TWO-LEVEL CONTROL OF
Processes with Dead Time and Input Constraints

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Outline

- Background information
- Motivation
- Two-level control signal
- Controller design
- Implementation of the controller
- Simulation examples
Background information

- regulation \textit{v.s.} set-point responses
  - regulation: the major task
  - set-point responses: often necessary
  - 2DOF controller
  - fast set-point response
  - fast but without overshoot
  - dead time
  - input constraint

In general, this is difficult. However, for some processes, this can be done.
Systems under consideration

The most common chemical processes: the first-order plus dead time (FOPDT)

\[ G(s) = \frac{Ke^{-\tau s}}{Ts + 1}, \]

where \( K \) is the static gain, \( \tau \) is the dead time and \( T \) is the apparent time constant.
Motivation

A typical control signal using a PI controller with $F(z) = 1$

Three stages:
I: due to the integrating effect, the control signal increases until the actuator saturates;
II: the integrator “winds up” and the actuator saturates;
III: the control signal settles down.
Why is the response slow?

Stage I: The proportional gain cannot be too large otherwise the actuator saturates very quickly. This means that the potential of the controller is often not fully used to speed up the system response; see the shaded area in the figure.

Stage II: The integrator windup requires the error signal to go opposite for a long period to drag the integrator back to normal. This causes a large overshoot and long settling time.

Stage III: The oscillation is not desirable either, which causes a long settling time.

⇒ The desired control signal
Two-level control signal

The desired control signal when the set-point change is the bound $\bar{r}$ of all step changes

\[ u \]

\[ (n + 1)T_s \quad lT_s \]

\[ \bar{r}/K \]

Time (sec)
Two-level control signal (cont’d)

The signal can be expressed as:

\[
1 - a^{n+1}z^{-n-1} \cdot \frac{1}{K(1 - a^{n+1})} \cdot \bar{r},
\]

which gives the desired transfer function from \( r \) to \( u \):

\[
T_{ur}^d(z) = \frac{1 - a^{n+1}z^{-n-1}}{K(1 - a^{n+1})}.
\]

The first part should be under the saturation bound \( \bar{u} \):

\[
\frac{\bar{r}}{K(1 - a^{n+1})} \leq \bar{u} \Rightarrow n \geq \frac{T}{T_s} \ln \frac{K\bar{u}}{K\bar{u} - \bar{r}} - 1.
\]
Controller design

\[ G(z) = K \frac{1 - a}{z - a} z^{-l}, \quad C(z) = \frac{(1 - az^{-1})N(z)}{D(z)} \]

\( l = \frac{\tau}{T_s} \) is a positive integer and \( a = e^{-T_s/T} \). The order of polynomials \( N(z) \) and \( D(z) \) in \( z^{-1} \) is \( n \) and \( m \), respectively.

\[ T_{yr}(z) = F(z) \frac{K(1 - a)N(z)z^{-(l+1)}}{D(z) + K(1 - a)N(z)z^{-(l+1)}} \]

\[ T_{ur}(z) = F(z) \frac{(1 - az^{-1})N(z)}{D(z) + K(1 - a)N(z)z^{-(l+1)}}. \]
Controller design: $F(z)$

Since the closed-loop system is stable, $F(z)$ can be simply chosen to cancel the closed-loop poles.

$$F(z) = \frac{D(z)}{K(1-a)} + N(z)z^{-(l+1)}.$$  

Then,

$$T_{yr}(z) = N(z)z^{-(l+1)}$$

$$T_{ur}(z) = N(z)\frac{1 - az^{-1}}{K(1-a)}.$$  

The output $y$ is expected to start just after the dead time, $N(0) \neq 0$. Hence, $D(0) \neq 0$.  

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Controller design: \( N(z) \)

The desired transfer function:

\[
T_{ur}^d(z) = \frac{1 - a^{n+1} z^{-n-1}}{K(1 - a^{n+1})}.
\]

The actual transfer function:

\[
T_{ur}(z) = N(z) \frac{1 - az^{-1}}{K(1 - a)}.
\]

\[
N(z) = \frac{\sum_{i=0}^{n} a^i z^{-i}}{\sum_{i=0}^{n} a^i}.
\]
Controller design: $D(z)$

$D(z)$ is designed to guarantee the stability of the closed-loop system. One possibility is to choose

$$D(z) = \frac{1 - z^{-1}}{K_I} N(z)$$

to offer a PI controller:

$$C(z) = \frac{(1 - a z^{-1}) N(z)}{D(z)} = K_I \frac{1 - a z^{-1}}{1 - z^{-1}}.$$

The corresponding open-loop transfer function is

$$L(z) = C(z) G(z) = \frac{K_I K (1 - a)}{(z - 1) z^l}.$$
A typical root-locus diagram
Tuning of the controller: $K_I$

**Theorem** The closed-loop system is stable if

$$0 < K_I < \frac{2}{K(1 - a)} \sin \frac{\pi}{4l + 2}.$$ 

To obtain a phase margin of $\phi_m$, $K_I$ can be chosen as

$$K_I = \frac{2}{K(1 - a)} \sin \frac{\pi - 2\phi_m}{4l + 2}.$$ 

To obtain a gain margin of $g_m$, $K_I$ can be chosen as

$$K_I = \frac{2}{K(1 - a)g_m} \sin \frac{\pi}{4l + 2}.$$
Some comments

the settling time is approximately

\[(l + n + 1)T_s \approx \tau + T \ln \frac{K\bar{u}}{K\bar{u} - \bar{r}}.\]

It is independent of the control parameter \(K_I\) and the sampling period. It depends on the saturation bound \(\bar{u}\) and is hence an inherent property of the system. There is no way to make the response any faster.

the static error is 0 because \(N(1) = 1\).

There is no braking control.

there is no need for such a brake because the response reaches the steady state in finite time and there is no overshoot;

the benefit of a large negative action is very small when \(\bar{u}\) is not very large, which is the common case in practice,

the control strategy is more sensitive when there is a large negative control action.
An alternative implementation

\[ F_u(z) = \frac{1 - a^{n+1}z^{-n-1}}{K(1 - a^{n+1})}, \quad G_m(z) = K \frac{1 - a}{1 - az^{-1}}z^{-(l+1)}. \]

This structure appeared in [Wallén and Åström, 2002].
Advantages of this implementation

The control signal $u$ is split into two parts:

$$u = u_o + u_c,$$

with the desired (open-loop) control signal $u_o$ and the contribution $u_c$ of the feedback controller $C$ resulted from disturbances and model uncertainties.

There is strong connection with the input-shaping technique.

It is clearer that the desired control signal can be designed in an open-loop way if the plant is stable. What’s extra is to inject this desired control signal into the model $G_m$ of the process and to obtain the error between the model output $y_m$ and the process output $y$ for error feedback.

The feedback controller $C$ does not affect the shape of the control signal, which is not explicit in the case discussed before (where $N(z)$ is a part of the controller). This means that the controller may not be limited to a PI controller as designed above. In other words, the proposed technique can be regarded as a “bolt-on” to any standard well-tuned PID controllers.

The sampling periods for the feedforward controller $F_u(z)$ and the feedback loop can be different to give more freedom to the design of the feedback controller.
Internal model control

If $C(z)$ is designed to be

$$C(z) = \frac{F_u(z)}{1 - G_m(z)F_u(z)},$$

then the system is actually the well-known IMC.
An example

\[ G(s) = \frac{e^{-5s}}{s + 1} \]

\[ T_s = 0.25s \quad \Rightarrow \quad a = 0.7788, \quad l = 20 \]

\[ \bar{u} = 1.45, \quad \bar{r} = 1 \quad \Rightarrow \quad n \geq 3.68 \]

\[ N(z) = 0.31 + 0.2414z^{-1} + 0.1881z^{-2} + 0.1464z^{-3} + 0.1141z^{-4}. \]

\[ \phi_m = 45^\circ \quad \Rightarrow \quad K_I = 0.173. \]
The converted set point $r'$

\[ m = n + 1 = 5, \quad l = 20, \quad T_s = 0.25 \]
The system response

The error signal $e = r' - y$  
The output $y$ and the control signal $u$
Another example

\[ G(s) = \frac{e^{-0.5s}}{s + 1} \]
Comparative studies

(a) the system outputs

(b) the control signals
Robustness

(a) $T$ increased by 20%

(b) $\tau$ increased by 20%
Summary

A controller is designed to obtain a two-level control signal and a deadbeat set-point response. The controller is tuned to obtain the desired stability margin.