This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.

Scaling Laws of Multicast Capacity for Power-Constrained Wireless Networks under Gaussian Channel Model

Cheng Wang, Changjun Jiang, *Member, IEEE,* Xiang-Yang Li, *Senior Member, IEEE,* Shaojie Tang, Yuan He, *Member, IEEE,* Xufei Mao, and Yunhao Liu, *Senior Member, IEEE*

Abstract—We study the asymptotic *networking-theoretic* multicast capacity bounds for *random extended networks* (REN) under *Gaussian channel model*, in which all wireless nodes are individually power-constrained. During the transmission, the power decays along path with attenuation exponent $\alpha > 2$. In REN, n nodes are randomly distributed in the square region of side length $\sqrt{n}$. There are n_s randomly and independently chosen multicast sessions. Each multicast session has n_d+1 randomly chosen terminals, including one source and n_d destinations. By effectively combining two types of routing and scheduling strategies, we analyze the asymptotic achievable throughput for all $n_s = \omega(1)$ and n_d. As a special case of our results, we show that for $n_s = \Theta(n)$, the per-session multicast capacity for REN is of order $\Theta(\frac{1}{\sqrt{n_d n}})$ when $n_d = O(\frac{n}{(\log n)^{\alpha+1}})$ and is of order $\Theta(\frac{1}{n_d} \cdot (\log n)^{-\frac{\alpha}{2}})$ when $n_d = \Omega(\frac{n}{\log n})$.

Index Terms—Multicast Capacity, Percolation, Wireless ad hoc networks, Random networks, Achievable throughput

1 INTRODUCTION

THE capacity scaling laws of wireless networks have received much attention from the researchers, especially after the pioneer work by Gupta and Kumar [2]. There are generally two kinds of capacity bounds. The first kind is called *information-theoretic* bound, which is obtained by allowing arbitrary (physical layer) cooperative relay strategies, [3]. The issue was first addressed by Xie and Kumar [4]. The second kind is called *networking-theoretic* bound [2], which is derived under the assumption that the signals received from nodes other than one particular transmitter are regarded as interference degrading the communication. It is intuitive that the optimal strategy for *networking-theoretic* bounds is to confine to nearest neighbor communication and maximize spatial reuse, because the interference generated by long communication would prevent most of the other nodes from communicating simultaneously, [2], [5]. In this paper, we focus on the networking-theoretic capacity that depends on the adopted network models, including deployment models, scaling models, communication models, and the pattern of traffic sessions (unicast, broadcast, or multicast). Here we study *random networks*, where the nodes are randomly placed and their destinations are also randomly chosen.

Generally, two types of communication models are used. (1) The first is the *binary-rate communication model* under which if the value of a given conditional expression is beyond the threshold, the transmitter can send successfully to the receiver at a specific constant data rate; otherwise, it can not send any. The *protocol model* (ProM) and *physical model* (PhyM) defined in [2] both belong to the binary-rate communication model. The conditional expression of ProM is the fraction of the distances from the particular transmitter and other transmitters to the intended receiver; the conditional expression of PhyM is SINR (signal to interference plus noise ratio). This model is simple, thus, analytically attractive. Many work therein are based on this model, e.g., [6]–[15]. (2) The second is the *continuous-rate communication model* that determines the transmission rate at which the transmitter can communicate with receiver reliably, based on a continuous function of the receiver's SINR. Generally, two nodes v_i and v_j can establish a direct communication link, over a channel of bandwidth B, of rate $R(v_i, v_j) = B\log_2(1+(1/\eta)\,\mathrm{SINR}(v_j))$. When $\eta > 1$, the receiver can achieve the maximum rate that meets a given BER requirement under a specific modulation and coding scheme; When $\eta = 1$, the receiver achieves the Shannon's capacity with additive Gaussian white noise, see [16], [17]. Then, in the case of $\eta = 1$, the *continuous-rate communication model* can be called *Gaussian channel model* (GCM), also called *generalized physical model* [18], [19].

About the relations among the ProM, PhyM and GCM, we observe that: (i) For random dense networks (RDN) (where the area of deployment region is fixed and the node density increases to infinity), the ProM and PhyM can act as the reasonable simplifications of GCM; and if multiple commu-

- *Wang and Jiang are with the Department of Computer Science and Engineering, Tongji University, and with the Key Laboratory of Embedded System and Service Computing, Ministry of Education, China. (E-mail: 3chengwang@gmail.com, cjjiang@tongji.edu.cn)*
- *Li is with the Tsinghua National Lab. for Information Sci. and Tech. (TNLIST), Tsinghua University. He is also with the Department of Computer Science, Illinois Institute of Technology. (E-mail: xli@cs.iit.edu)*
- *Shaojie Tang is with the Department of Computer Science, Illinois Institute of Technology, Chicago, IL. (Email: tangshaojie@gmail.com)*
- *Mao is with Beijing Key Lab of Intelligent Telecommunications Software and Multimedia, Beijing University of Posts and Telecommunications, Beijing China. (Email: maoxufei@bupt.edu.cn)*
- *Liu and He are with TNLIST, School of Software, Tsinghua University, and with the Department of Computer Science and Engineering, Hong Kong Univ. of Sci. and Tech. (Email: yunhaoliu@gmail.com, heyust@gmail.com)*
- *The preliminary result was published at IEEE INFOCOM 2009 [1].*

Digital Object Indentifier 10.1109/TC.2011.63

nication and interference radii (or the thresholds of SINR) are permitted under the ProM (or the PhyM), the capacity derived under GCM can be equally derived under the ProM and PhyM, and vice versa. (ii) For random extended networks (REN) (where the node density is fixed to a constant and the area of the deployment region increases to infinity), the ProM and PhyM are both over-optimistic and unrealistic, while the GCM can capture the nature of wireless channels better.

In this paper, we study the *networking-theoretic* multicast capacity for REN under GCM. We present both improved lower bound and improved upper bound on multicast capacity, compared with previous literature. See Section 3 for our main results. Some existing results can be derived by our results as the special cases, such as [3], [20], [21].

For studying the lower bound of multicast capacity, we design two types of multicast strategies for REN. In one type of scheme, we construct the routing based on percolation theory and schedule short-hops and long-hops respectively. In the other type, we construct the routing without using the percolation theory, to avoid the bottleneck on the accessing path into *highways* [20]. Combining the two types of schemes, we obtain the achievable throughput as the lower bound of multicast capacity, which improves the previously best known results. We design our routing and schedule schemes based on several innovative techniques: using both backbone highway and second highway systems based on percolation theory, and parallel scheduling of nearby links. Second highway systems and parallel scheduling of nearby links, to the best of our knowledge, are not used in previous studies. In this work, we consider all cases in terms of the number of multicast sessions $n_s = \omega(1)$ and that of destinations n_d per session, while most known results put constraints on n_s and n_d.

On the other hand, for deriving upper bounds on multicast capacity, we apply several novel concepts such as *lattice view* and *island*. One approach is to study the bottleneck on some links. We show that there exist some special links terminating in certain islands that will be used by many multicast sessions (thus high load) and its own data rate is small, thus, implying an upper bound on per-session multicast capacity. Furthermore, for the lattice view consisting of cells of constant side length, by bounding the aggregated capacities and loads of such cells under any routing and scheduling schemes, we obtain another upper bound on per-session multicast capacity.

The rest of the paper is structured as follows. In Section 2, we introduce the network model. Main results are presented in Section 3. In Section 4, we present our general analysis techniques. In Section 5, we study upper bounds on multicast capacity. We design multicast strategies and analyze the achievable throughput for *random extended networks* in Section 6. We review existing results in Section 7, and conclude the paper in Section 8.

2 Network Model

We construct a *random network* $\mathcal{N}(a^2, n)$ by placing nodes according to a Poisson point process (p.p.p.) of intensity $\lambda(a^2, n) = \frac{n}{a^2}$ on the two-dimension plane and focusing on the square region $\mathcal{A}(a^2) = [0, a] \times [0, a]$. Thus, let $a = \sqrt{n}$, we obtain the *random extended network* (REN). According to Chebyshev's inequality, we get that the number of nodes in $\mathcal{A}(a^2)$ is within $((1-\epsilon)n, (1+\epsilon)n)$ with high probability, where $\epsilon > 0$ is an arbitrarily small constant. To simplify the description, we assume that the number of nodes is exactly n, without changing our results in order sense, [3], [20]. We are mainly concerned with the events that occur inside these squares with high probability (w.h.p.); that is, with probability tending to one as $n \to \infty$.

2.1 Multicast Capacity Definition

We first give the formal definition of capacity in our model. Let $\mathcal{V} = \{v_1, v_2, \cdots, v_n\}$ denote the set of all ad hoc nodes. Assume that a subset $\mathcal{S} \subseteq \mathcal{V}$ of $n_s = |\mathcal{S}|$ random nodes will serve as the source nodes of n_s multicast sessions. We *randomly* and *independently* choose n_s multicast sessions as follows. To generate the k-th $(1 \le k \le n_s)$ multicast session, denoted by $\mathcal{M}_{\mathcal{S},k}$, $n_d + 1$ points $p_{\mathcal{S},k_i}$ $(0 \le i \le n_d$, and $1 \le n_d \le n-1)$ are randomly and independently chosen from the deployment region $\mathcal{A}(a^2)$. Denote the set of these n_d+1 points by $\mathcal{P}_{\mathcal{S},k} = \{p_{\mathcal{S},k_0}, p_{\mathcal{S},k_1}, \cdots, p_{\mathcal{S},k_{n_d}}\}$. Let $v_{\mathcal{S},k_i}$ be the nearest ad hoc node from $p_{\mathcal{S},k_i}$ (ties are broken randomly). In $\mathcal{M}_{\mathcal{S},k}$, the node $v_{\mathcal{S},k_0}$, serving as a source, intends delivering data to n_d destinations $\mathcal{D}_{\mathcal{S},k} = \{v_{\mathcal{S},k_1}, v_{\mathcal{S},k_2}, \cdots, v_{\mathcal{S},k_{n_d}}\}$ at an arbitrary data rate $\lambda_{\mathcal{S},k}$. Let $\mathcal{U}_{\mathcal{S},k} = \{v_{\mathcal{S},k_0}\} \cup \mathcal{D}_{\mathcal{S},k}$ be the *spanning set* of nodes for the multicast session $\mathcal{M}_{\mathcal{S},k}$.

Let $\Lambda_{\mathcal{S},n_d} = (\lambda_{\mathcal{S},1}, \lambda_{\mathcal{S},2}, \cdots, \lambda_{\mathcal{S},n_s})$ denote a *rate vector* of the multicast data rate of all multicast sessions. We follow the standard definition of a feasible rate vector $\Lambda_{\mathcal{S},n_d} = (\lambda_{\mathcal{S},1}, \lambda_{\mathcal{S},2}, \cdots, \lambda_{\mathcal{S},n_s})$ in [12], [21]. A multicast rate vector $\Lambda_{\mathcal{S},n_d}$ is *feasible* if there is a $T < \infty$ such that in every time interval (with unit seconds) $[(t-1)\cdot T, t \cdot T]$, every node $v_{\mathcal{S},k_0} \in \mathcal{S}$ can send $T \cdot \lambda_{\mathcal{S},k}$ bits to all its n_d destinations. For a multicast rate vector, we define the *minimum per-session multicast throughput* (or *per-session multicast throughput* for simplicity) as $\Lambda^{\mathrm{p}}_{\mathcal{S},n_d}(n) = \min_{v_{\mathcal{S},k_0} \in \mathcal{S}} \lambda_{\mathcal{S},k}$.

Definition 1 (Achievable Per-Session Multicast Throughput): A per-session multicast throughput $\Lambda^{\mathrm{p}}_{\mathcal{S},n_d}(n)$ is *achievable* for n_s multicast sessions (each session with n_d destinations) if there is a feasible rate vector $\Lambda_{\mathcal{S},n_d} = (\lambda_{\mathcal{S},1}, \lambda_{\mathcal{S},2}, \cdots, \lambda_{\mathcal{S},n_s})$ such that $\Lambda^{\mathrm{p}}_{\mathcal{S},n_d}(n) = \min_{v_{\mathcal{S},k_0} \in \mathcal{S}} \lambda_{\mathcal{S},k}$.

Definition 2 (Multicast Capacity for Random Networks): The per-session multicast capacity for a class of random networks is of order $\Theta(g(n))$ if there are constants $c > 0$ and $c < c' < +\infty$ such that

$$\lim_{n \to +\infty} \Pr(\Lambda^{\mathrm{p}}_{\mathcal{S},n_d}(n) = c \cdot g(n) \text{ is achievable}) = 1,$$
$$\liminf_{n \to +\infty} \Pr(\Lambda^{\mathrm{p}}_{\mathcal{S},n_d}(n) = c' \cdot g(n) \text{ is achievable}) < 1.$$

2.2 Communication Model

We assume all nodes are individually power-constrained, i.e., for any node v_i, it transmits at a constant power $P_i \in [P_{min}, P_{max}]$, where P_{min} and P_{max} are some positive constants. Node v_j receives the transmitted signal from node v_i with power $P_i \cdot \ell(v_i, v_j)$, where $\ell(v_i, v_j)$ indicates the path loss between v_i and v_j. We restrict ourselves to a model where the interference at the receiver is simply regarded as noise, i.e., we focus on the *networking-theoretic bounds* instead of the *information-theoretic bounds*, [4], [22]–[27]. Hence, any two nodes can establish a direct communication link, over a channel of bandwidth B, of rate

$$R(v_i, v_j) = B\log_2(1 + \frac{P_i \cdot \ell(v_i, v_j)}{N_0 + \sum_{v_k \in \mathcal{S}(i)/v_i} P_k \cdot \ell(v_k, v_j)}),$$

where N_0 is the ambient noise power at the receiver, and $\mathcal{S}(i)$ is the set of nodes transmitting when v_i is scheduled.

NOTATIONS: Throughout this paper, we use following notations:

- For a two-dimension line segment $\mathcal{L} = uv$, $|\mathcal{L}|$ represents the Euclidean distance between u and v. For a discrete set $\mathcal{U}$, $|\mathcal{U}|$ represents its cardinality.
- For a continuous region $\mathcal{A}$, we use $\|\mathcal{A}\|$ to denote its area. For an Euclidean tree $\mathcal{T}$, we use $\|\mathcal{T}\|$ to denote its total Euclidean edge lengths.
- For a multicast session $\mathcal{M}_{\mathcal{S},k}$ with spanning set $\mathcal{U}_{\mathcal{S},k}$, let $\mathrm{EMST}(\mathcal{M}_{\mathcal{S},k})$ or $\mathrm{EMST}(\mathcal{U}_{\mathcal{S},k})$ denote the Euclidean minimum spanning tree (EMST) over $\mathcal{U}_{\mathcal{S},k}$, and $\mathrm{EST}(\mathcal{M}_{\mathcal{S},k})$ or $\mathrm{EST}(\mathcal{U}_{\mathcal{S},k})$ represent an Euclidean spanning tree (EST) over $\mathcal{U}_{\mathcal{S},k}$.

To make the expression more concise,

- define two functions as

$$\max_{\text{order}}\{\varphi(n),\ \phi(n)\} = \begin{cases} \Theta(\varphi(n)), & \text{if } \varphi(n) = \Omega(\phi(n)) \\ \Theta(\phi(n)), & \text{if } \phi(n) = \Omega(\varphi(n)) \end{cases}$$

$$\min_{\text{order}}\{\varphi(n),\ \phi(n)\} = \begin{cases} \Theta(\varphi(n)), & \text{if } \varphi(n) = O(\phi(n)) \\ \Theta(\phi(n)), & \text{if } \phi(n) = O(\varphi(n)) \end{cases}$$

- let $\theta(n)$:$[\varphi(n), \phi(n)]$ represent that $\theta(n) = \Omega(\varphi(n))$ and $\theta(n) = O(\phi(n))$, and let $\theta(n)$:$(\varphi(n), \phi(n)]$ represent that $\theta(n) = \omega(\varphi(n))$ and $\theta(n) = O(\phi(n))$.

3 MAIN RESULTS

Let the power attenuation function be

$$\ell(v_i, v_j) = \min\left\{1,\ |v_i v_j|^{-\alpha}\right\}$$

with $\alpha > 2$ and $N_0 > 0$. We study the multicast throughput by taking all cases of $n_s = \omega(1)$ and $n_d : [1, n]$ into account. The general results are shown in Theorem 7. In this section, we summarize our results under the assumption that $n_s = \Theta(n)$, as a special case of our general results.

For the upper bounds, we have that

Theorem 1: The per-session multicast capacity for random extended networks is at most of order

$$\begin{cases} O(\frac{1}{\sqrt{n_d n}}) & \text{when} \quad n_d : [1, \frac{n}{(\log n)^{\alpha}}] \\ O(\frac{1}{n_d (\log n)^{\frac{\alpha}{2}}}) & \text{when} \quad n_d : [\frac{n}{(\log n)^{\alpha}}, n] \end{cases} \quad (1)$$

For the lower bounds, we have that

Theorem 2: The per-session multicast capacity for random extended networks is at least of order

$$\begin{cases} \Omega(\frac{1}{\sqrt{n_d n}}) & \text{when} \quad n_d : [1, \frac{n}{(\log n)^{\alpha+1}}] \\ \Omega(\frac{1}{n_d (\log n)^{\frac{\alpha+1}{2}}}) & \text{when} \quad n_d : [\frac{n}{(\log n)^{\alpha+1}}, \frac{n}{(\log n)^2}] \\ \Omega(\frac{1}{\sqrt{n n_d} \cdot (\log n)^{\frac{\alpha-1}{2}}}) & \text{when} \quad n_d : [\frac{n}{(\log n)^2}, \frac{n}{\log n}] \\ \Omega(\frac{1}{n_d (\log n)^{\frac{\alpha}{2}}}) & \text{when} \quad n_d : [\frac{n}{\log n}, n] \end{cases} \quad (2)$$

Combining Theorem 1 and Theorem 2, we obtain that

Theorem 3: The per-session multicast capacity for random extended networks is of order

$$\begin{cases} \Theta(\frac{1}{\sqrt{n_d n}}) & \text{when} \quad n_d : [1, \frac{n}{(\log n)^{\alpha+1}}] \\ \Theta(\frac{1}{n_d (\log n)^{\frac{\alpha}{2}}}) & \text{when} \quad n_d : [\frac{n}{\log n}, n] \end{cases} \quad (3)$$

Observe that there is a gap between our upper bound and lower bound when $n_d : [\frac{n}{(\log n)^{\alpha+1}}, \frac{n}{\log n}]$. The gap would be closed by presenting possibly new tighter upper bound and lower bound, and designing algorithms to achieve it.

4 TECHNICAL LEMMAS

4.1 Techniques for Upper Bounds

We first give a new notion called *lattice view* by which some upper bounds can be derived.

Definition 3 (Lattice View): Partition a square deployment region $\mathcal{A}(a^2) = [0, a]^2$ into $\lceil \frac{a}{g} \rceil^2$ cells of side length g:$[\frac{a}{\sqrt{n}}, a)$, we call the produced lattice graph *lattice view*, and denote it by $\mathbb{V}(a, g)$.

Definition 4 (Island): In a lattice view $\mathbb{V}(a, g)$, a cell is called *island* if it contains $\Theta(\frac{n}{a^2} \cdot g^2)$ nodes and all its eight neighbor cells are empty.

Lemma 1: There exists w.h.p. an island in the lattice view $\mathbb{V}(a, g)$, if $g \le \frac{a}{2} \cdot \sqrt{\frac{(1-\epsilon)\cdot \log n}{2n}}$, where $\epsilon \in (0, 1)$ is constant.

Based on a given lattice view $\mathbb{V}(a, g)$, we next propose a useful result about arbitrary multicast trees.

Lemma 2: Given a multicast session $\mathcal{M}_{\mathcal{S},k}$, let $\mathcal{T}_{\mathcal{S},k}$ be a multicast tree for $\mathcal{M}_{\mathcal{S},k}$, and let $N(\mathcal{T}_{\mathcal{S},k}, a, g)$ denote the number of cells used by $\mathcal{T}_{\mathcal{S},k}$ in $\mathbb{V}(a, g)$, then it holds that $N(\mathcal{T}_{\mathcal{S},k}, a, g) = \Omega(\frac{1}{g} \cdot \| \mathrm{EMST}(\mathcal{M}_{\mathcal{S},k}) \|)$ when $n_d = O(\frac{a^2}{g^2})$.

Under any multicast strategy $\mathscr{F}$, the load of each cell in a lattice view $\mathbb{V}(a, g)$ can be classified into two types, i.e., *initial transmission load* and *relay burden*.

Definition 5: For any cell $\mathcal{C}_j \in \mathbb{V}(a, g)$, $j = 1, 2, \cdots, \lceil \frac{a^2}{g^2} \rceil$, define the *load* of $\mathcal{C}_j$ as the number of the links whose transmitters or receivers are located in $\mathcal{C}_j$; among those links, call the number of the links whose transmitters or receivers belong to any spanning sets $\mathcal{U}_{\mathcal{S},k}$ (for $v_{\mathcal{S},k_0} \in \mathcal{S}$) the *initial transmission load* of $\mathcal{C}_j$, and call the number of the other links the *relay burden* of $\mathcal{C}_j$.

4.2 Techniques for Lower Bounds

In general, the lower bounds of network capacity are obtained by designing some specific strategies. Denote a multicast strategy by $\mathscr{F}$, and denote the corresponding routing and transmission scheduling scheme by $\mathscr{F}^r$ and $\mathscr{F}^t$, respectively.

A routing scheme $\mathscr{F}^r$ may have a hierarchical structure consisting of τ phases corresponding to *sub-routing schemes* $\mathscr{F}^{r_1}, \mathscr{F}^{r_2}, \cdots, \mathscr{F}^{r_\tau}$, where $\tau \ge 1$ is a constant. Let $\mathcal{V}(\mathscr{F}^{r_j})$ denote the set of nodes passed through by some multicast sessions under the routing scheme $\mathscr{F}^{r_j}$, for $j \in [1, \tau]$.

Definition 6 (Sufficient Region): For a node $v_i^j \in \mathcal{V}(\mathscr{F}^{r_j})$, $1 \le j \le \tau$, and a multicast session $\mathcal{M}_{\mathcal{S},k}$, $1 \le k \le n_s$, we call a region $\mathcal{Q}(\mathscr{F}^{r_j}, \mathcal{M}_{\mathcal{S},k}, v_i^j)$ *sufficient region* if

$$\Pr(E(\mathscr{F}^{r_j}, \mathcal{M}_{\mathcal{S},k}, v_i^j)) \le \Pr(\tilde{E}(\mathscr{F}^{r_j}, \mathcal{M}_{\mathcal{S},k}, v_i^j)), \quad (4)$$

where the event $E(\mathscr{F}^{r_j}, \mathcal{M}_{\mathcal{S},k}, v_i^j)$ is defined as: $\mathcal{M}_{\mathcal{S},k}$ is routed through v_i^j under the sub-routing scheme $\mathscr{F}^{r_j}$; and the event $\tilde{E}(\mathscr{F}^{r_j}, \mathcal{M}_{\mathcal{S},k}, v_i^j)$ is defined as: A Poisson node is located in the region $\mathcal{Q}(\mathscr{F}^{r_j}, \mathcal{M}_{\mathcal{S},k}, v_i^j)$.

Lemma 3 (Achievable Throughput in Phase j): For a random network $\mathcal{N}(a^2, n)$, if all nodes in $\mathcal{V}(\mathscr{F}^{r_j})$ can sustain the rate of R_j under the transmission scheduling $\mathscr{F}^t$, and for $k \in [1, n_s]$, the areas of sufficient regions satisfy w.h.p. that

$$\|\mathcal{Q}(\mathscr{F}^{r_j}, \mathcal{M}_{\mathcal{S},k}, v_i^j)\| \leq Q_j \tag{5}$$

where Q_j is independent of k and i, then the achievable per-session throughput during Phase j is of

$$\Lambda_j = \begin{cases} \Omega(\frac{R_j}{n_s} \cdot \frac{a^2}{Q_j}) & \text{when} \quad n_s = [\frac{a^2}{Q_j} \cdot \log n, n] \\ \Omega(R_j \cdot \frac{1}{\log n}) & \text{when} \quad n_s = (1, \frac{a^2}{Q_j} \cdot \log n] \end{cases} \tag{6}$$

According to the principle of network bottleneck, we have,

Lemma 4: The throughput under a multicast strategy $\mathscr{F}$, consisting of τ phases, is achieved of $\Lambda = \min\{\Lambda_j, \text{ for } 1 \leq j \leq \tau\}$, where Λ_j is the throughput during Phase j.

5 Upper Bounds of Multicast Capacity

We study the upper bounds for *random extended networks* (REN) under Gaussian channel model.

5.1 Lattice View $\mathbb{V}(\sqrt{n}, \frac{1}{3}\sqrt{\log n})$

From Lemma 1, for $\epsilon = \frac{1}{9}$, there is an island in the lattice view $\mathbb{V}(\sqrt{n}, \frac{1}{3}\sqrt{\log n})$. Thus, we get the following lemma.

Lemma 5: Under Gaussian channel model, the per-session multicast capacity for REN is of order $O(\frac{n}{n_s n_d}\left(\log n\right)^{-\frac{\alpha}{2}})$.

Proof: Denote an island in $\mathbb{V}(\sqrt{n}, \frac{1}{3}\sqrt{\log n})$ by $\mathcal{I}$. For a link, say $u \to v$, where the receiver v is located in $\mathcal{I}$, its length is $|uv| = \Omega(\sqrt{\log n})$, then the capacity of this link is

$$C_{u,v} \leq B\log_2(1 + \frac{P_{max}|uv|^{-\alpha}}{N_0}) = O((\log n)^{-\frac{\alpha}{2}})$$

Consider the initial transmission load of $\mathcal{I}$. According to Equation (2) in the appendices and *union bounds*, we have that the initial transmission loads of all cells in $\mathbb{V}(\sqrt{n}, \frac{1}{3}\sqrt{\log n})$ are w.h.p. of order $\Omega(\frac{n_s \cdot n_d \log n}{n})$. In addition, there are at most $\Theta(\log n)$ simultaneous links terminating (or initiating) in $\mathcal{I}$ since it contains $\Theta(\log n)$ nodes inside. By the *pigeonhole principle*, there exists a link whose load is $\Omega(\frac{n_s n_d}{n})$. Then, the lemma follows from $C_{u,v} = O((\log n)^{-\frac{\alpha}{2}})$. □

5.2 Lattice View $\mathbb{V}(\sqrt{n}, c)$

We adopt the lattice view $\mathbb{V}(\sqrt{n}, c)$ to derive a new upper bound of the multicast capacity, where c is a constant such that $m = \frac{n}{c^2}$ is an integer. First, we consider the throughput capacity of the cells in this lattice view.

Lemma 6: The throughput capacity of any cell in $\mathbb{V}(\sqrt{n}, c)$ is at most of order $O(1)$.

Proof: For any cell $\mathcal{C}_i$ in $\mathbb{V}(\sqrt{n}, c)$, denote the set of all links initiating (or terminating) in $\mathcal{C}_i$ that are scheduled simultaneously in time t by $\Pi_i(t)$. Since the number of nodes in any cell of $\mathbb{V}(\sqrt{n}, c)$ is at most of order $O(\log n)$ (by Lemma B in the appendices), we get that $\max_{\mathcal{C}_i \in \mathbb{V}(\sqrt{n}, c)}\{|\Pi_i(t)|\} = O(\log n)$. Denote the transmitting power, length and rate of the jth link in $|\Pi_i(t)|$ as $P_{i,j} \in [P_{min}, P_{max}]$, $l_{i,j}$ and $\lambda_{i,j}$ for $1 \leq j \leq |\Pi_i(t)|$. Then, it holds that

$$\lambda_{i,j} \leq B\log_2\left(1 + \frac{P_{i,j} \cdot \min\{1, l_{i,j}^{-\alpha}\}}{N_0 + \min\{1, (l_{i,j} + \sqrt{2}c)^{-\alpha}\} \sum\limits_{k \neq j} P_{i,k}}\right)$$

Thus, we have $\lambda_{i,j} = O(\frac{P_{i,j} \cdot \min\{1, l_{i,j}^{-\alpha}\}}{N_0 + \min\{1, (l_{i,j} + \sqrt{2}c)^{-\alpha}\} \sum_{k \neq j} P_{i,k}})$. Since $c > 0$ is a constant, it holds that

$$\lambda_{i,j} = O(\frac{P_{i,j} \cdot \min\{1, l_{i,j}^{-\alpha}\}}{N_0 + \min\{1, l_{i,j}^{-\alpha}\} \sum_{k \neq j} P_{i,k}}).$$

Then, we obtain that $\lambda_{i,j} = O(\frac{2P_{i,j}}{\sum_{k=1}^{|\Pi_i(t)|} P_{i,k}})$. Furthermore, we get that $\sum_{j=1}^{|\Pi_i(t)|} \lambda_{i,j} = O(1)$, which completes the proof. □

Lemma 7: For all multicast sessions $\mathcal{M}_{\mathcal{S},k}$ $(1 \leq k \leq n_s)$, it holds that when $n_d = o(\frac{n}{\log n})$,

$$\sum\nolimits_{k=1}^{n_s} \|\operatorname{EMST}(\mathcal{M}_{\mathcal{S},k})\| = \Omega(n_s \cdot \sqrt{n_d \cdot n}).$$

Proof: Recall that for any multicast session, say $\mathcal{M}_{\mathcal{S},k}$, a set of $n_d + 1$ points are chosen *randomly* and *independently* from the deployment region $\mathcal{A}(n)$, denoted by $\mathcal{P}_{\mathcal{S},k} = \{p_{\mathcal{S},k_0}, p_{\mathcal{S},k_1}, \cdots, p_{\mathcal{S},k_{n_d}}\}$. Let $\operatorname{EMST}(\mathcal{P}_{\mathcal{S},k})$ denote the Euclidean minimum spanning tree (EMST) based on the set $\mathcal{P}_{\mathcal{S},k}$. Then, by Lemma D in the appendices, it holds almost surely that

$$\sum\nolimits_{k=1}^{n_s} \|\operatorname{EMST}(\mathcal{P}_{\mathcal{S},k})\| = \Theta(n_s\sqrt{n_d n}). \tag{7}$$

Next, we build an Euclidean spanning tree (EST) $\mathcal{T}_0 = \operatorname{EST}_0(\mathcal{P}_{\mathcal{S},k})$ on the basis of $\operatorname{EMST}(\mathcal{U}_{\mathcal{S},k})$, as illustrated in

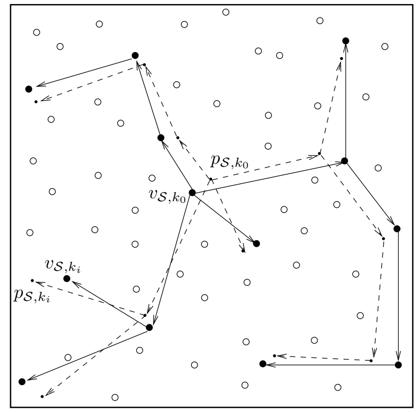


Fig. 1. Multicast session $\mathcal{M}_{\mathcal{S},k}$. The tree consisting of solid lines represents the Euclidean minimum spanning tree (EMST) over $\mathcal{U}_{\mathcal{S},k} = \{v_{\mathcal{S},k_i} \mid 0 \leq i \leq n_d\}$, denoted by $\operatorname{EMST}(\mathcal{U}_{\mathcal{S},k})$. The tree consisting of dashed lines represents an Euclidean spanning tree (EST) over $\mathcal{P}_{\mathcal{S},k} = \{p_{\mathcal{S},k_i} \mid 0 \leq i \leq n_d\}$, denoted by $\operatorname{EST}_0(\mathcal{P}_{\mathcal{S},k})$, where for any $0 \leq i, j \leq n_d$, $p_{\mathcal{S},k_i} \to p_{\mathcal{S},k_j} \in \operatorname{EST}_0(\mathcal{P}_{\mathcal{S},k})$ if and only if $v_{\mathcal{S},k_i} \to v_{\mathcal{S},k_j} \in \operatorname{EMST}(\mathcal{U}_{\mathcal{S},k})$.

Fig.1. Then, $\|\,\mathrm{EST}_0(\mathcal{P}_{\mathcal{S},k}\| \geq \|\,\mathrm{EMST}(\mathcal{P}_{\mathcal{S},k}\|$. Denote any edge $p_{\mathcal{S},k_i} \to p_{\mathcal{S},k_j}$ by $< i,j >$ without confusion, then

$$\begin{aligned}&\|\,\mathrm{EMST}(\mathcal{U}_{\mathcal{S},k})\| \\ &\geq \sum_{<i,j>\in\mathcal{T}_0} (|p_{\mathcal{S},k_i}p_{\mathcal{S},k_j}| - |p_{\mathcal{S},k_i}v_{\mathcal{S},k_i}| - |p_{\mathcal{S},k_j}v_{\mathcal{S},k_j}|) \\ &= \|\,\mathrm{EST}_0(\mathcal{P}_{\mathcal{S},k}\| - \sum_{<i,j>\in\mathcal{T}_0} (|p_{\mathcal{S},k_i}v_{\mathcal{S},k_i}| + |p_{\mathcal{S},k_j}v_{\mathcal{S},k_j}|) \\ &\geq \|\,\mathrm{EMST}(\mathcal{P}_{\mathcal{S},k}\| - 2n_d \cdot \max\{|p_{\mathcal{S},k_i}v_{\mathcal{S},k_i}|, \text{ for } 0 \leq i \leq n_d\}.\end{aligned}$$

Let $\mathcal{D}(p_{\mathcal{S},k_i}, r(n))$ denote the disk centered at the point $p_{\mathcal{S},k_i}$ with a radius $r(n)$. Then, the number of nodes in $\mathcal{D}(p_{\mathcal{S},k_i}, r(n))$, denoted by $N(p_{\mathcal{S},k_i}, r(n))$, follows a Poisson distribution of mean $\pi \cdot (r(n))^2$. Let $r(n) = 3\sqrt{\log n}$, according to Equation (2) in the appendices, we have

$$\Pr\left(N(p_{\mathcal{S},k_i}, 3\sqrt{\log n}) \leq \frac{9\pi}{2} \cdot \log n\right) \leq \frac{1}{n^3}.$$

Define $N_{\min} := \min\{N(p_{\mathcal{S},k_i}, r(n)), \text{ for all } 0 \leq i \leq n_d, 1 \leq k \leq n_s\}$. By *union bounds*, we get

$$\Pr\left(N_{\min} \leq \frac{9\pi}{2} \cdot \log n\right) \leq n_s \cdot (n_d+1) \cdot \frac{1}{n^3} \leq \frac{1}{n} \to 0,$$

which implies that for any $1 \leq k \leq n_s$ and $0 \leq i \leq n_d$, $|p_{\mathcal{S},k_i}v_{\mathcal{S},k_i}| \leq r(n) = 3\sqrt{\log n}$, w.h.p.. Hence,

$$\sum_{k=1}^{n_s} \|\,\mathrm{EMST}(\mathcal{U}_{\mathcal{S},k})\| \geq \sum_{k=1}^{n_s} \|\,\mathrm{EMST}(\mathcal{P}_{\mathcal{S},k}\| - 6n_s \cdot n_d \cdot \sqrt{\log n}.$$

Following $n_d = o(\frac{n}{\log n})$ and Equation (7), it holds that $\sum_{k=1}^{n_s} \|\,\mathrm{EMST}(\mathcal{P}_{\mathcal{S},k}\| = \omega(6n_s \cdot n_d \cdot \sqrt{\log n})$. Thus,

$$\sum_{k=1}^{n_s} \|\,\mathrm{EMST}(\mathcal{U}_{\mathcal{S},k})\| = \Omega(\sum_{k=1}^{n_s} \|\,\mathrm{EMST}(\mathcal{P}_{\mathcal{S},k}\|).$$

Combining with Equation (7), we complete the proof. □

Lemma 8: The per-session multicast capacity for REN is of order $O(\frac{\sqrt{n}}{n_s\sqrt{n_d}})$ when $n_d = o(\frac{n}{\log n})$.

Proof: For each multicast tree $\mathcal{T}_{\mathcal{S},k}$, denote the number of cells in $\mathbb{V}(\sqrt{n}, c)$ used by it as $N(\mathcal{T}_{\mathcal{S},k}, \sqrt{n}, c)$. According to Lemma 2, it holds that

$$\sum_{k=1}^{n_s} N(\mathcal{T}_{\mathcal{S},k}, \sqrt{n}, c) = \Omega\left(\sum_{k=1}^{n_s} \|\,\mathrm{EMST}(\mathcal{M}_{\mathcal{S},k})\|\right) \tag{8}$$

Case 1: When $n_d : (1, n/\log n)$.

Combining Lemma 7 with Equation (8), we obtain that $\sum_{k=1}^{n_s} N(\mathcal{T}_{\mathcal{S},k}, \sqrt{n}, c) = \Omega(n_s\sqrt{n_d n})$ when $n_d = o(\frac{n}{\log n})$.

By pigeonhole principle, there is at least one cell that will be used by at least $\Omega(\frac{n_s\sqrt{n_d}}{\sqrt{n}})$ sessions. By Lemma 6, the total throughput capacity of any cell in $\mathbb{V}(\sqrt{n}, c)$ is of order $O(1)$. Thus, under any strategy, due to the congestion in some cells, the multicast throughput is at most of order $O(\frac{\sqrt{n}}{n_s\sqrt{n_d}})$.

Case 2: When $n_d = \Theta(1)$.

The problem degenerates into the case of unicast sessions. From the result in [21], the per-session unicast capacity for REN is of order $O(\frac{\sqrt{n}}{n_s})$, i.e., $O(\frac{\sqrt{n}}{n_s\sqrt{n_d}})$ for $n_d = \Theta(1)$.

Combining two cases, we complete the proof. □

Based on Lemma 5 and Lemma 8, we obtain Theorem 4 by performing some simple algebraic manipulations.

Theorem 4: The per-session multicast capacity for random extended networks is of order

$$\begin{cases} O(\frac{\sqrt{n}}{n_s\sqrt{n_d}}) & \text{when} \quad n_d : [1, \frac{n}{(\log n)^\alpha}] \\ O(\frac{n}{n_s n_d} \cdot (\log n)^{-\frac{\alpha}{2}}) & \text{when} \quad n_d : [\frac{n}{(\log n)^\alpha}, n] \end{cases}$$

By letting $n_s = \Theta(n)$, we get Theorem 1.

6 Lower Bounds of Multicast Capacity

We derive the lower bounds on multicast capacity for REN by proposing two multicast strategies, denoted by $\mathscr{F}$ and $\mathscr{S}$, respectively. Our multicast strategies are cell-based, then we first recall a new notion called *scheme lattice* [28] for succinctness of the description.

Definition 7 (Scheme Lattice): Divide a square deployment region $\mathcal{A}(a^2) = [0,a]^2$ into a lattice consisting of square cells of side length g, we call the lattice *scheme lattice* and denote it by $\mathbb{L}(a, g, \theta)$, where $\theta \in [0, \frac{\pi}{4}]$ is the minimum angle between the sides of the deployment region and produced cells.

6.1 Highways System

The highway system consists of highways of two levels. The first are the *first-class highways* (FHs), indeed the *highways* in [20]. The second are the *second-class highways* (SHs) that are built without using percolation theory.

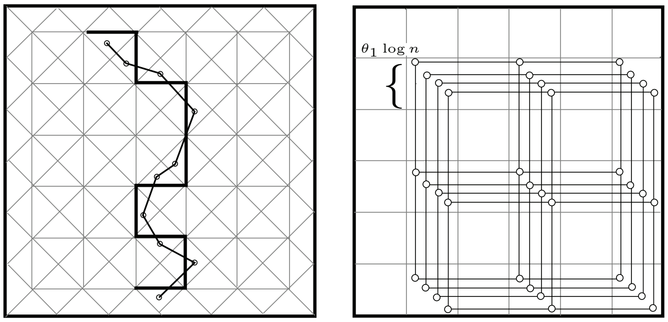
6.1.1 First-Class Highways (FHs)

We recall the construction of FHs based on percolation theory [20], and introduce the transmission scheduling by which each FH can sustain a rate of constant order.

Construction of FHs: The FHs are built based on the scheme lattice $\mathbb{L}(\sqrt{n}, c, \frac{\pi}{4})$, as depicted in Fig.2(a). Then, there are m^2 cells in $\mathbb{L}(\sqrt{n}, c, \frac{\pi}{4})$, where $m = \lceil \sqrt{n}/\sqrt{2}c \rceil$ (we can adjust the value of c such that $\sqrt{n}/\sqrt{2}c$ is an integer). Let $N(\mathcal{C}_i)$ denote the number of Poisson points inside cell $\mathcal{C}_i$, which is a Poisson random variable with mean c^2. For all i, the probability that a square $\mathcal{C}_i$ contains at least one Poisson point ($N(\mathcal{C}_i) \geq 1$) is $p \equiv 1 - e^{-c^2}$. We say a square is *open* if it contains at least one point, and *closed* otherwise. Then any square is open with probability p, independently from each other. Based on $\mathbb{L}(\sqrt{n}, c, \frac{\pi}{4})$, we draw a horizontal edge across half of the squares, and a vertical edge across the others, to obtain a scheme lattice $\mathbb{L}(\sqrt{n}, \sqrt{2}c, 0)$, as shown in Fig.2(a). We say a given edge $\hbar$ in $\mathbb{L}(\sqrt{n}, \sqrt{2}c, 0)$ is *open* if the cell in $\mathbb{L}(\sqrt{n}, c, \frac{\pi}{4})$, crossed by $\hbar$, is *open*, and call a path comprised of edges in $\mathbb{L}(\sqrt{n}, \sqrt{2}c, 0)$ *open* if it contains only open edges. Based on an open path connecting the left side of $\mathcal{A}(n)$ with its right side (or connecting the upper side of $\mathcal{A}(n)$ with its bottom side), as illustrated in Fig.2(a), choose a node from each cell in $\mathbb{L}(\sqrt{n}, c, \frac{\pi}{4})$ corresponding to the open edges of the open path, and connect a pair of nodes from two adjacent cells, we finally obtain a crossing path. We call those crossing paths *first-class highways* (FHs).

Density of FHs: For a given $\kappa > 0$, partition the scheme lattice $\mathbb{L}(\sqrt{n}, c, \frac{\pi}{4})$ into horizontal (or vertical) rectangle slabs of size $m \times (\kappa \log m - \epsilon_m)$ (or $(\kappa \log m - \epsilon_m) \times m$), denoted by $\mathcal{R}_i^h$ (or $\mathcal{R}_i^v$). Denote the number of disjoint horizontal (or vertical) FHs within $\mathcal{R}_i^h$ (or $\mathcal{R}_i^v$) by N_i^h (or N_i^v). It holds that

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.



(a) First-class highways (b) Second-class highways

Fig. 2. (a) The bold polygonal line represents an *open* path consisting of open edges. A vertical *first-class highway* (FH) is illustrated by a polygonal line whose inflexions are called *first-class stations*. (b) The bold lines connecting the nodes, called *second-class stations*, represent the second-class highways (SHs).

Lemma 9: ([20]) For every κ and $p \in (5/6, 1)$ satisfying $2 + \kappa \log(6(1-p)) < 0$, there exists a $\delta = \delta(\kappa, p)$ such that

$$\lim_{m\to\infty} \Pr(N^h \geq \delta \log m) = 1, \lim_{m\to\infty} \Pr(N^v \geq \delta \log m) = 1,$$

where $N^h = \min_i N_i^h$ and $N^v = \min_i N_i^v$.

Notations for FHs: To simplify the description, we assume that there are exactly $\delta \log m$ horizontal (or vertical) FHs in each horizontal (or vertical) slab, which does not change the results in order sense. According to lemma 9, we can further divide every slab into $\delta \log m$ slices of size $l \times \sqrt{n}$, where $l = \frac{(\kappa \log m - \varepsilon_m)}{\delta \log m}$. Hence, we can define the mapping among the slabs, slices, and FHs. Please see the details in Table 1. The following are some remarks.

1) Any slice can and only can project to an FH contained by the slab that posses the slice, which ensures that the distance from any points in the slice to the corresponding highway is at most of $\kappa \log m - \varepsilon_m$.
2) For a node v and horizontal slice $\mathcal{S}_i^h \in \mathbb{S}^h$ (or vertical slice $\mathcal{S}_i^v \in \mathbb{S}^v$), if v is located in $\mathcal{S}_i^h$ (or $\mathcal{S}_i^v$), then $\mathbf{f}^h(v) = \mathbf{g}^h(\mathcal{S}_i^h)$ (or $\mathbf{f}^v(v) = \mathbf{g}^v(\mathcal{S}_i^v)$).
3) $\psi^h(\mathfrak{h}_k^h)$ (or $\psi^v(\mathfrak{h}_k^v)$) denotes the horizontal (or vertical) slab completely containing the horizontal (or vertical) FH $\mathfrak{h}_k$ (or $\mathfrak{h}_k^v$).

Transmission Scheduling for FHs: To schedule the FHs, we use a 9-TDMA scheduling scheme based on the scheme lattice $\mathbb{L}(\sqrt{n}, c, \frac{\pi}{4})$ by letting $K = 3$ and $d = 1$ in Fig. 4 of [20]. According to Theorem 3 in [20], all FHs can sustain w.h.p. the rate of order $\Omega(1)$.

TABLE 1
Notations for FHs

Notation	Meaning
$\mathcal{R}_i^h$	i-th horizontal *slab* of size $m \times (\kappa \log m - \epsilon_m)$
$\mathcal{R}_i^v$	i-th vertical *slab* of size $(\kappa \log m - \epsilon_m) \times m$
$\mathbb{R}^h$ (or $\mathbb{R}^v$)	the *set* of all horizontal (or vertical) slabs
$\mathcal{S}_j^h$	j-th horizontal *slice* of size $m \times \frac{\kappa \log m - \varepsilon_m}{\delta \log m}$
$\mathcal{S}_j^v$	j-th vertical *slice* of size $\frac{\kappa \log m - \varepsilon_m}{\delta \log m} \times m$
$\mathbb{S}^h$ (or $\mathbb{S}^v$)	the *set* of all horizontal (or vertical) slices
$\mathfrak{h}_k^h$ (or $\mathfrak{h}_k^v$)	k-th horizontal (or vertical) *FH*
$\mathbb{H}^h$ (or $\mathbb{H}^v$)	the *set* of all horizontal (or vertical) FHs
$\mathbf{g}^h$:$\mathbb{S}^h \to \mathbb{H}^h$	a *bijection* from horizontal slices to horizontal FHs
$\mathbf{g}^v$:$\mathbb{S}^v \to \mathbb{H}^v$	a *bijection* from vertical slices to vertical FHs
$\mathbf{f}^h : \mathcal{V} \to \mathbb{H}^h$	a *function* from nodes to horizontal FHs
$\mathbf{f}^v : \mathcal{V} \to \mathbb{H}^v$	a *function* from nodes to vertical FHs
$\psi^h : \mathbb{H}^h \to \mathbb{R}^h$	a *function* from horizontal FHs to horizontal slabs.
$\psi^v : \mathbb{H}^v \to \mathbb{R}^v$	a *function* from vertical FHs to vertical slabs.

6.1.2 Second-Class Highways (SHs)

We build the SHs and design the transmission scheduling to achieve the rate of order $\Omega((\log n)^{-\frac{\alpha}{2}})$ along each SH.

Construction of SHs: The SHs are constructed based on the scheme lattice $\mathbb{L}(\sqrt{n}, \sigma\sqrt{\log n} - \epsilon_n, 0)$, as depicted in Fig.2(b), where $\sigma > 0$ is a constant and we choose $\epsilon_n > 0$ as the smallest value such that $\sqrt{n}/(\sigma\sqrt{\log n} - \epsilon_n)$ is an integer. It is obvious that $\epsilon_n = o(1)$. Then there are $n/(\sigma\sqrt{\log n} - \epsilon_n)^2$ cells. Let $N(\bar{\mathcal{C}}_j)$ be the number of Poisson nodes inside a cell $\bar{\mathcal{C}}_j$, which is a Poisson random variable with mean $(\sigma\sqrt{\log n} - \epsilon_n)^2$. Furthermore, we define the uniform lower bound of $N(\bar{\mathcal{C}}_j)$ as $N_{\bar{\mathcal{C}}}$.

To ensure the feasibility of the method to construct SHs, we give the following lemma.

Lemma 10: For any ϱ, $\varrho > 1 + \log \varrho$, and σ, $\sigma^2 \geq \frac{4\varrho}{(2\varrho - \log \varrho - 1)}$, each cell in $\mathbb{L}(\sqrt{n}, \sigma\sqrt{\log n} - \epsilon_n, 0)$ contains w.h.p. no less than $\theta_1 \log n$ nodes, where θ_1 is a constant with $\theta_1 = \frac{\sigma^2}{2\varrho}$.

Proof: Since $(\sigma\sqrt{\log n} - \epsilon_n)^2 > \frac{1}{2}\sigma^2 \log n$, as $n \to \infty$, according to Lemma B in the appendices and *union bounds*, we have

$$\Pr\left(N_{\bar{\mathcal{C}}} \leq \frac{\sigma^2 \cdot \log n}{2\varrho}\right) \leq \frac{2n}{\sigma^2 \cdot \log n} \Pr\left(N(\bar{\mathcal{C}}_j) \leq \frac{\sigma^2 \cdot \log n}{2\varrho}\right)$$
$$\leq \frac{2n}{\sigma^2 \cdot \log n} \frac{n^{\frac{\sigma^2}{2\varrho}}}{n^{\frac{\sigma^2}{2}}} \cdot n^{\frac{\sigma^2 \cdot \log \varrho}{2\varrho}} = \frac{2}{\sigma^2 \cdot \log n \cdot n^{\frac{\sigma^2}{2} - 1 - \frac{(1+\log \varrho)\sigma^2}{2\varrho}}}$$

Thus, when we choose ϱ with $\varrho > 1 + \log \varrho$ and σ with $\sigma^2 \geq \frac{2\varrho}{\rho - (1 + \log \rho)}$, it holds that $\Pr(N_{\bar{\mathcal{C}}} \leq \frac{\sigma^2 \cdot \log n}{2\varrho}) \to 0$. □

We call each row (or column) of $\mathbb{L}(\sqrt{n}, \sigma\sqrt{\log n} - \epsilon_n, 0)$ *row-slab* (or *column-slab*), denoted by $\bar{\mathcal{R}}_i^h$ (or $\bar{\mathcal{R}}_i^v$). In each row-slab (or column-slab), we give each cell an order number, e.g., increasing from top to bottom (or from left to right), and call the cells with odd (or even) order number *odd-order (or even-order) cells*. We construct the horizontal SHs in $\bar{\mathcal{R}}_i^h$ by the following operations: First, for $\sqrt{n}/(\sigma\sqrt{\log n} - \epsilon_n)$ cells in $\bar{\mathcal{R}}_i^h$, choose one node from each cell. Second, connect the selected nodes from consecutive odd-order cells to derive a path called horizontal *odd* SH; connect the nodes from consecutive even-order cells to derive a horizontal *even* SH. Similarly, we can construct the vertical *odd* SH and *even* SH.

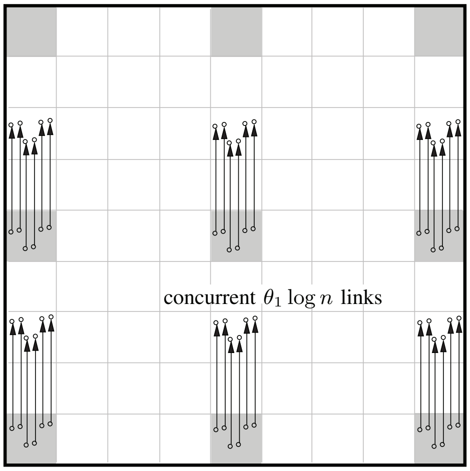


Fig. 3. Second-class transmission scheduling. Gray squares can be scheduled simultaneously. In any time slot, there are $\Theta(\log n)$ concurrent links initiating from every activated cell.

TABLE 2
Notations for SHs

Notation	Meaning
$\bar{\mathcal{R}}_i^h$	i-th *row-slab* of area $\sqrt{n} \times (\sigma\sqrt{\log n} - \epsilon_n)$
$\bar{\mathcal{R}}_i^v$	i-th *column-slab* of area $(\sigma\sqrt{\log n} - \epsilon_n) \times \sqrt{n}$
$\bar{\mathbb{R}}^h$ (or $\bar{\mathbb{R}}^v$)	the *set* of all row-slabs (or column-slabs)
$\bar{\mathcal{S}}_j^h$	j-th *row-slice* of area $\sqrt{n} \times (\sigma/(2\theta_1\sqrt{\log n}))$
$\bar{\mathcal{S}}_j^v$	j-th *column-slice* of area $(\sigma/(2\theta_1\sqrt{\log n})) \times \sqrt{n}$
$\bar{\mathbb{S}}^h$ (or $\bar{\mathbb{S}}^v$)	the *set* of all all row-slices (or column-slices)
$\bar{\mathfrak{h}}_k^h$ (or $\bar{\mathfrak{h}}_k^v$)	k-th horizontal (or vertical) *SH*
$\bar{\mathbb{H}}^h$ (or $\bar{\mathbb{H}}^v$)	the *set* of all horizontal (or vertical) SHs
$\bar{\mathbf{g}}^h$:$\bar{\mathbb{S}}^h \to \bar{\mathbb{H}}^h$	a *bijection* from row-slices to horizontal SHs
$\bar{\mathbf{g}}^v$:$\bar{\mathbb{S}}^v \to \bar{\mathbb{H}}^v$	a *bijection* from column-slices to vertical SHs
$\bar{\mathbf{f}}^h : \mathcal{V} \to \bar{\mathbb{H}}^h$	a *function* from nodes to horizontal SHs
$\bar{\mathbf{f}}^v : \mathcal{V} \to \bar{\mathbb{H}}^v$	a *function* from nodes to vertical SHs

Density of SHs: We say two SHs are disjoint if they do not share a common node. Next, we consider the density of SHs, that is, the number of the disjoint SHs in unit area. Denote the number of disjoint SHs within a row-slab $\bar{\mathcal{R}}_i^h$ (or column-slab $\bar{\mathcal{R}}_i^v$) by $\bar{N}_i^h$ (or $\bar{N}_i^v$). Let $\bar{N}^h = \inf \bar{N}_i^h$, $\bar{N}^v = \inf \bar{N}_i^v$. According to Lemma 10, each cell in $\mathbb{L}(\sqrt{n}, \sigma\sqrt{\log n} - \epsilon_n, 0)$ contains w.h.p. at least $\theta_1 \log n$ nodes. Since the SHs include odd SHs and even SHs, the following lemma clearly holds.

Lemma 11: For any ϱ, $\varrho > 1 + \log \varrho$ and σ, $\sigma^2 \geq \frac{4\varrho}{(2\varrho - \log \varrho - 1)}$, there exists a constant $\theta_1 = \frac{\sigma^2}{2\varrho}$ such that

$$\lim_{n\to\infty} \Pr(\bar{N}^h \geq 2\theta_1 \log n) = 1; \ \lim_{n\to\infty} \Pr(\bar{N}^v \geq 2\theta_1 \log n) = 1.$$

Notations for SHs: For simplicity, we assume that there are exactly $2\theta_1 \log n$ horizontal SHs in each $\bar{\mathcal{R}}_j^h$ (or $\bar{\mathcal{R}}_j^v$), including $\theta_1 \log n$ odd horizontal SHs and $\theta_1 \log n$ even horizontal SHs, without changing the results in order sense. According to Lemma 11, we can further divide every row-slab $\bar{\mathcal{R}}_j^h$ into $2\theta_1 \log n$ slices of width $\bar{l}$ and length $\sqrt{n}$, where $\bar{l} = \sigma/(2\theta_1\sqrt{\log n})$. We call these produced slices *row-slices*. Then, we can define the mappings among the row-slab, column-slab, row-slice, column-slice, and SHs. Please see the details in Table.2. The following remarks are made:

- Any slice can and only can project to the SH in the slab containing it, which ensures the distance from any nodes to the corresponding SH is at most of $\sigma\sqrt{\log n}$.
- For a node v and row-slice $\bar{\mathcal{S}}_i^h \in \bar{\mathbb{S}}^h$, if v is located in $\bar{\mathcal{S}}_i^h$, then $\bar{\mathbf{f}}^h(v) = \bar{\mathbf{g}}^h(\bar{\mathcal{S}}_i^h)$.

Second-Class Transmission Scheduling: We adopt a 16-TDMA scheme to schedule the transmissions along the SHs. The main technique called *parallel scheduling* here is described as following: Instead of scheduling only one link in each activated cell in each time slot, we consider scheduling a set of links initiating from the same cell together. Specially, we divide time into a sequence of 16 successive slots. In each time slot, we consider disjoint sets of cells in $\mathbb{L}(\sqrt{n}, \sigma\sqrt{\log n} - \epsilon_n, 0)$ that are allowed to be activated simultaneously, as depicted in Fig. 3. Notice that if a cell is activated, $2\theta_1 \log n$ links that initiate from this cell can transmit simultaneously. Obviously, compared with scheduling only one link in each cell, this modification increases the total rate by order of $\Theta(\log n)$ if the total interference is still bounded. So can we prove that the total interference is still bounded? Fortunately, the proof of the following lemma gives us a positive answer. We further prove that, the rate of any SH is of order $\Omega((\log n)^{-\frac{\alpha}{2}})$. It is easy to see that the length of every hop in the SHs is at most of $\sqrt{10} \cdot (\sigma\sqrt{\log n} - \epsilon_n)$ and at least of $\sigma\sqrt{\log n} - \epsilon_n$.

Lemma 12: Along each SH, the rate can be sustained of order $\Omega((\log n)^{-\frac{\alpha}{2}})$.

Proof: For any link along the SHs in any time slot, since its length is at least of $\sigma\sqrt{\log n} - \epsilon_n$, the sum of interferences to the receivers is bounded by

$$\begin{aligned} I(n) &\leq P \cdot (\theta_1 \log n - 1) \cdot \ell(\sigma\sqrt{\log n} - \epsilon_n) \\ &\quad + \sum_{i=1}^{n} 8iP(\theta_1 \log n) \cdot \ell((4i-3) \cdot (\sigma\sqrt{\log n} - \epsilon_n)) \\ &\leq 2^\alpha \cdot P\theta_1\sigma^{-\alpha}(\log n)^{1-\frac{\alpha}{2}} \cdot \left(1 + \lim_{n\to\infty}\sum_{i=1}^{n} \frac{8i}{(4i-3)^\alpha}\right) \end{aligned}$$

The latest limitation obviously converges to a constant when $\alpha > 2$. On the other hand, since the length of the link is at most of $\sqrt{10}\cdot(\sigma\sqrt{\log n} - \epsilon_n)$, the signal strength at the receiver can be bounded by

$$S(n) \geq P \cdot \ell \cdot (\sqrt{10}(\sigma\sqrt{\log n} - \epsilon_n)) \geq P \cdot 10^{-\frac{\alpha}{2}} \sigma^{-\alpha} (\log n)^{-\frac{\alpha}{2}}$$

Then, the achievable rate along the SH is at least of

$$R(n) = \frac{1}{16} \cdot B \cdot \log_2 \left(1 + \frac{S(n)}{N_0 + I(n)}\right)$$

Since $\alpha > 2$ and $N_0 > 0$, we have $\frac{S(n)}{N_0 + I(n)} \to 0$, as $n \to \infty$. Hence, $R(n) = \Omega((\log n)^{-\frac{\alpha}{2}})$. □

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.

Algorithm 1 Multicast Routing Scheme $\mathscr{F}^r$

Input: The multicast session $\mathcal{M}_{\mathcal{S},k}$ and $\mathrm{EST}(\mathcal{U}_{\mathcal{S},k})$.
Output: A multicast routing tree $\mathcal{T}(\mathcal{U}_{\mathcal{S},k})$.

1: For each link $v_i \to v_j$ of $\mathrm{EST}(\mathcal{U}_{\mathcal{S},k})$, implement the following sub-steps to realize the routing from v_i to v_j. (Please See the illustration in Fig. 4.)
(1) By a single long hop, v_i drains the packets into the vertical SH $\bar{\mathbf{f}}^v(v_i)$ via a node $\bar{w}_i^v$ that is the closest second-class station in $\bar{\mathbf{f}}^v(v_i)$ to v_i with the distance of $|v_i\bar{w}_i^v| = \Theta(\sqrt{\log n})$.
(2) Along the vertical SH $\bar{\mathbf{f}}^v(v_i)$, the packets are drained into the horizontal FH $\mathbf{f}^h(v_i)$ via w_i^h that is the closest first-class station to the intersection of $\bar{\mathbf{f}}^v(v_i)$ and $\mathbf{f}^h(v_i)$.
(3) The packets are transported along $\mathbf{f}^h(v_i)$ to u_{ij}^h that is the closest station on $\mathbf{f}^h(v_i)$ to u_{ij}, where u_{ij} denotes the intersection of $\mathbf{f}^h(v_i)$ and $\mathbf{f}^v(v_j)$.
(4) By a single short hop, the packets are transmitted from u_{ij}^h to u_{ij}^v that is the closest station on $\mathbf{f}^v(v_j)$ to u_{ij}.
(5) The packets are transported along $\mathbf{f}^h(v_i)$ to w_j^v that is the closest first-class station to the intersection of the horizontal SH $\bar{\mathbf{f}}^h(v_j)$ and the vertical FH $\mathbf{f}^v(v_j)$.
(6) Along the horizontal SH $\bar{\mathbf{f}}^h(v_j)$, the packets are delivered to $\bar{w}_j^h$ that is the closest second-class station in $\bar{\mathbf{f}}^h(v_j)$ to v_j with the distance of $|v_j\bar{w}_j^h| = \Theta(\sqrt{\log n})$.
(7) By a single long hop, $\bar{w}_j^h$ delivers the packets to v_j.

2: Consider the next link of $\mathrm{EST}(\mathcal{U}_{\mathcal{S},k})$ (go to step 1), until all the links in $\mathrm{EST}(\mathcal{U}_{\mathcal{S},k})$ are checked.

3: For the resulted graph, we merge the same edges (hops), and remove those circles which have no impact on the connectivity of the communications for $\mathrm{EST}(\mathcal{U}_{\mathcal{S},k})$; we finally obtain the multicast routing tree $\mathcal{T}(\mathcal{U}_{\mathcal{S},k})$.

6.2 Multicast Strategy Based on FHs and SHs: $\mathscr{F}$

Let $\mathscr{F}$ denote the multicast strategy based on FHs and SHs, $\mathscr{F}^r$ and $\mathscr{F}^t$ denote its routing and transmission scheduling schemes, respectively.

6.2.1 Routing scheme $\mathscr{F}^r$

For a multicast session $\mathcal{M}_{\mathcal{S},k}$ $(k \in [1, n_s])$ with the spanning set $\mathcal{U}_{\mathcal{S},k}$, we first construct an Euclidean spanning tree (EST) for $\mathcal{M}_{\mathcal{S},k}$, denoted by $\mathrm{EST}(\mathcal{U}_{\mathcal{S},k})$ or $\mathrm{EST}(\mathcal{M}_{\mathcal{S},k})$, by using a similar method to that in [12]. We call the nodes on FHs *first-class stations*, and call the nodes on SHs *second-class stations*. Please see the illustrations in Fig.2. Based on $\mathrm{EST}(\mathcal{U}_{\mathcal{S},k})$, we propose Algorithm 1 to construct the multicast routing tree $\mathcal{T}(\mathcal{U}_{\mathcal{S},k})$.

6.2.2 Transmission scheduling scheme $\mathscr{F}^t$

During the realization of routing between each communication pairs in the EST, say $v_i \to v_j$, there are seven phases for a packet from v_i to v_j, corresponding to 7 substeps of Step 1 in Algorithm 1. Please see the illustration in Fig. 4.

Throughout all seven phases, there are two types of links in terms of hop length. The first are the short links, along the FHs, of the length $O(1)$, we call them *first-class links*. The links in Phases 3, 4 and 5 are first-class links. The second are the longer links of length $\Theta(\sqrt{\log n})$, we call them *second-class links*. The links in Phases 1, 2 and Phases 6,7 are second-class links. Divide the time into two disjoint parts, and call them *first-class phase* and *second-class phase*, in which we schedule respectively the first-class links and second-class links.

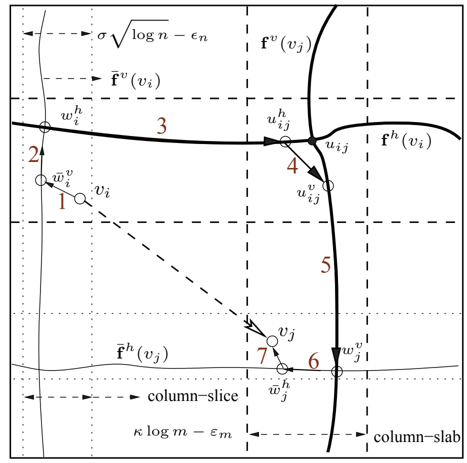


Fig. 4. Routing of communication-pairs $v_i \to v_j$. Two bold solid curves represent the FH $\mathbf{f}^h(v_i)$ and FH $\mathbf{f}^v(v_j)$. Two thin solid curves represent the SH $\bar{\mathbf{f}}^v(v_i)$ and SH $\bar{\mathbf{f}}^h(v_j)$.

6.3 Multicast Throughput Under Strategy $\mathscr{F}$

For the first-class phase consisting of Phases 3, 4 and 5, the links in Phase 4 have no difference from those in Phase 3 and Phase 5. Thus, we do not analyze them individually, while we regard Phase 3, 4 and 5 as a single phase called *Phase-(3; 4; 5)*. Then, we propose Lemma 13 for this phase.

Lemma 13: During Phase-(3; 4; 5), the per-session throughput can be achieved of order

$$\Lambda_{3;4;5} = \begin{cases} \Omega(\frac{1}{n_s} \cdot \frac{n}{Q_{3;4;5}}) & \text{when} \quad n_s = [\frac{n \cdot \log n}{Q_{3;4;5}}, n] \\ \Omega(1/\log n) & \text{when} \quad n_s = (1, \frac{n \cdot \log n}{Q_{3;4;5}}] \end{cases}$$

where

$$Q_{3;4;5} = \begin{cases} \Theta(\sqrt{n_d n}) & \text{when} \quad n_d : [1, \frac{n}{(\log n)^2}] \\ \Theta(n_d \log n) & \text{when} \quad n_d : [\frac{n}{(\log n)^2}, \frac{n}{\log n}] \\ \Theta(n) & \text{when} \quad n_d : [n/\log n, n] \end{cases} \quad (9)$$

Subsequently, we consider Phase 2 and Phase 6.

Lemma 14: In Phase 2, the per-session throughput can be achieved of order

$$\Lambda_2 = \begin{cases} \Omega(\frac{R_2}{n_s} \cdot \frac{n}{Q_2}) & \text{when} \quad n_s = [\frac{n}{Q_2} \cdot \log n, n] \\ \Omega(R_2/\log n) & \text{when} \quad n_s = (1, \frac{n}{Q_2} \cdot \log n] \end{cases} \quad (10)$$

where $R_2 = \Omega((\log n)^{-\frac{\alpha}{2}})$, $Q_2 = \min_{\text{order}}\{n_d\sqrt{\log n}, n\}$.

By a similar procedure to the proof of Lemma 14, we can get the following result for Phase 6.

Lemma 15: In Phase 6, the per-session multicast throughput can be achieved of the same order as in Phase 2.

During Phases 1 and 7, like Phases 2 and 6, we also use a 16-TDMA scheme to schedule the links of length $\Theta(\sqrt{\log n})$ in parallel, by which we can ensure that every link achieves

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.

Algorithm 2 Multicast Routing Scheme $\mathscr{S}^r$

Input: The multicast session $\mathcal{M}_{\mathcal{S},k}$ and $\mathrm{EST}(\mathcal{U}_{\mathcal{S},k})$.
Output: A multicast routing tree $\bar{\mathcal{T}}(\mathcal{U}_{\mathcal{S},k})$.

1: For each link $v_i \to v_j$ of $\mathrm{EST}(\mathcal{U}_{\mathcal{S},k})$, implement the following sub-steps to realize the routing $v_i \to v_j$.
(1) By a single hop, v_i drains the packets into the vertical SH $\bar{\mathbf{f}}^v(v_i)$ via $\bar{w}_i^v$ that is the closest second-class station to v_i on $\bar{\mathbf{f}}^v(v_i)$ with the distance of $|v_i\bar{w}_i^v| = \Theta(\sqrt{\log n})$).
(2) The packets are transported along $\bar{\mathbf{f}}^v(v_i)$ to $\bar{u}_{ij}^v$ that is the closest second-class station on $\bar{\mathbf{f}}^v(v_i)$ to $\bar{u}_{ij}$, where $\bar{u}_{ij}$ denotes the intersection of $\bar{\mathbf{f}}^v(v_i)$ and $\bar{\mathbf{f}}^h(v_j)$;
(3) By a single hop, the packets are transmitted from $\bar{u}_{ij}^v$ to $\bar{u}_{ij}^h$ that is the closest station on $\bar{\mathbf{f}}^h(v_j)$ to $\bar{u}_{ij}$.
(4) The packets are transported along horizontal SH $\bar{\mathbf{f}}^h(v_j)$ to $\bar{w}_j^h$ that is the closest second-class station to v_j on $\bar{\mathbf{f}}^h(v_j)$ with the distance of $|v_j\bar{w}_j^h| = \Theta(\sqrt{\log n})$.
(5) By a single hop, $\bar{w}_j^h$ delivers the packets to v_j.
2: Consider the next link of $\mathrm{EST}(\mathcal{U}_{\mathcal{S},k})$ (go to step 1), until all the links in $\mathrm{EST}(\mathcal{U}_{\mathcal{S},k})$ are checked.
3: Use the same method as step 3 of $\mathscr{F}^r$ to obtain the final multicast routing tree $\bar{\mathcal{T}}(\mathcal{U}_{\mathcal{S},k})$.

the rate of order $\Omega((\log n)^{-\frac{\alpha}{2}})$. On the other hand, there is no relay burden on the nodes in Phases 1 and 7 due to the single-hop pattern, thus, it is easy to obtain the following result.

Lemma 16: $\min_{\text{order}}\{\Lambda_1, \Lambda_7\} = \Omega(\max_{\text{order}}\{\Lambda_2, \Lambda_6\})$.

Combining Lemmas 13, 14, 15 and 16, we obtain the following result according to Lemma 4.

Theorem 5: By using the multicast strategy $\mathscr{F}$, the per-session multicast throughput is achieved of order:
When $n_d = O(n/\sqrt{\log n})$,

$$\begin{cases} \Omega(\frac{1}{(\log n)^{1+\frac{\alpha}{2}}}) & \text{when } n_s : (1, \frac{n\log n}{\Gamma}] \\ \Omega(\min_{\text{order}}\{\frac{n}{n_s\Gamma}, \frac{1}{(\log n)^{1+\frac{\alpha}{2}}}\}) & \text{when } n_s : [\frac{n\log n}{\Gamma}, \frac{n\sqrt{\log n}}{n_d}] \\ \Omega(\min_{\text{order}}\{\frac{n}{n_s\Gamma}, \frac{n}{n_s n_d(\log n)^{\frac{\alpha+1}{2}}}\}) & \text{when } n_s : [\frac{n\sqrt{\log n}}{n_d}, n] \end{cases}$$

When $n_d = \Omega(n/\sqrt{\log n})$,

$$\begin{cases} \Omega(\frac{n}{n_s n_d(\log n)^{\frac{\alpha+1}{2}}}) & \text{when } n_s : (1, n\sqrt{\log n}/n_d] \\ \Omega(\frac{1}{(\log n)^{1+\frac{\alpha}{2}}}) & \text{when } n_s : [n\sqrt{\log n}/n_d, n] \end{cases}$$

where $\Gamma := Q_{3;4;5}$ is defined in Equation (9).

6.4 Multicast Strategy Based on Only SHs: $\mathscr{S}$

Now, we devise another multicast strategy, denoted by $\mathscr{S}$, which is only based on the SHs. The routing scheme is denoted by $\mathscr{S}^r$, and is described in Algorithm 2. For the transmission scheduling, denoted by $\mathscr{S}^t$, we only need to implement the second-class transmission scheduling since no other types of links exist. It can be shown that if the bottleneck of $\mathscr{F}^r$ lies in its second-class phase, the multicast throughput under $\mathscr{S}$ is not less than that under $\mathscr{F}$. Specifically, we have

Theorem 6: By using the multicast strategy $\mathscr{S}$, the per-session multicast throughput is achieved of order

$$\begin{cases} \Omega(\frac{n}{(\log n)^{\frac{\alpha}{2}} n_s\cdot\bar{Q}}) & \text{when } n_s = \Omega(\frac{n\cdot\log n}{\bar{Q}}) \\ \Omega(1/(\log n)^{1+\frac{\alpha}{2}}) & \text{when } n_s = O(\frac{n\cdot\log n}{\bar{Q}}) \end{cases}$$

where $\bar{Q} = \begin{cases} \Theta(\sqrt{\frac{n_d n}{\log n}}) & \text{when } n_d = O(\frac{n}{\log n}) \\ \Theta(n_d) & \text{when } n_d = \Omega(\frac{n}{\log n}). \end{cases}$

Proof: The throughputs during Phase 1 and Phase 5 are not less than those during other phases, implying that the bottleneck of the entire routing lies on SHs. According to Lemma 12, the rate of each SH can be achieved of order $\Omega((\log n)^{-\frac{\alpha}{2}})$. On the other hand, by using a similar procedure to the proof of Lemma 13, we can obtain that the burden of the second-class stations is w.h.p. at most of order

$$\begin{cases} O(n_s\bar{Q}/n) & \text{when } n_s\bar{Q}/n = \Omega(\log n) \\ O(\log n) & \text{when } n_s\bar{Q}/n = O(\log n) \end{cases} \tag{11}$$

with $\bar{Q} = \min_{\text{order}}\{\sqrt{nn_d}/\sqrt{\log n} + n_d,\ n\}$. Hence, we complete the proof. □

6.5 General Result for Random Extended Networks

Combining Theorem 5 and Theorem 6, we obtain the general result in Theorem 7.

Theorem 7: The per-session multicast throughput for random extended networks can be achieved of order $\Omega(\lambda(n))$ as described in Table 3.

Theorem 2 can be obtained based on Theorem 7 by letting $n_s = \Theta(n)$.

As in [12], we design multicast routing schemes based on the construction of Euclidean spanning trees. Note that this way of constructing the spanning tree is not symmetric, which leads that most paths will go through the center area of the network. Under such routing, some parts of the network will be under a relatively large load, therefore, those parts would become a bottleneck for the multicast sessions, called *local bottleneck*. In fact, in the derivation of Theorem 2, it is just this local bottleneck that limits the network throughput under our schemes, since we take the maximum load of any part of the network into account. Then, the existence of the local bottleneck makes our schemes look *non-optimal*. While, combining with the upper bounds in Theorem 1, we obtain that our scheme is *optimal* (in order sense) in the regimes of $n_d : [1, \frac{n}{(\log n)^{\alpha+1}}]$ and $n_d : [\frac{n}{\log n}, n]$. That implies that in these regimes of n_d, the load at the local bottleneck is at most a constant times of that at other parts of the network. Furthermore, the local bottleneck issue should be fully studied in the future work. Designing a multicast scheme without the local bottleneck is a possible solution to close the remaining gap between the upper and lower bounds.

7 Literature Reviews

In this section, we mainly review the *networking-theoretic* capacity scaling laws for random ad hoc network. We summarize the classifications of this issue in Table 4, and indicate the scope of a related work by a 3-dimensional coordinate (x, y, z) based on Table 4, where $x \in$\{U, B, M\}, $y \in$\{D, E\} and $z \in$\{O, Y, G}. For instance, (U,E,G) denotes the per-session **u**nicast capacity for random extended networks (R**E**N) under Gaussian channel model (**G**CM).

In REN, there must be some links of length $\omega(1)$ under any routing scheme in order to ensure the connectivity of network. For those links, when the ProM or PhyM is adopted, the rate will be set as a constant order if they can be scheduled, which is over-optimistic and unrealistic for power-constrained wireless networks. This can explain why we hardly introduce the works on (x,E,z), $x \in$ {U, B, M} and $z \in$ {O, Y}. Moreover, for RDN, the throughput under the GCM can be equivalently achieved under the ProM and PhyM, if multiple communication and interference radii (or the thresholds of SINR) are permitted under the ProM (or the PhyM). In the following review, we use *session patterns* as the main index.

Unicast Sessions: In the pioneering work of capacity scaling laws, Gupta and Kumar [2] showed that the order of **(U,D,O)** is $\Theta(1/\sqrt{n \log n})$; and they derived the lower bound and upper bound of **(U,D,Y)** as $\Omega(1/\sqrt{n \log n})$ and $O(1/\sqrt{n})$, respectively, leaving a gap between the upper and lower bounds of unicast capacity for RDN under PhyM.

Franceschetti et al. [20] proposed the hierarchical schemes based on *bond percolation model*, under which the lower bounds of **(U,D,G)** and **(U,E,G)** can be both achieved of order $\Omega(1/\sqrt{n})$; later, Keshavarz-Haddad et al. [29] derived the upper bound of **(U,D,G)** as $O(1/\sqrt{n})$, Li et al. [21] proved that the upper bound of **(U,E,G)** is also of $O(1/\sqrt{n})$. Combining the works in [20], [21], [29], one can get that the unicast capacities for both RDN and REN under Gaussian channel model (GCM) are of order $\Theta(1/\sqrt{n})$.

Broadcast Sessions: According to [13], [30] done by Keshavarz-Haddad et al., and Tavli, respectively, the order of **(B,D,O)** is of $\Theta(\frac{1}{n})$. Keshavarz-Haddad and Riedi [31] analyzed the essential impact of topology and interference on the broadcast capacity under the PhyM and GCM. As a part of the contributions of [31], the **(B,D,Y)** and **(B,D,G)** are proved to be both of order $\Theta(\frac{1}{n})$ when the the bandwidth is of a constant order. For **(B,E,G)**, Zheng [3] proved that the order is of $\Theta(\frac{(\log n)^{-\frac{\alpha}{2}}}{n})$.

Multicast Sessions: Earlier, Jacquet and Rodolakis [32] showed that the upper bound of **(M,D,O)** is of $O(1/\sqrt{n_d n \log n})$. Shakkottai et al. [14] designed a novel multicast scheme called *comb*, by which the lower bound of **(M,D,O)** can be achieved of order $\Omega(1/\sqrt{n_d n \log n})$ when the number of multicast sources, denoted by n_s, is n^{ε} for some $\varepsilon > 0$, and the number of destinations per multicast session, denoted by n_d, is $n^{1-\varepsilon}$. Li [12] proved that the order of **(M,D,O)** is of $\Theta(1/\sqrt{n_d n \log n})$ when $n_d = O(n/\log n)$, and is of $\Theta(1/n)$ when $n_d = \Omega(n/\log n)$. By using a novel technique called *arena*, Keshavarz-Haddad and Riedi [19] proved all the upper bounds of **(M,D,O)**, **(M,D,Y)** and **(M,D,G)** are of order $O(\frac{1}{\sqrt{n n_d}})$ when $n_d : [1, \frac{n}{(\log n)^2}]$, are of $O(\frac{1}{n_d \log n})$ when $n_d : [\frac{n}{(\log n)^2}, \frac{n}{\log n}]$, and are of $O(1)$ when $n_d : [\frac{n}{\log n}, n]$. Furthermore, they derived the lower bounds of **(M,D,Y)** and **(M,D,G)** are of order $\Omega(\frac{1}{\sqrt{n n_d}})$ when n_d : $[1, \frac{n}{(\log n)^3}]$, are of $\Omega(\frac{(\log n)^{-3/2}}{n_d})$ when $n_d : [\frac{n}{(\log n)^3}, \frac{n}{(\log n)^2}]$,

TABLE 3
Achievable Per-Session Multicast Throughput for REN

Range of n_d	Better Strategy and Order of $\lambda(n)$
$[1, \frac{n}{(\log n)^{1+\alpha}}]$	$\mathscr{F}: \begin{cases} (\log n)^{-1-\frac{\alpha}{2}} & \text{if } n_s : (1, \frac{\sqrt{n}}{\sqrt{n_d}} \cdot (\log n)^{1+\frac{\alpha}{2}}] \\ \frac{\sqrt{n}}{n_s\sqrt{n_d}} & \text{if } n_s : [\frac{\sqrt{n}}{\sqrt{n_d}} \cdot (\log n)^{1+\frac{\alpha}{2}}, n] \end{cases}$
$[\frac{n}{(\log n)^{1+\alpha}}, \frac{n}{(\log n)^2}]$	$\mathscr{F}: \begin{cases} (\log n)^{-1-\frac{\alpha}{2}} & \text{if } n_s : (1, \frac{n\cdot\sqrt{\log n}}{n_d}] \\ \frac{n}{n_s n_d (\log n)^{\frac{\alpha+1}{2}}} & \text{if } n_s : [\frac{n\cdot\sqrt{\log n}}{n_d}, n] \end{cases}$
$[\frac{n}{(\log n)^2}, \frac{n}{\log n}]$	$\mathscr{S}: \begin{cases} (\log n)^{-1-\frac{\alpha}{2}} & \text{if } n_s : (1, \frac{\sqrt{n}\cdot(\log n)^{\frac{3}{2}}}{\sqrt{n_d}}] \\ \frac{\sqrt{n}}{n_s\sqrt{n_d}\cdot(\log n)^{\frac{\alpha-1}{2}}} & \text{if } n_s : [\frac{\sqrt{n}\cdot(\log n)^{\frac{3}{2}}}{\sqrt{n_d}}, n] \end{cases}$
$[\frac{n}{\log n}, n]$	$\mathscr{S}: \begin{cases} (\log n)^{-1-\frac{\alpha}{2}} & \text{if } n_s : (1, \frac{n \log n}{n_d}] \\ \frac{n}{n_s \cdot n_d \cdot (\log n)^{\frac{\alpha}{2}}} & \text{if } n_s : [\frac{n \log n}{n_d}, n] \end{cases}$

are of $\Omega(\frac{1}{\sqrt{n n_d \log n}})$ when $n_d : [\frac{n}{(\log n)^2}, \frac{n}{\log n}]$, and are of $\Omega(1)$ when $n_d : [\frac{n}{\log n}, n]$.

For **(M,E,G)**, Li et al. [21] proposed a lower bound as $\Omega(\frac{\sqrt{n}}{n_s\sqrt{n_d}})$ when $n_d = O(\frac{n}{(\log n)^{2\alpha+6}})$ and $n_s = \Omega(n^{\frac{1}{2}+\theta})$, where $\theta > 0$ is any positive constant. Note that we focus on **(M,E,G)** in this paper. We derive the more tight lower bounds of **(M,E,G)** for *all cases* of $n_s : (1, n]$ by introducing the *two-level highway system* and *parallel transmission scheduling*, and propose the upper bounds based on some new arguments.

There are some other types of sessions, such as *gathercast* (*many-to-one sessions*) [33], [34], *anycast* [35] and *manycast* [36], etc. We omit the review of works for those sessions, since they are not directly relevant to the scope of this paper.

8 Conclusion

We study the networking-theoretic multicast capacity bounds for random extended networks (REN) under Gaussian Channel model. Based on percolation theory, we propose two multicast strategies for REN and derive the achievable multicast throughput by considering all cases of $n_s : (1, n]$ and $n_d : [1, n]$. We show that under the assumption of $n_s = \Theta(n)$, the per-session multicast throughput derived by our scheme is order-optimal when $n_d = O(\frac{n}{(\log n)^{\alpha+1}})$ or $n_d = \Omega(\frac{n}{\log n})$. There are still gaps between the lower bounds and upper bounds on multicast capacity of REN for some regimes of n_d, i.e., $n_d : [\frac{n}{(\log n)^{\alpha+1}}, \frac{n}{\log n}]$. An interesting and challenging issue is to close the gaps on multicast capacity by presenting possibly new tighter upper bounds, and lower bounds, and designing corresponding algorithms to achieve the asymptotic multicast capacity.

Acknowledgments

The authors would like to thank the anonymous reviewers for their constructive comments. The research of authors is partially supported by the National Basic Research Program

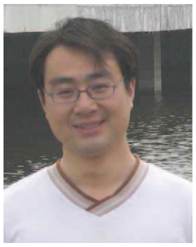
TABLE 4
Networking-Theoretic Capacity Scaling Laws for Random Ad Hoc Networks

	Capacity of Session Patterns	Scaling Models (Density)	Communication Models
1	**U:** **U**nicast Capacity	**D:** Random **D**ense Networks (R**D**N)	**O:** Pr**o**tocol Model (Pr**o**M)
2	**B:** **B**roadcast Capacity	**E:** Random **E**xtended Networks (R**E**N)	**Y:** Ph**y**sical Model (Ph**y**M)
3	**M:** **M**ulticast Capacity		**G:** **G**aussian Channel Model (**G**CM)

of China (973 Program) under grants No. 2010CB328101 and No. 2010CB334707, the Program for Changjiang Scholars and Innovative Research Team in University, the Shanghai Key Basic Research Project under grant No. 10DJ1400300, the Expo Science and Technology Specific Projects of China under grant No. 2009BAK43B37, the NSF CNS-0832120 and CNS-1035894, the National Natural Science Foundation of China under grant No. 60828003, the Program for Zhejiang Provincial Key Innovative Research Team, and the Program for Zhejiang Provincial Overseas High-Level Talents (One-hundred Talents Program).

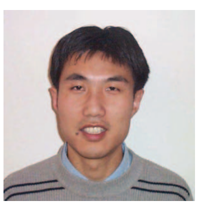
REFERENCES

[1] C. Wang, X.-Y. Li, C. Jiang, S. Tang, and Y. Liu, "Scaling laws on multicast capacity of large scale wireless networks," in *Proc. IEEE INFOCOM 2009*.
[2] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Transactions on Information Theory*, vol. 46, no. 2, pp. 388–404, 2000.
[3] R. Zheng, "Asymptotic bounds of information dissemination in power-constrained wireless networks," *IEEE Transactions on Wireless Communications*, vol. 7, no. 1, pp. 251–259, Jan. 2008.
[4] L. Xie and P. Kumar, "A network information theory for wireless communication: scaling laws and optimal operation," *IEEE Transactions on Information Theory*, vol. 50, no. 5, pp. 748–767, 2004.
[5] A. ÖzgÜr, O. LÉvÊque, and D. Tse, "Hierarchical Cooperation Achieves Optimal Capacity Scaling in Ad Hoc Networks," *IEEE Transactions on Information Theory*, vol. 53, no. 10, pp. 3549–3572, 2007.
[6] S. Yi, Y. Pei, and S. Kalyanaraman, "On the capacity improvement of ad hoc wireless networks using directional antennas," in *Proc. ACM MobiHoc 2003*.
[7] S. R. Kulkarni and P. Viswanath, "A deterministic approach to throughput scaling in wireless networks," *IEEE Transactions on Information Theory*, vol. 50, no. 6, pp. 1041–1049, 2004.
[8] B. Liu, Z. Liu, and D. Towsley, "On the capacity of hybrid wireless networks," in *Proc. IEEE INFOCOM 2003*.
[9] U. C. Kozat and L. Tassiulas, "Throughput capacity of random ad hoc networks with infrastructure support," in *Proc. ACM MobiCom 2003*.
[10] A. Zemlianov and G. de Veciana, "Capacity of ad hoc wireless networks with infrastructure support," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 3, pp. 657–667, 2005.
[11] J. Gomez and A. T. Campbell, "Variable-range transmission power control in wireless ad hoc networks," *IEEE Transactions on Mobile Computing*, vol. 6, no. 1, pp. 87–99, 2007.
[12] X.-Y. Li, "Multicast capacity of wireless ad hoc networks," *IEEE/ACM Transactions on Networking*, vol. 17, no. 3, pp. 950–961, 2009.
[13] A. Keshavarz-Haddad, V. Ribeiro, and R. Riedi, "Broadcast capacity in multihop wireless networks," in *Proc. ACM MobiCom 2006*.
[14] X. Shakkottai, S. Liu, and R. Srikant, "The multicast capacity of large multihop wireless networks," in *Proc. ACM MobiHoc 2007*.
[15] X. Wang, Y. Bei, Q. Peng, and L. Fu, "Speed improves delay-capacity tradeoff in motioncast," *to appear in IEEE Transactions on Parallel and Distributed Systems*, 2011.
[16] S. Toumpis and A. J. Goldsmith, "Capacity regions for wireless ad hoc networks," *IEEE Transactions on Wireless Communications*, vol. 2, no. 4, pp. 736–748, 2003.
[17] T. Cover and J. Thomas, *Elements of information theory*. Wiley, 2006.
[18] A. Agarwal and P. Kumar, "Capacity bounds for ad hoc and hybrid wireless networks," *ACM SIGCOMM Computer Communication Review*, vol. 34, no. 3, pp. 71–81, 2004.
[19] A. Keshavarz-Haddad and R. Riedi, "Multicast capacity of large homogeneous multihop wireless networks," in *Proc. IEEE WiOpt 2008*.
[20] M. Franceschetti, O. Dousse, D. Tse, and P. Thiran, "Closing the gap in the capacity of wireless networks via percolation theory," *IEEE Transactions on Information Theory*, vol. 53, no. 3, pp. 1009–1018, 2007.
[21] X.-Y. Li, Y. Liu, S. Li, and S. Tang, "Multicast capacity of wireless ad hoc networks under Gaussian channel model," *IEEE/ACM Transactions on Networking*, vol. 18, no. 4, pp. 1145–1157, 2010.
[22] L. Xie and P. Kumar, "On the path-loss attenuation regime for positive cost and linear scaling of transport capacity in wireless networks," *IEEE/ACM Transactions on Networking*, vol. 14, pp. 2313–2328, 2006.
[23] O. Dousse, M. Franceschetti, and P. Thiran, "Information theoretic bounds on the throughput scaling of wireless relay networks," in *Proc. IEEE INFOCOM 2005*.
[24] A. ÖzgÜr, O. LÉvÊque, and E. Preissmann, "Scaling Laws for One-and Two-Dimensional Random Wireless Networks in the Low-Attenuation Regime," *IEEE Transactions on Information Theory*, vol. 53, no. 10, pp. 3573–3585, 2007.
[25] S. Ahmad, A. Jovičić, and P. Viswanath, "On outer bounds to the capacity region of wireless networks," *IEEE/ACM Transactions on Networking*, vol. 14, pp. 2770–2776, 2006.
[26] O. Leveque and I. Telatar, "Information-theoretic upper bounds on the capacity of large extended ad hoc wireless networks," *IEEE Transactions on Information Theory*, vol. 51, no. 3, pp. 858–865, 2005.
[27] C. Hu, X. Wang, D. Nie, and J. Zhao, "Multicast scaling laws with hierarchical cooperation," in *Proc. IEEE INFOCOM 2010*.
[28] C. Wang, X.-Y. Li, C. Jiang, S. Tang, and Y. Liu, "Multicast throughput for hybrid wireless networks under Gaussian channel model," *to appear in IEEE Transactions on Mobile Computing*, Oct. 2010.
[29] A. Keshavarz-Haddad and R. Riedi, "Bounds for the capacity of wireless multihop networks imposed by topology and demand," in *Proc. ACM MobiHoc 2007*.
[30] B. Tavli, "Broadcast capacity of wireless networks," *IEEE Communications Letters*, vol. 10, no. 2, pp. 68–69, 2006.
[31] A. Keshavarz-Haddad and R. Riedi, "On the broadcast capacity of multihop wireless networks: Interplay of power, density and interference," in *Proc. IEEE SECON 2007*.
[32] P. Jacquet and G. Rodolakis, "Multicast scaling properties in massively dense ad hoc networks," in *Proc. IEEE ICPADS 2005*.
[33] D. Marco, E. Duarte-Melo, M. Liu, and D. Neuhoff, "On the many-to-one transport capacity of a dense wireless sensor network and the compressibility of its data," in *Proc. IEEE IPSN 2003*.
[34] A. Giridhar and P. Kumar, "Computing and communicating functions over sensor networks," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 4, pp. 755–764, 2005.
[35] V. Lenders, M. May, and B. Plattner, "Density-based anycast: a robust routing strategy for wireless ad hoc networks," *IEEE/ACM Transactions on Networking*, vol. 16, no. 4, pp. 852–863, 2008.
[36] C. Carter, S. Yi, P. Ratanchandani, and R. Kravets, "Manycast: exploring the space between anycast and multicast in ad hoc networks," in *Proc. ACM MobiCom 2003*.

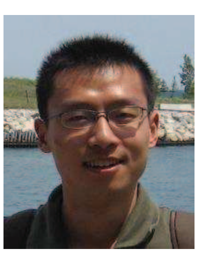


Cheng Wang received his BS degree at Department of Mathematics and Physics from Shandong University of Technology in 2002, his MS degree at Department of Applied Mathematics from Tongji University in 2006, and his PhD degree in Department of Computer Science at Tongji University in 2011. His research interests include wireless communications and networking, network coding, and distributed computing.

Changjun Jiang received the Ph.D. degree from the Institute of Automation, Chinese Academy of Sciences, Beijing, China, in 1995 and conducted post-doctoral research at the Institute of Computing Technology, Chinese Academy of Sciences, in 1997. Currently he is a Professor with the Department of Computer Science and Engineering, Tongji University, Shanghai. He is also a council member of China Automation Federation and Artificial Intelligence Federation, the Vice Director of Professional Committee of Petri Net of China Computer Federation, the Vice Director of Professional Committee of Management Systems of China Automation Federation, and an Information Area Specialist of Shanghai Municipal Government. His current areas of research are concurrent theory, Petri net, and formal verification of software, concurrency processing and intelligent transportation systems.



Xiang-Yang Li (M'99, SM'08) is an associate professor of Computer Science at the Illinois Institute of Technology. He received MS (2000) and PhD (2001) degree at Department of Computer Science from University of Illinois at Urbana-Champaign. He received the Bachelor degree at Department of Computer Science and Bachelor degree at Department of Business Management from Tsinghua University, China, both in 1995. He serves as an Editor of of several journals, including "IEEE Transactions on Parallel and Distributed Systems (TPDS)", and "Networks: An International Journal". He also serves in Advisory Board of "Ad Hoc & Sensor Wireless Networks: An International Journal" from 2005, and "IEEE CN" from 2011. He was a guest editor of several special issues, e.g., "ACM Mobile Networks and Applications", "IEEE JSAC". He published a monograph "Wireless Ad Hoc and Sensor Networks: Theory and Applications", in June 2008 by Cambridge University Press. He also co-edited the book "Encyclopedia of Algorithms", by Springer publisher, as the area editor for mobile computing.



Shaojie Tang has been a PhD student of Computer Science Department at the Illinois Institute of Technology since 2006. He received BS degree in Radio Engineering from Southeast University, China, in 2006. His current research interests include algorithm design and analysis for wireless ad hoc networks, wireless sensor networks, and online social networks.

Yuan He received his BE degree in University of Science and Technology of China, his ME degree in Institute of Software, Chinese Academy of Sciences, and his PhD degree in Hong Kong University of Science and Technology. He is a member of Tsinghua National Lab for Information Science and Technology. He now works as a PostDoc Fellow with Prof. Yunhao Liu in the Department of Computer Science and Engineering at Hong Kong University of Science and Technology. His research interests include sensor networks, peer-to-peer computing, and pervasive computing. He is a member of the IEEE and ACM.

Dr. Xufei Mao is an assistant professor at Department of Computer Science, Beijing University of Posts and Telecommunications, China. He received B.S. from Shenyang Univ. of Tech., M.S. from Northeastern University, China, and PhD degree in Computer Science from Illinois Institute of Technology. His research interests include design and analysis of algorithms for wireless sensor networks and the design and implementation of large-scale wireless sensor network systems. He is a student member of IEEE.

Yunhao Liu (SM'06) received his BS degree in Automation Department from Tsinghua University, China, in 1995, and an MA degree in Beijing Foreign Studies University, China, in 1997, and an MS and a Ph.D. degree in Computer Science and Engineering at Michigan State University in 2003 and 2004, respectively. He is a member of Tsinghua National Lab for Information Science and Technology, and the Director of Tsinghua National MOE Key Lab for Information Security. He is also a faculty at the Department of Computer Science and Engineering, the Hong Kong University of Science and Technology. Being a senior member of IEEE, he is also the ACM Distinguished Speaker.