2. Transformation to a distributed linear system

Define

\[ D(Q(t)) = \left\{ f \in W^2(-\tau_r, 0; U) | f(0) = 0 \right\} \]

and

\[ B = (\lambda (Q(t)) - \lambda)u \in \mathbb{C} \]

Another operator \( T_X \) is defined similarly. We assume:

\\((A1)) \quad (Q(t), X, U) \) generates a regular system on the state space \( D^2(-\tau_r, 0; U) \), the control space \( X \) and the observation space \( X \).

\\((A2)) \quad (Q(t); X, U) \) generates a regular system on the state space \( D^2(-\tau_r, 0; U), \) the control space \( X \) and the observation space \( X \).

Define the operators:

\[ A_{\lambda, X} = (A + \lambda)^{-1}(\lambda I - B) \]

\[ D(A_{\lambda, X}) = \mathbb{C} \times D^2(-\tau_r, 0; U) \]

and

\[ B = (0 \in \mathbb{C}) \]

where

\[ A_{\lambda, X} = A - \lambda I \]

\[ D(A_{\lambda, X}) = \mathbb{C} \times D^2(-\tau_r, 0; U) \]

and

\[ B = (0 \in \mathbb{C}) \]

Now if we set \( \xi = (\xi(t), \xi_u, \xi_{tu}) \) thenAffected by \((2), \) the result is equivalent to

\[ \left\{ \xi(t) = A_{\lambda, X}(t) + B_0(t) \right\} \]

The following result describes the form of the stabilizing operator for the delay system (2).

Theorem 3. Assume the conditions \((A1) \) and \((A2) \) are satisfied. If \( \xi \in L^2(A_{\lambda, X}) \), the delay system (2) then it is of the form

\[ (C \in \mathbb{C}, D) \]

We recall that if \( T(\lambda) \) is compact for \( \lambda > 0 \) then \( \delta_{\lambda} \) is the generator of an eventually compact semigroup. Moreover, the unstable set

\[ \sigma^+ = \{ \lambda \in \sigma(A_{\lambda, X}) \mid \text{Re} \lambda \geq 0 \} \]

is finite, which is denoted by \( \sigma^+ = \{ \lambda_1, \lambda_2, \ldots, \lambda_r \} \). For \( \lambda_1 \in \sigma^+, \) if \( i = 1, 2, \ldots, r \), we can set the dimension of Ker\(A_{\lambda_i} \) as

\[ d_i = \text{dim}\Sigma_{\lambda_i} \]

and the basis of Ker\(A_{\lambda_i} \) by \( \{ \phi_{i1}, \phi_{i2}, \ldots, \phi_{im} \} \). Therefore, we extend a generator of an eventually compact semigroup to \( \delta_{\lambda} \) as an admissible feedback operator.

5. A necessary condition

We now focus on the characterization of feedback stabilizability of the delay system (2). For each \( \lambda \in \mathbb{C} \) we redefine

\[ \mathbb{C} \times A_{\lambda, X} \]

The closed loop system associated with \( \mathbb{C} \times A_{\lambda, X} \) is extended to a \( C \times \)–semigroup \( V \) respectively.

\begin{align*}
\text{Conditions on feedback stabilization of systems with state and input delays in Banach spaces} \\
\text{Said Hadd and Qing-Chang Zhong} \\
\text{Department of Electrical Engineering and Electronics} \\
\text{The University of Liverpool}
\end{align*}

8. Examples

Example 1. Consider the system (8) with

\[ A = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \quad P = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}, \quad P_2 = \begin{pmatrix} p_{21} & p_{22} \end{pmatrix} \]

Then, \( \Delta(\lambda) = \lambda I - A, \sigma(A) = \{ -1, 1 \} \) and \( \sigma^+ = \{ 1 \} \). Hence

\[ \text{Ker}(\Delta(\lambda)) = T(\lambda) = \sigma(\lambda) \]

which implies that \( d_1 = \text{dim}\Sigma_{\lambda_i} = 1 \). Now (9) is reduced to

\[ \text{Rank}(\begin{pmatrix} p_{11} & p_{12} \\ 2p_{11} & 2p_{12} \end{pmatrix}) \]

\[ = \text{Rank}(2p_{11} + p_{12} + e^{-\tau_r}(2p_{11} + 2p_{12})) = 1 \]

Hence, the delay system is stable if and only if \( \text{Rank}(2p_{11} + p_{12} + e^{-\tau_r}(2p_{11} + 2p_{12})) \neq 1 \). In other words, the system is not stabilizable

\[ r = \text{Rank}(2p_{11} + p_{12} + e^{-\tau_r}(2p_{11} + 2p_{12})) \]

Example 2. Consider the system (8) with

\[ A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad P = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad P_2 = 0 \]

Here

\[ \Delta(\lambda) = \begin{pmatrix} \lambda & -2 \\ -e^{-\tau_r} & \lambda \end{pmatrix} \]

and

\[ \text{Ker}(\Delta(\lambda)) = \text{span}\{ 1 \} \text{ and } \text{Ker}(\Delta(\lambda)) = \text{span}\{ 1 \} \]

Now we have

\[ \text{Rank}(\begin{pmatrix} \lambda & -2 \\ -e^{-\tau_r} & \lambda \end{pmatrix}) = 0 \]

Thus, \( \lambda_0 \) is a stabilizable eigenvalue but \( \lambda_2 = 1 \) is not.

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