MMAE 530 Advanced Mechanics of Solids

Midterm Exam Formula Sheet

1. Coordinate Transformation (Change of Basis), $Q_{ij} = \underline{e}'_i \bullet \underline{e}_j$ and $Q_{ik}Q_{jk} = \delta_{ij}$

   For Vectors, $u'_i = Q_{ij}u_j$

   For Two-Tensors, $\sigma'_{ij} = Q_{ik}Q_{jl}\sigma_{kl}$

   For Three-Tensors, $\alpha'_{ijk} = Q_{il}Q_{jm}Q_{kn}\alpha_{lmn}$

   For Four-Tensors, $C'_{ijkl} = Q_{im}Q_{jn}Q_{kp}Q_{lq}C_{mnpq}$

2. Deformation Mapping, $x'_i = \chi(x_i) = x_i + u_i(x_j)$

3. Deformation Gradient Tensor Components, $F_{ij} = \dfrac{\partial x'_i}{\partial x_j} = u_{i,j} + \delta_{ij}$

4. Lagrangian (Green's) Strain Tensor, $E_{ij} = \dfrac{1}{2}(F_{ki}F_{kj} - \delta_{ij}) = \dfrac{1}{2}(u_{i,j} + u_{j,i} + u_{k,i}u_{k,j})$

5. Lagrangian Strain in Direction $\hat{n}$, $E_{nn} = \hat{n}_i E_{ij}\hat{n}_j$

6. Stretch in Direction $\hat{n}$, $\lambda(n) = \left|F_{ij}\hat{n}_j\right| = \dfrac{1}{2}(1+2E_{nn})^{1/2}$

7. Small Strain Tensor Components, $\varepsilon_{ij} = \dfrac{1}{2}(u_{i,j} + u_{j,i})$

8. Extensional Strain in Direction $\hat{n}$, $\varepsilon_{nn} = \hat{n}_i\varepsilon_{ij}\hat{n}_j$

9. Shear Strain Between the Directions $\hat{n}$ and $\hat{t}$, $\varepsilon_{nt} = \dfrac{\gamma_{nt}}{2} = \hat{n}_i\varepsilon_{ij}\hat{t}_j$

10. Compatibility in 3-D, $\varepsilon_{ij,kl} + \varepsilon_{kl,ij} - \varepsilon_{ik,jl} - \varepsilon_{jl,ik} = 0$

11. Compatibility in 2-D (plane strain), $\varepsilon_{11,22} + \varepsilon_{22,11} - 2\,\varepsilon_{12,12} = 0$

12. Stress Tensor Components, $\sigma_{ij} = \sigma_{ji}$

13. Traction components, $t_i = \sigma_{ij}n_j$

14. Equilibrium, $\sigma_{ij,j} + b_i - \rho a_i = 0$ where b_i is body force, and a_i is acceleration (in rotational motion

    radial inertia is $a_r = w^2 r$ where w is angular velocity, r is distance from the axis of rotation)

15. Constitutive Relation, $\sigma_{ij} = C_{ijkl}\varepsilon_{kl}$

16. Linear Isotropic Stress-Strain Relation, $\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij}$

17. Linear Isotropic Strain-Stress Relation, $\varepsilon_{ij} = \dfrac{1}{E}\left[(1+\nu)\sigma_{ij} - \nu\sigma_{kk}\delta_{ij}\right]$

18. Volumetric Strain (Dilatation), $\dfrac{\Delta V}{V} = d = \varepsilon_{kk}$

19. Pressure, $p = -\dfrac{\sigma_{kk}}{3}$

20. Principal Values (Eigenvalues) of Tensor $\underline{\underline{A}}$, $\det(A_{ij} - \lambda\delta_{ij}) = \left| A_{ij} - \lambda\delta_{ij} \right| = 0$

21. Invariants of Tensor $\underline{\underline{A}}$, (single subscripted quantities stands for eigenvalues of tensor $\underline{\underline{A}}$)

$$I_1 = tr(\underline{\underline{A}}) = A_{kk} = A_1 + A_2 + A_3$$

$$I_2 = \frac{1}{2}\{(tr\underline{\underline{A}})^2 - tr(\underline{\underline{A}})^2\} = \frac{1}{2}(A_{ii}A_{jj} - A_{ij}A_{ji}) = A_1A_2 + A_1A_3 + A_2A_3$$

$$I_3 = \det(\underline{\underline{A}}) = e_{ijk}A_{i1}A_{j2}A_{k3} = A_1A_2A_3$$

22. Deviatoric Stress Tensor Components, $S_{ij} = \sigma_{ij} - \dfrac{\sigma_{kk}}{3}\delta_{ij}$

23. Deviatoric Strain Tensor Components, $e_{ij} = \varepsilon_{ij} - \dfrac{\varepsilon_{kk}}{3}\delta_{ij}$

24. $S_{ij} = 2\mu e_{ij}$, $p = -Kd$, Bulk Modulus $K = \dfrac{E}{3(1-2\nu)}$

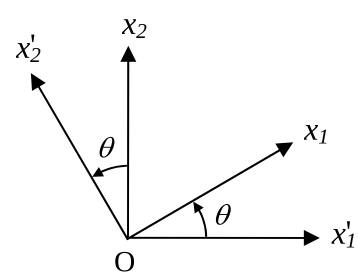
25. Lame's Constants, $\mu = \dfrac{E}{2(1+\nu)}$, $\lambda = \dfrac{\nu E}{(1+\nu)(1-2\nu)}$, $\lambda + 2\mu = \dfrac{(1-\nu)E}{(1+\nu)(1-2\nu)}$

26. Transformation of Stress Tensor Components Between Two Coordinates (2-D)

$$\sigma'_{11} = \left(\frac{\sigma_{11}+\sigma_{22}}{2}\right) + \left(\frac{\sigma_{11}-\sigma_{22}}{2}\right)\cos 2\theta + \tau_{12}\sin 2\theta$$

$$\sigma'_{22} = \left(\frac{\sigma_{11}+\sigma_{22}}{2}\right) - \left(\frac{\sigma_{11}-\sigma_{22}}{2}\right)\cos 2\theta - \tau_{12}\sin 2\theta$$

$$\tau'_{12} = -\left(\frac{\sigma_{11}-\sigma_{22}}{2}\right)\sin 2\theta + \tau_{12}\cos 2\theta$$



27. Principal Stresses (2-D)

$$\sigma_{1,2} = \left(\frac{\sigma_{11}+\sigma_{22}}{2}\right) \pm \sqrt{\left(\frac{\sigma_{11}-\sigma_{22}}{2}\right)^2 + \tau_{12}^2}$$

28. Maximum Shear Stress (2-D)

$\tau_{\max} = \dfrac{\sigma_{\max} - \sigma_{\min}}{2}$, where $\sigma_{\max}$, $\sigma_{\min}$ are the max. and min. principal stresses respectively

29. Stresses on Octahedral Planes

$$\sigma_{oct} = \frac{1}{3}[\sigma_1 + \sigma_2 + \sigma_3]$$

$$\tau_{oct} = \frac{1}{3}[(\sigma_1-\sigma_2)^2 + (\sigma_1-\sigma_3)^2 + (\sigma_2-\sigma_3)^2]^{1/2}$$

Good Luck,
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