - \beta(\Delta_t, \tau_b)) \Pi^I(\Delta_{t-1}, \Delta_t, A) \right]$$

$$\Pi^E(\Delta_{t-1}, \Delta_t, L) = \gamma(\Delta_{t-1}, \tau_v) (1 - \beta(\Delta_t, \tau_b)) \Pi^E(\Delta_{t-1}, \Delta_t, A) + (1 - \gamma(\Delta_{t-1}, \tau_v)) \Pi(\Delta_t)$$

The above payoffs reflect the possibility that the incumbent’s patent is invalidated by the court, in which case the incumbent’s profits are fully dissipated by free entry into his product variant. Then entrant’s resulting profit is $\Pi(\Delta_t)$. Upon upholding the incumbent’s patent, with probability $\gamma(\Delta_{t-1}, \tau_v)$, and finding the entrant’s product infringing, with probability $\beta(\Delta_t, \tau_b)$, the incumbent realizes profits of $\Pi(\Delta_{t-1})$, while the entrant’s realizes 0 profits as his production is blocked until the incumbent’s patent expires. If the entrant’s product is found non-infringing, the two firms will compete in Bertrand fashion with corresponding profits given by equations (4) and (5).

Since an entrant in period $t$ plays the role of an incumbent in period $t + 1$, the total expected payoff for an entrant also includes his continuation payoff as an incumbent in the next period given by

$$E_{\Delta_{t+1}}(\Pi^I(\Delta_t, \Delta_{t+1}, \sigma_{t+1})) = p(R_{t+1}) \Pi^I(\Delta_t, \Delta_{t+1}, \sigma_{t+1}) + (1 - p(R_{t+1})) \Pi(\Delta_t)$$

$\Pi^I(\Delta_t, \Delta_{t+1}, \sigma_{t+1})$ denotes the incumbent’s payoff in period $t + 1$ in the presence of a new entry in period $t + 1$, which occurs with probability $p(R_{t+1})$. Otherwise, the incumbent in period $t + 1$ becomes the only firm with a patented product and earns the profits given by equation (2).

In section 3, we characterize the equilibrium of the subgame in which a new entry occurs on the market.

3 Equilibrium Derivation

3.1 When do accommodation and litigation occur?

We consider a Markov Perfect Equilibrium of the subgame, in which the incumbent chooses how to respond to new entry. In such equilibrium, players’ strategies depend only on the payoff relevant variables $\Delta_{t-1}, \Delta_t, \tau_v$ and $\tau_b$ and not on the calendar time.

As described above, an incumbent responds to entry by either litigating, accommodating or settling with the entrant. The entrant and incumbent may settle outside the court only if the entrant has a credible threat to litigate. Proposition 1 characterizes the conditions under which such credible threat exists.

**Proposition 1** There exists an unique $\Delta^A_t(\Delta_{t-1}, \tau_v, \tau_b) \in [0, \Delta_{max}]$ such that the incumbent prefers accommodation over litigation if and only if $\Delta_t \geq \Delta^A_t(\Delta_{t-1}, \tau_v, \tau_b)$ where $\Delta^A_t(0, \tau_v, \tau_b) = \Delta^A_t(\Delta_{t-1}, 0, \tau_b) = \Delta^A_t(\Delta_{t-1}, \tau_v, 0) = 0$ and $\Delta^A_t(\Delta_{t-1}, 1, \tau_b) = \Delta_{max}$. 

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Proposition 1 states that there exists a threshold level of improvement, \( \Delta^t_{i-1} (\Delta_t, \tau_v, \tau_b) \), above which the incumbent never litigates. Given equations (4) and (6), the incumbent prefers litigation if \( \Pi^t (\Delta_{t-1}, \Delta_t, L) > \Pi^t (\Delta_{t-1}, \Delta_t, A) \). Or equivalently, the incumbent holds a credible threat to litigate if

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\frac{\gamma (\Delta_{t-1}, \tau_v) \beta (\Delta_t, \tau_b) \left[ \Pi (\Delta_{t-1}) - \Pi^t (\Delta_{t-1}, \Delta_t, A) \right]}{(1 - \gamma (\Delta_{t-1}, \tau_v)) \Pi^t (\Delta_{t-1}, \Delta_t, A)} > 1
\]

(8)

The left-hand size of equation (8) represents the benefit-cost ratio of litigation relative to accommodation. The numerator denotes the incumbent’s expected benefit from litigation. From the incumbent’s point of view, the benefit of litigation lies in his ability to block the entrant’s production whenever the entrant’s product is found infringing, which limits market competition and increases the incumbent’s profit by \( \Pi (\Delta_{t-1}) - \Pi^t (\Delta_{t-1}, \Delta_t, A) \). The denominator denotes the incumbent’s expected costs of litigation. The drawback of litigation for the incumbent consists of the risk of patent invalidation, resulting in profit loss of \( \Pi^t (\Delta_{t-1}, \Delta^t_t, A) \). For low values of \( \Delta_t \), the likelihood of an infringement verdict \( \beta (\Delta_t, \tau_b) \) is high, making litigation an appealing option. In particular, for \( \Delta_t = 0 \), \( \Pi^t (\Delta_{t-1}, 0, A) = 0 \) and thus the benefit-cost ratio exceeds 1. An increase in \( \Delta_t \) has two effects. First, it decreases the likelihood \( \beta (\Delta_t, \tau_b) \) of successful litigation. Second, it decreases the relative profitability of litigation over accommodation, \( \frac{\Pi (\Delta_{t-1})}{\Pi^t (\Delta_{t-1}, \Delta^t_t, A)} \), as greater product differentiation between the entrant and the incumbent mitigates competition and improves the accommodation profit. Both effects reduce the benefit-cost ratio of litigation, explaining the lack of litigation incentives above \( \Delta^t_{i-1} (\Delta_{t-1}, \tau_v, \tau_b) \).

As anticipated, \( \Delta^t_{i-1} (\Delta_{t-1}, \tau_v, \tau_b) \) is affected by the patent strength. At the extreme of no protection (i.e. either \( \Delta_{t-1} = 0 \) or \( \tau_v = 0 \) or \( \tau_b = 0 \)), the benefit-cost ratio is 0 and the incumbent never finds it optimal to litigate, i.e. \( \Delta^t_{i-1} (\Delta_{t-1}, \tau_v, \tau_b) = 0 \). At the other extreme of iron-clad patent (i.e. \( \gamma (\Delta_{t-1}, \tau_v) = 1 \) and \( \beta (\Delta_t, \tau_b) > 0 \)), the benefit-cost ratio goes to infinity and thus the incumbent always prefers litigation over accommodation, i.e. \( \Delta^t_{i-1} (\Delta_{t-1}, \tau_v, \tau_b) = \Delta^{\text{max}} \).

Having characterized the equilibrium of the stage game, we next study how the incumbent’s innovation size and the strength of his patent protection impacts the likelihood of litigation and market profits. The effect that these variables have on the market outcome is important not only because of their impact on the incumbent’s and entrant’s profits, but also because of their role in incentivizing new product improvements. In Section 4, we analyze the comparative static effects of \( \Delta_{t-1}, \tau_v \) and \( \tau_b \) on the market outcome and firm profitability. Section 5 discusses how changes in these variables impact innovation incentives.

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5We break the incumbent’s indifference in the favor of accommodation over litigation. Such tie-breaking rule is efficient whenever litigation is costlier than accommodation.
4 The impact of the incumbent’s size and patent policy on the equilibrium outcome.

4.1 The impact of the incumbent’s size

The more significant the incumbent’s innovation, the stronger is his patent validity claim. Intuitively, this should make the incumbent more willing to engage in litigation. Proposition 2 confirms this intuition.

Proposition 2 \( \Delta_t^A \) is increasing in \( \Delta_{t-1} \).

Proposition 2 states that as the incumbent’s quality improvement increases, he is less willing to accommodate new entry. To understand this result, note that at \( \Delta_t = \Delta_t^A > 0 \), the benefit-cost ratio of litigation over accommodation given by equation (8) is exactly equal to 1. An increase in \( \Delta_{t-1} \) has two effects on this benefit-cost ratio. First, it increases the incumbent’s patent validity claim, \( \gamma(\Delta_{t-1}, \tau_v) \), which clearly increases the benefit-cost ratio. Second, \( \Delta_{t-1} \) increases the relative profitability of litigation over accommodation, \( \frac{\Pi(\Delta_{t-1})}{\Pi(\Delta_{t-1}, \Delta_t, A)} \), as the incumbent’s profits are more responsive to the size of the innovation when facing less competition, i.e. \( \frac{d\Pi(\Delta_{t-1})}{d\Delta_{t-1}} > \frac{d\Pi(\Delta_{t-1}, \Delta_t, A)}{d\Delta_{t-1}} > 0 \). Therefore, both effects work in the same direction of increasing the benefit-cost ratio of litigation over accommodation, making litigation relatively more appealing.

The following Corollary is an immediate consequence of Propositions 1 and 2.

Corollary 1 Given \( \Delta_t \), there exist cutoff \( \Delta_{t-1}^A(\Delta_t, \tau_v, \tau_b) = (\Delta_{t-1}^A(\Delta_t, \tau_v, \tau_b))^\ast \) such that \( \sigma^\ast = A \) for \( \Delta_{t-1} \leq \Delta_{t-1}^A(\Delta_t, \tau_v, \tau_b) \) and \( \sigma^\ast = L \) for \( \Delta_{t-1} > \Delta_{t-1}^A(\Delta_t, \tau_v, \tau_b) \) where \( \Delta_{t-1}^A(0, \tau_v, \tau_b) = \Delta_{t-1}^A(\Delta_t, 1, \tau_b) = 0 \), and \( \Delta_{t-1}^A(\Delta_t, 0, \tau_b) = \Delta_{t-1}^A(\Delta_t, \tau_v, 0) = \Delta_{t-1}^\max \).

Corollary 1 is useful in studying the impact of the incumbent’s innovation size on the market profits given by the following Proposition.

Proposition 3 (Equilibrium payoffs) Given \( \tau_v \) and \( \tau_b \), the impact of \( \Delta_{t-1} \) on the equilibrium payoffs is as follows:

- Incumbent’s payoff \( \Pi^I(\Delta_{t-1}, \Delta_t, \sigma_t^\ast) \) is strictly increasing in \( \Delta_{t-1} \) and is maximized for \( \Delta_{t-1} = \Delta_{t-1}^\max \).
- Entrant’s stage game payoff \( \Pi^E(\Delta_{t-1}, \Delta_t, \sigma_t^\ast) \) is non-monotonic in \( \Delta_{t-1} \). If \( \Pi^E(\Delta_{t-1}^A, \Delta_t, A) > \Pi^E(\Delta_{t-1}^\max, \Delta_t, L) \), then \( \Pi^E(\Delta_{t-1}, \Delta_t, \sigma_t^\ast) \) is maximized for \( \Delta_{t-1} = \Delta_{t-1}^A \). Otherwise, \( \Pi^E(\Delta_{t-1}, \Delta_t, \sigma_t^\ast) \) is maximized for \( \Delta_{t-1} = \Delta_{t-1}^\max \).

Proposition 3 states that both the incumbent and the entrant may benefit from stronger incumbent in terms of the incumbent’s product size. As highlighted in Corollary 1, given entrant’s innovation size \( \Delta_t \), the incumbent accommodates if his own innovation size is
sufficiently small. Moreover, we know from equations (4) and (5) that the accommodation profits of both the incumbent and the entrant are increasing in $\Delta_{t-1}$ since being further away from the best non-patented technology reduces market competition, which in turn allows for higher market profits. Therefore, higher $\Delta_{t-1}$ can benefit both the patent-holding incumbent as well as the entrant, explaining Proposition 3.

As $\Delta_{t-1}$ increases beyond $\Delta^A_{t-1}(\Delta_t, \tau_v, \tau_b)$, the incumbent switches his response to entry from accommodation to litigation. As a result, entrant’s payoffs exhibit a discontinuity at this point and jump downward. Interestingly, entrant’s payoff from being litigated may be increasing in $\Delta_{t-1}$. This is due to the fact that a larger product improvement by the incumbent mitigates competition by the best non-patented technology, which has positive spillover effect on $\Pi^E(\Delta_{t-1}, \Delta_t, A)$. Further, a higher $\Delta_{t-1}$ translates to a lower likelihood of patent invalidation, in which case change in entrant’s payoff is given by $(1 - \beta(\Delta_t, \tau_b)) \Pi^E(\Delta_{t-1}, \Delta_t, A) - \Pi(\Delta_t)$, which may be positive or negative depending on the values of other parameters. Depending upon which of the aforementioned two effects dominate, $\Pi^E(\Delta_{t-1}, \Delta_t, L)$ may increase or decrease in $\Delta_t$. The event that $\Pi^E(\Delta_{t-1}, \Delta_t, L)$ is increasing in $\Delta_{t-1}$ gives rise to possibility that $\Pi^E(\Delta^A_{t-1}, \Delta_t, L)$ may be higher than $\Pi^E(\Delta^A_{t-1}, \Delta_t, A)$ and hence $\Pi^E(\Delta_{t-1}, \Delta_t, \sigma^*_i)$ will be maximized at $\Delta_{t-1} = \Delta^\text{max}$.

4.2 The impact of the patentability standard

In this Section, we study the impact of (stochastically) lower patentability standard on the market outcome and profits. Recall from Section 1 that higher $\tau_v$ increases the likelihood of the incumbent’s patent being upheld in court, i.e. $\frac{\partial \gamma(\Delta_{t-1}, \tau_v)}{\partial \tau_v} > 0$. The following Proposition describes the impact of higher $\tau_v$ on the incumbent’s incentives to litigate versus accommodate new market entry.

Proposition 4 (Equilibrium cutoffs) $\Delta^A_t$ is increasing in $\tau_v$.

A weaker patentability standard reduces the likelihood of patent invalidation, which increases the benefit-cost ratio of litigation over accommodation. As a result, the accommodation region decreases, while the litigation region goes up. In contrast, the settlement region responds non-monotonically to stronger $\tau_v$. Clearly, for $\tau_v = 0$, $\gamma(\Delta_{t-1}, 0) = 0$ and the incumbent has no credible threat to litigate, precluding a settlement. As $\tau_v$ increases above 0, the accommodation cutoff $\Delta^A_t$ grows with a higher rate than the settlement cutoff $\Delta^S_t$ because when choosing between accommodation and litigation, the incumbent does not account for the negative impact of litigation on the entrant’s profit, making him more willing to litigate than jointly optimal. Similar to the comparative statics with respect to $\Delta_{t-1}$, there exists $\tau''_v$ such that $\Delta^A_t = \Delta^\text{max}$. Thus, above this value, the incumbent always has a credible threat to litigate, causing the settlement region to shrink as $\tau_v$ increases beyond $\tau''_v$.

Corollary 2 There exists $\tau^A_v(\Delta_{t-1}, \Delta_t, \tau_b)$ such that $\sigma^*_i = A$ for all $\tau_v \leq \tau^A_v(\Delta_{t-1}, \Delta_t, \tau_b)$ and $\sigma^*_i = L$ for all $\tau_v > \tau^A_v(\Delta_{t-1}, \Delta_t, \tau_b)$.
Next, we study the impact of a weaker patentability standard on the stage game equilibrium payoffs. In particular, we look at how the incumbent’s expected payoff $E_{\Delta_i}[\Pi^I(\Delta_{i-1}, \Delta_i, \sigma^*_i)]$ changes with the patentability standard while keeping the entrant’s innovation incentives, and thus the likelihood of new entry $p(R_i)$, constant. This payoff sheds light on the expected benefit of being an incumbent, which impacts the entrant’s expected payoff from market entry in period $t$ since he plays a dual role of an entrant in period $t$ and an incumbent in period $t+1$.

**Proposition 5** (Equilibrium payoffs) The impact of $\tau_v$ on the equilibrium payoffs is as follows:

- Incumbent’s equilibrium payoff $\Pi^I(\Delta_{i-1}, \Delta_i, \sigma^*_i)$ is (weakly) increasing in $\tau_v$.
- Entrant’s stage game equilibrium payoff $\Pi^E(\Delta_{i-1}, \Delta_i, \sigma^*_i)$ is non-monotonic in $\tau_v$ and is maximized for $\tau_v = \tau^A_v$.

Proposition 5 points out that from the incumbent’s point of view, a weaker patentability standard is payoff enhancing at least for lower values of $\tau_v$. Note that for $\tau_v = 0$, $\gamma(\Delta_i, 0) = \gamma(\Delta_{i+1}, 0) = 0$, so that accommodation always takes place in equilibrium, i.e. $\Delta^A(\Delta_{i-1}, 0, \tau_v) = 0$. By Proposition 4 increasing the patentability standard above $\tau_v = 0$, gives the incumbent a credible threat to litigate, which (weakly) increase the incumbent’s payoff as it enables him to pursue litigation against incremental innovations. While the incumbent’s accommodation payoff is independent of $\tau_v$, his litigation payoff is always increasing in the patent strength.

Proposition 5 also states that the entrant may also benefit from weaker patentability standard, despite the fact that this increases his chance of being litigated. While entrant’s payoff exhibits a downward jump at $\tau^A_v$, $\Pi^E(\Delta_{i-1}, \Delta_i, \Lambda)$ may be increasing or decreasing in $\tau_v$. The intuition behind this result is similar to that in Proposition 3. Having a patent holding incumbent results in expected change of $(1 - \beta(\Delta_i, \tau_b)) \Pi^E(\Delta_{i-1}, \Delta_i, \Lambda) - \Pi(\Delta_i)$ in entrant’s payoff. Depending upon whether this change is positive or negative, entrant’s payoff from being litigated is increasing or decreasing in $\tau_v$. This gives rise to the possibility that despite being litigated, entrant would benefit from a weaker patentability standard.

The possibility that the entrant benefits from weaker patentability standard has important implications on the entrant’s innovation incentives as we discuss in Section 5.

### 4.3 The impact of patent breadth

In this Section, we study the impact of (stochastically) wide patent breadth on the market outcome and profits. Recall from Section 1 that higher $\tau_b$ increases the likelihood of the entrant product being infringing on incumbent’s patent in court, i.e. $\frac{\partial \beta(\Delta_i, \tau_b)}{\partial \tau_b} > 0$. The following Proposition describes the impact of higher $\tau_b$ on the incumbent’s incentives to litigate and payoffs of market participants.

**Proposition 6**

1. $\Delta^A_i$ is increasing in $\tau_b$.
2. Incumbent’s equilibrium payoff $\Pi^I(\Delta_{i-1}, \Delta_i, \sigma^*_i)$ is (weakly) increasing in $\tau_b$.  

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3. Entant’s state game equilibrium payoff $\Pi^E(\Delta_{t-1}, \Delta_t, \sigma^*_t)$ is (weakly) decreasing in $\tau_b$.

Proposition 6 states that incumbent’s incentives to litigate the incumbent are increasing in $\tau_b$. While incumbent unambiguously benefits from higher $\tau_b$, entrant is always adversely affected by higher $\tau_b$. As entrant becomes an incumbent in next period, Proposition 6 highlights the conflicting impact of strengthening $\tau_b$ on entrant’s payoffs in current period payoffs versus next period. The possibility that an entrant stands to benefit from stochastically wider patent breadth in next period has important implications on the his innovation incentives as discussed in Section 5.

5 Optimal Innovation Incentives

In this section, we consider the entrant’s problem of choosing the optimal investment in innovation. Recall that the entrant with innovation capacity in period $t$ chooses investment amount $R_t$ and realizes a new innovation with probability $p(R_t)$. The new innovation is drawn from a stationary distribution with cdf $H(\Lambda)$. While the success probability has increasing and diminishing returns, i.e. $p'(R_t) > 0$ and $p''(R_t) \leq 0$, investment is associated with convex cost function $c(R)$.

In period $t$, the potential entrant faces the following optimization problem:

$$\max_{R_t} p(R_t) E_{\Delta_t} \left[ \Pi^E(\Delta_{t-1}, \Delta_t, \sigma^*, \Lambda_t) + p(R_{t+1}) E_{\Delta_{t+1}} \left[ \Pi^I(\Delta_t, \Delta_{t+1}, \sigma^*, 1) \right] + (1 - p(R_{t+1})) \Pi(\Delta_t) \right] - c(R_t) \quad (9)$$

where $\Lambda_t = 0$ denotes a market without a patent-holding incumbent and $\Lambda_t = 1$ denotes a market with a patent-holding incumbent. The objective function above captures the fact that with probability $p(R_t)$ the entrant succeeds in generating an innovation, in which case he obtains an entrant’s equilibrium payoff of $\Pi^E(\Delta_{t-1}, \Delta_t, \sigma^*, \Lambda_t)$ in the current payoff and an incumbent’s continuation payoff in the next period $p(R_{t+1}) E_{\Delta_{t+1}} \left[ \Pi^I(\Delta_t, \Delta_{t+1}, \sigma^*, 1) \right] + (1 - p(R_{t+1})) \Pi(\Delta_t)$, which is a function of the next period’s investment level $R_{t+1}$. In period $t$, the first order optimization condition can be written as:

$$p'(R^*(\Delta_{t-1}, \Lambda_t)) E_{\Delta_t} \left[ \Pi^E(\Delta_{t-1}, \Delta_t, \sigma^*, \Lambda_t) + \Pi(\Delta_t) - p(R^*(\Delta_t, 1)) \left[ \Pi(\Delta_t) - E_{\Delta_{t+1}} \left[ \Pi^I(\Delta_t, \Delta_{t+1}, \sigma^*, 1) \right] \right] \right] = c'(R^*(\Delta_{t-1}, \Lambda_t)) \quad (10)$$

The above condition simply states that at the optimum, the entrant equates the marginal benefit of investment, which is the marginal increase in expected profits due to the higher investment level, and the marginal cost of investment, captured by $c'(R^*(\Delta_{t-1}, \Lambda_t))$. We are looking for a stationary Markov equilibrium, $R^*(\Delta_{t-1}, \Lambda_t)$, which is only a function of the current state of the market captured by the best technology characteristic $\Delta_{t-1}$ and its patent status $\Lambda_t$, and does not depend on the calendar time. This allows us to rewrite equation (10) as
\[
E_{\Delta_t-1} [\Pi (\Delta_t-1)] + E_{\Delta_t-1} \left[ p(R^*(\Delta_{t-1},1)) \left[ E_{\Delta_t} \left[ \Pi^I (\Delta_{t-1}, \Delta_t, \sigma^*, 1) \right] - \Pi (\Delta_{t-1}) \right] \right] = \frac{c' (R^*(\Delta_{t-1},1))}{p'(R^*(\Delta_{t-1},1))} - E_{\Delta_t} \left[ \Pi^E (\Delta_{t-1}, \Delta_t) \right].
\] (11)

The above equation completely determines the equilibrium value of \( R^*(\Delta_{t-1},1) \). Having obtained \( R^*(\Delta_{t-1},1) \), the equilibrium value of \( R^*(\Delta_{t-1},0) \) follows from equation (10).

Given equations (10) and (11) that characterize the optimal investment level by the entrant, we are interested in how this investment level varies with the incumbent’s size, \( \Delta_{t-1} \), and the patent policy, captured by \( \tau_0 \) and \( \tau_b \).

From Proposition 2, we know that a stronger incumbent in terms of his innovation size, increases the likelihood of the entrant facing litigation upon entry as it improves the incumbent’s patent validity claim and increases his relative profitability of litigation over accommodation, \( \Pi(\Delta_{t-1}) \). Intuitively, this should have a discouraging impact on innovation investment by the entrant. However, we also know that the entrant’s accommodation profit \( \Pi^E(\Delta_{t-1}, \Delta_t, A) \) is increasing in the incumbent’s product quality as a stronger incumbent also implies a less competitive market. This tends to increase the benefit of entry, and thus innovation incentives. The following proposition states that for a sufficiently small incumbent, the entrant’s benefit from softer competition outweighs the cost of higher litigation likelihood, causing the entrant to increase his investment level in response to higher \( \Delta_{t-1} \).

**Proposition 7** The entrant’s investment level in the presence of a patent-holding incumbent, \( R^*(\Delta_{t-1},1) \), is maximized for a positive value of \( \Delta_{t-1} \). Formally, \( \lim_{\Delta_{t-1} \to 0} \frac{\partial R^*(\Delta_{t-1},1)}{\partial \Delta_{t-1}} > 0 \).

Proposition 7 is an immediate consequence of Proposition 3, which shows that the entrant’s current payoff is increasing in \( \Delta_{t-1} \) for sufficiently small values of \( \Delta_{t-1} \). This is due to the fact that as \( \Delta_{t-1} \) goes to 0, the litigation threat disappears and the entrant is more concerned about the impact on the incumbent’s size on the market competition. A stronger incumbent results in a greater product differentiation between the best patented and non-patented technology, which makes entry more attractive.

An immediate Corollary of Proposition 7 is that the entrant’s innovation incentives may be higher if the existing best technology on the market is protected by a patent.

**Corollary 3** There exists \( \tilde{\Delta}_{t-1} > 0 \) such that for all \( \Delta_{t-1} \in (0, \tilde{\Delta}_{t-1}] \), \( R^*(\Delta_{t-1},1) > R^*(\Delta_{t-1},0) \).

Note that for \( \Delta_t = 0 \), the entrant does not face a litigation threat by the best non-patented technology and realizes a profit of \( \Pi(\Delta_t) \) in period \( t \). This payoff is equivalent to the payoff when \( \Delta_t = 1 \) and \( \Delta_{t-1} = 0 \) since litigation never takes place and \( \Pi^E(0, \Delta_t, A) = \Pi(\Delta_t) \). Since the entrant’s payoff is increasing in \( \Delta_{t-1} \) for sufficiently small values of \( \Delta_{t-1} \), it follows that the entrant would favor a patent holding incumbent as long as the incumbent is not too powerful to generate a significant litigation threat.

Corollary 3 highlights an important role of the patent system that has been overlooked by the existing literature. While Scotchmer [1991] and Bessen and Meurer [2006] emphasize the negative impact of patents of generating substantial litigation threat for future
innovators, and thus discouraging entry, we find that patents also play the role of reducing existing market competition, which can in turn make entry of improved products more profitable. As a result, innovation incentives for future entrants may be higher if the current leading product on the market is protected by a patent.

Next, we turn our attention to the impact of strengthening patent protection via weaker patentability standard $\tau_v$. As pointed out by Proposition 4, this increases the incentives of the incumbent to litigate since he faces a lower threat of patent invalidation. This, however, is not necessarily bad news for the entrant. As pointed out by Proposition 5, the entrant may benefit from weaker patentability standard, and thus stronger incumbent, since he takes into account the impact of such policy not only on his current payoff, but also on his future payoff as an incumbent. The following Proposition states that this might translate into stronger innovation incentives for the entrant.

**Proposition 8** There exists $\hat{\Delta}_{t-1} > 0$ such that for all $\Delta_{t-1} \in [0, \hat{\Delta}_{t-1}]$, the entrant’s investment level in the presence of a patent-holding incumbent is maximized for a positive value of $\tau_v$. Formally, 

$$\lim_{\tau_v \to 0} \frac{\partial R^*(\Delta_{t-1}, 1)}{\partial \tau_v} > 0 \text{ for } \Delta_{t-1} \in [0, \hat{\Delta}_{t-1}]$$

Proposition 8 states that the entrant’s investment level increases in $\tau_v$ if the patentability standard is sufficiently weak and the entrant faces a relatively small incumbent. Intuitively, with a weak incumbent, strengthening the patentability standard will have little effect on the current entrant’s profit as it would provide only moderate litigation incentives in the current period. However, an increase in the patentability standard can have a significant positive effect on the entrant’s continuation payoff by strengthening his litigation threat against incremental future innovations. As a result, the trade-off between the current adverse effect and the future positive effect of stronger patent protection is realized in favor of stronger protection when the current incumbent is sufficiently weak.

### 6 Conclusion

This paper studies the litigation choices made by the existing patent holder when facing the entrant into the market. It shows that the incumbent inclines to litigate incremental improvement, settle with intermediate and accommodate significant innovations. An important finding of this paper, which is overlooked by most existing literature, is that a stronger incumbent may act as an incentive to potential innovator, since he can help with mitigating the competition from existing technology the entrant has to face. The findings also suggests that the entrant’s innovation incentives can be higher if the incumbent is protected by a patent. A fruitful topic for future work includes exploring the effect of longer patent term on the innovation incentive and social welfare.

### Appendix: Proofs

**Proof of Proposition 1** Define

$$G^I(\Delta_{t-1}, \Delta_t,A) \equiv \Pi^I(\Delta_{t-1}, \Delta_t,A) - \Pi^I(\Delta_{t-1}, \Delta_t,L) \quad (A-1)$$
$G^I(\Delta_{t-1}, \Delta_t, A)$ denotes the net gain in incumbent’s payoff from choosing *Accommodation*. Incumbent would find it optimal to choose *Accommodation* if and only if $G^I(\Delta_{t-1}, \Delta_t, A) \geq 0$. Depending on the values of $\gamma(\cdot)$ and $\beta(\cdot)$, there are 3 possibilities might arise:

1. Either $\gamma(\cdot) = 0$ or $\beta(\cdot) = 0$. It is immediate that $G^I(\Delta_{t-1}, \Delta_t, A) \geq 0$ for all $\Delta_t$ resulting in $\Delta^A_t = 0$.

2. $\gamma(\cdot) = 1$ and $\beta(\cdot) > 0$. Notice that $\Pi^I(\Delta_{t-1})$ is higher than $\Pi^I(\Delta_{t-1}, \Delta_t, A)$ and hence, $G^I(\Delta_{t-1}, \Delta_t, A) \leq 0$ for all $\Delta_t$ resulting in $\Delta^A_t = \Delta^{\max}$. 

3. $0 < \gamma(\cdot) < 1$ and $\beta(\cdot) > 0$. For $\Delta_t = 0$, $\Pi^I(\Delta_{t-1}, 0, A) = 0$ while litigation results in a positive expected payoff $\gamma(\Delta_{t-1}, \sigma_v) \Pi^I(\Delta_{t-1})$ resulting in $G^I(\Delta_{t-1}, 0, A) < 0$ implying that Incumbent prefers litigation for $\Delta_t = 0$ and $\Delta^I_t > 0$. The derivative of $G^I(\Delta_{t-1}, \Delta_t, A)$ with respect to $\Delta_t$ is given by:

$$
\frac{\partial G^I(\Delta_{t-1}, \Delta_t, A)}{\partial \Delta_t} = (1 - \gamma(\Delta_{t-1}, \tau_v) + \gamma(\Delta_{t-1}, \tau_v)\beta(\Delta_t, \tau_b)) \frac{\partial \Pi^I(\Delta_{t-1}, \Delta_t, A)}{\partial \Delta_t} - \gamma \frac{\partial \beta(\Delta_t, \tau_b)}{\partial \Delta_t} (\Pi(\Delta_{t-1}) - \Pi^I(\Delta_{t-1}, \Delta_t, A))
$$

Notice that $\Pi^I(\Delta_{t-1}, \Delta_t, A) \leq \Pi(\Delta_{t-1})$. This is so because $\Pi(\Delta_{t-1})$ reflects incumbent’s profits when he produces the best patented innovation in the market, while $\Pi^I(\Delta_{t-1}, \Delta_t, A)$ reflects incumbent’s profits when he faces additional competition from an entrant with improved innovation $\Delta_t$. Combining this with the facts that $0 \leq \gamma, \beta \leq 1$, $\beta(\Delta_t, \tau_b)$ is decreasing in $\Delta_t$ and $\Pi^I(\Delta_{t-1}, \Delta_t, A)$ is increasing in $\Delta_t$, it can be seen that $G^I(\Delta_{t-1}, \Delta_t, A)$ is increasing in $\Delta_t$. Hence, as $\Delta_t$ increases, incumbent’s incentives to choose accommodation over litigation strengthen thereby yielding a threshold $\Delta^A_t(\Delta_{t-1}, \gamma, \beta)$ such that the incumbent no longer finds it optimal to litigate for $\Delta_t > \Delta^A_t(\Delta_{t-1}, \gamma, \beta) > 0$.

**Proof of Proposition**

Recall that $\Delta^A_t$ solves $G^I(\Delta_{t-1}, \Delta_t, A) = 0$ (given by equation (A-1)). Implicit Function Theorem yields that $\frac{\partial \Delta^A_t}{\partial \Delta_{t-1}} = -\frac{\partial G^I(\Delta_t, \Delta_t, A)}{\partial \Delta_{t-1}} |_{\Delta_t = \Delta^A_t}$ We have already shown in proof of Proposition that $\partial G^I(\Delta_t, \Delta_{t-1}, A)/\partial \Delta_{t-1} > 0$ for all $\Delta_t$. The derivative of $G^I(\Delta_t, \Delta_{t-1}, A)$ with respect to $\Delta_{t-1}$ given by:

$$
\frac{\partial G^I(\Delta_t, \Delta_{t-1}, A)}{\partial \Delta_{t-1}} = -\frac{\partial \gamma(\Delta_{t-1}, \tau_v)}{\partial \Delta_{t-1}} \left( \beta(\Delta_t) \left( \Pi(\Delta_{t-1}) - \Pi^I(\Delta_{t-1}, \Delta_t, A) \right) \right) + M(\Delta_t, \Delta_{t-1})
$$

where $M(\Delta_t, \Delta_{t-1}) \equiv (1 - \gamma(\Delta_{t-1})) \frac{\partial \Pi^I(\Delta_{t-1}, \Delta_t, A)}{\partial \Delta_{t-1}} - \gamma(\Delta_{t-1}) \beta(\Delta_t) \left( \frac{\partial \Pi(\Delta_{t-1})}{\partial \Delta_{t-1}} - \frac{\partial \Pi^I(\Delta_{t-1}, \Delta_t, A)}{\partial \Delta_{t-1}} \right)$. As $\gamma(\Delta_{t-1}, \tau_v)$ is increasing in $\Delta_{t-1}$ and $\Pi(\Delta_{t-1}) - \Pi^I(\Delta_{t-1}, \Delta_t, A)$ is positive, the first term of above expression is always negative. Next, we determine the sign of $M(\Delta_t, \Delta_{t-1})$
when evaluated at \( \Delta_t = \Delta_t^A \). Recall that \( G^i(\Delta_t^A, \Delta_t - 1, A) = 0 \), or equivalently \( \gamma(\Delta_t - 1) \beta(\Delta_t^A) = \frac{(1 - \gamma(\Delta_t - 1))\Pi^i(\Delta_t, \Delta_t - 1, \Delta_t^A)}{\Pi^i(\Delta_t, \Delta_t - 1, \Delta_t^A)} \). Substituting for \( \gamma(\Delta_t - 1) \beta(\Delta_t^A) \), and the derivatives of \( \Pi^i(\Delta_t, \Delta_t - 1, A) \) and \( \Pi(\Delta_t - 1) \) simplifies \( M(\Delta_t^A, \Delta_t - 1) \) to 
\[
\Delta_t^A(1 - \gamma(\Delta_t - 1))\Pi^i(\Delta_t, \Delta_t - 1, \Delta_t^A) = \frac{(2\Delta_t^A + 3\Delta_t - 1)\Delta_t^A}{(4\Delta_t^A + 3\Delta_t - 1)} \frac{\Pi^i(\Delta_t, \Delta_t - 1, A)}{\Pi^i(\Delta_t, \Delta_t - 1, \Delta_t^A)} \cdot \Delta_t^A \frac{\Pi^i(\Delta_t, \Delta_t - 1, \Delta_t^A)}{\Pi^i(\Delta_t, \Delta_t - 1, A)}
\]
which is clearly negative. This proves that \( G^i(\Delta_t^A, \Delta_t - 1, A) \) is decreasing in \( \Delta_t - 1 \) which, in turn, implies that \( \Delta_t^A \) is increasing in \( \Delta_t - 1 \).

**Proof of Proposition 3**

As highlighted by corollary \( \sigma^* = A \) for \( \Delta_t - 1 \leq \Delta_t^A(\Delta_t, \gamma, \beta) \). In this region, entrant’s payoffs are \( \Pi^E(\Delta_t - 1, \Delta_t, A) + E_{\Delta_t - 1}[\Pi^i(\Delta_t, \Delta_t - 1, \sigma^*_{t + 1})] \) and incumbent’s payoffs are \( \Pi^i(\Delta_t - 1, \Delta_t, A) \). The result follows immediately by observing that both \( \Pi^E(\Delta_t - 1, \Delta_t, A) \) and \( \Pi^i(\Delta_t - 1, \Delta_t, A) \) increasing in \( \Delta_t - 1 \), while \( E_{\Delta_t - 1}[\Pi^i(\Delta_t, \Delta_t - 1, \sigma^*_{t + 1})] \) is independent of \( \Delta_t - 1 \).

**Proof of Proposition 4**

Implicit Function Theorem yields that, \( \frac{\partial \Delta_t^A}{\partial \tau_0} = -\frac{\partial G^i(\Delta_t, \Delta_t - 1, A)}{\partial \tau_0} \), \( \frac{\partial G^i(\Delta_t, \Delta_t - 1, A)}{\partial \Delta_t} > 0 \) for all \( \Delta_t \). The derivative of \( G^i(\Delta_t, \Delta_t - 1, A) \) with respect to \( \tau_0 \) given by:
\[
\frac{\partial G^i(\Delta_t, \Delta_t - 1, A)}{\partial \tau_0} = -\frac{\partial \gamma(\Delta_t - 1, \tau_0)}{\partial \tau_0} \left( \frac{\Pi^i(\Delta_t, \Delta_t - 1, A) + \beta(\Delta_t) \left( \Pi(\Delta_t - 1) - \Pi^i(\Delta_t, \Delta_t - 1, A) \right)}{\Pi^i(\Delta_t, \Delta_t - 1, A)} \right)
\]
As \( \gamma(\Delta_t - 1, \tau_0) \) is increasing in \( \tau_0 \) and \( \Pi(\Delta_t - 1) - \Pi^i(\Delta_t, \Delta_t - 1, A) \) is positive, the above expression is always negative.

**Proof of Proposition 5**

We assumed that \( \gamma(\Delta_t, 0) = 0 \). It then follows from Proposition 1 that \( \Delta_t^A(\tau_0 = 0) = \Delta_t^A(\tau_0 = 0) = 0 \). By continuity, there exists \( \tau_0^f \) such that \( \Delta_t^A(\Delta_t - 1, \gamma, \beta) = 0 \) for all \( \tau_0 \leq \tau_0^f \). In this region, incumbent receives \( \Pi^i(\Delta_t, \Delta_t - 1, A) \), which is independent of \( \tau_0 \). As highlighted in Proposition 4, \( \Delta_t^A \) becomes positive for higher values of \( \tau_0 \). If entrant has all the bargaining power in the settlement negotiation vis-a-vis the incumbent, then incumbent’s payoff is \( \Pi^i(\Delta_t, \Delta_t - 1, L) \) for \( \Delta_t < \Delta_t^A(\Delta_t - 1, \gamma, \beta) \) and \( \Pi^i(\Delta_t, \Delta_t - 1, A) \) otherwise. As \( \Pi^i(\Delta_t, \Delta_t - 1, L) \) and \( \Delta_t^A(\Delta_t - 1, \gamma, \beta) \) are both increasing in \( \tau_0 \), it is easy to see that incumbent’s equilibrium payoff is increasing in \( \tau_0 \). With probability \( p(R_t) > 0 \) entrant enters the market yielding an expected payoff \( E_{\Delta_t}[\Pi^i(\Delta_t, \Delta_t - 1, \sigma^*_{t})] \) to the incumbent where
\[
E_{\Delta_t}[\Pi^i(\Delta_t, \Delta_t - 1, \sigma^*_{t})] = p(R_t)\Pi^i(\Delta_t, \Delta_t - 1, \sigma^*_{t}) + (1 - p(R_t))\Pi(\Delta_t)
\]
As incumbent always accommodates an entrant for all \( \tau_v \leq \tau_v^I \), his expected payoff \( p(R_t) \Pi^I(\Delta_t, \Delta_{t-1}, A) + (1 - p(R_t)) \Pi(\Delta_t) \) becomes independent of \( \tau_v \). If \( \Delta_t^A (\Delta_{t-1}, \gamma, \beta) > 0 \) and entrant has all the bargaining power in the settlement negotiation vis-a-vis the incumbent, then

\[
\frac{\partial E_{\Delta_t} [\Pi^I(\Delta_t, \Delta_{t-1}, \sigma) \gamma]}{\partial \tau_v} = p(R_t) \left[ \int_{0}^{\Delta_t^A} \frac{\partial^2 \Pi^I(\Delta, \Delta_{t-1}, L)}{\partial \tau_v} f(\Delta) d\Delta_t + \Pi^I(\Delta_t^A, \Delta_{t-1}, L) f(\Delta) \frac{\partial \Delta_t^A}{\partial \tau_v} - \Pi^I(\Delta_t^A, \Delta_{t-1}, A) f(\Delta) \frac{\partial \Delta_t^A}{\partial \tau_v} \right]
\]

As \( \Pi^I(\Delta_t^A, \Delta_{t-1}, L) = \Pi^I(\Delta_t^A, \Delta_{t-1}, A) \) and \( \Pi^I(\Delta_t, \Delta_{t-1}, L) \) is increasing in \( \tau_v \), it is immediate that incumbent’s expected payoff is increasing in \( \tau_v \). ■

**Proof of Proposition 6.** Implicit Function Theorem yields that

\[
\frac{\partial \Delta_t^A}{\partial \tau_v} = -\frac{\partial G^I(\Delta_t, \Delta_{t-1}, A) / \partial \Delta_t}{\partial G^I(\Delta_t, \Delta_{t-1}, A) / \partial \Delta_t^A}.
\]

We have already shown in proof of Proposition 1 that \( \partial G^I(\Delta_t, \Delta_{t-1}, A) / \partial \Delta_t > 0 \) for all \( \Delta_t \). The derivative of \( G^I(\Delta_t, \Delta_{t-1}, A) \) with respect to \( \tau_b \) given by:

\[
-\gamma(\Delta_{t-1}, \tau_v) \frac{\partial \beta(\Delta_t, \tau_b)}{\partial \tau_b} \left( \Pi(\Delta_{t-1}) - \Pi^I(\Delta_t, \Delta_{t-1}, A) \right)
\]

As \( \beta(\Delta_t, \tau_b) \) is increasing in \( \tau_b \) and \( \Pi(\Delta_{t-1}) - \Pi^I(\Delta_t, \Delta_{t-1}, A) \) is positive, the above expression is always negative.

The result on payoffs follows immediately by observing that \( \Pi^E(\Delta_t, \Delta_{t-1}, A) \) and \( \Pi^E(\Delta_t, \Delta_{t-1}, A) \) are independent of \( \tau_b \), \( \Pi^E(\Delta_t, \Delta_{t-1}, L) \) is increasing in \( \tau_b \), \( \Pi^E(\Delta_t, \Delta_{t-1}, L) \) is decreasing in \( \tau_b \). ■

**Proof of Proposition 7.**

From our setting, we have \( c'(R_t^*) > 0, c''(R_t^*) > 0 \) and \( p'(R_t^*) > 0, p''(R_t^*) < 0 \).

Differentiate eq. (11) with respect to \( \Delta_{t-1} \) results in

\[
\frac{c''(R_t^*) p'(R_t^*) - p''(R_t^*) c'(R_t^*)}{[p'(R_t^*)]^2} \frac{\partial R_t^*(\Delta_{t-1}, 1)}{\partial \Delta_{t-1}} - \frac{\partial E_{\Delta_t} [\Pi^E(\Delta_{t-1}, \Delta_t, \sigma, \gamma, \beta, 1)]}{\partial \Delta_{t-1}} = 0
\]

Rearrange it, we have

\[
\frac{\partial R_t^*(\Delta_{t-1}, 1)}{\partial \Delta_{t-1}} = \frac{\partial E_{\Delta_t} [\Pi^E(\Delta_{t-1}, \Delta_t, \sigma, \gamma, \beta, 1)]}{\partial \Delta_{t-1}} \frac{[p'(R_t^*)]^2}{c''(R_t^*) p'(R_t^*) - p''(R_t^*) c'(R_t^*)}
\]

Note that \( \frac{[c'(R_t^*)]^2}{c''(R_t^*) p'(R_t^*) - p''(R_t^*) c'(R_t^*)} > 0 \) and
\[
\frac{\partial E_{\Delta_t}}{\partial \Delta_{t-1}} [\Pi^E(\Delta_{t-1}, \Delta_t, c^*, \gamma, \beta, 1)] = \int_{0}^{\Delta_t^x} \frac{\partial \Pi^E(\Delta_{t-1}, \Delta_t, L)}{\partial \Delta_{t-1}} f(\Delta_t) d\Delta_t + \int_{\Delta_t^x}^{\Delta_t^s} \frac{\partial \Pi^E(\Delta_{t-1}, \Delta_t, S)}{\partial \Delta_{t-1}} + [\Pi^E(\Delta_{t-1}, \Delta_t^A, S) - \Pi^E(\Delta_{t-1}, \Delta_t^A, A)] f(\Delta_t^A) \frac{\partial \Delta_t^A}{\partial \Delta_{t-1}} + \int_{\Delta_t^A}^{\Delta_{t-1}^{\max}} \frac{\partial \Pi^E(\Delta_{t-1}, \Delta_t, A)}{\partial \Delta_{t-1}} f(\Delta_t) d\Delta_t
\]

From Proposition 2, we know the both \(\Delta_t^A\) and \(\Delta_t^x\) increase in \(\Delta_{t-1}\), therefore, \(\lim_{\Delta_{t-1} \to 0} \Delta_t^A = \lim_{\Delta_{t-1} \to 0} \Delta_t^x = 0\), which makes the first two terms of the equation above approach to 0. Moreover, \(\lim_{\Delta_{t-1} \to 0} \Pi^E(\Delta_{t-1}, \Delta_t^A, S) = \lim_{\Delta_{t-1} \to 0} \Pi^E(\Delta_{t-1}, \Delta_t^A, A) = 0\). This implies that

\[
\lim_{\Delta_{t-1} \to 0} \frac{\partial E_{\Delta_t}}{\partial \Delta_{t-1}} [\Pi^E(\Delta_{t-1}, \Delta_t, c^*, \gamma, \beta, 1)] = \lim_{\Delta_{t-1} \to 0} \int_{0}^{\Delta_{t-1}^{\max}} \frac{\partial \Pi^E(\Delta_{t-1}, \Delta_t, A)}{\partial \Delta_{t-1}} f(\Delta_t) d\Delta_t > 0
\]

Plug the term above into (A-2) and take limit w.r.t \(\Delta_{t-1}\), we have

\[
\lim_{\Delta_{t-1} \to 0} \frac{\partial R_t}{\partial \Delta_{t-1}}(\Delta_{t-1}, 1) = \lim_{\Delta_{t-1} \to 0} \left[ c'(R_t^*)^2 p'(R_t^*) - p''(R_t^*) c'(R_t^*) \right] \int_{0}^{\Delta_{t-1}^{\max}} \frac{\partial \Pi^E(\Delta_{t-1}, \Delta_t, A)}{\partial \Delta_{t-1}} f(\Delta_t) d\Delta_t > 0
\]

\textbf{Proof of Corollary 3}.

The corollary follows immediately from Proposition 2 and the continuity of \(R^*\).

\textbf{References}


