HOMEWORK #2

MMAE 530 Advanced Mechanics of Solids
Fall 2009
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(Due 9/13/11)

Problem 1. Using index notation, show that

1) $\underline{\mathbf{a}}\times(\underline{\mathbf{b}}\times\underline{\mathbf{c}})=(\underline{\mathbf{a}}\bullet\underline{\mathbf{c}})\underline{\mathbf{b}}-(\underline{\mathbf{a}}\bullet\underline{\mathbf{b}})\underline{\mathbf{c}}$
2) $\underline{\mathbf{a}}\bullet(\underline{\mathbf{b}}\times\underline{\mathbf{c}})=\underline{\mathbf{b}}\bullet(\underline{\mathbf{c}}\times\underline{\mathbf{a}})$

where $\underline{\mathbf{a}}$, $\underline{\mathbf{b}}$, $\underline{\mathbf{c}}$ are vectors.

Problem 2. Suppose φ and $\underline{\mathbf{v}}$ are twice continuously differentiable scalar and vector functions, respectively. Using index notation, show that

1) $\underline{\nabla}\bullet(\underline{\nabla}\times\underline{\mathbf{v}})=\mathbf{0}$
2) $\underline{\nabla}\bullet(\underline{\nabla}\varphi)=\nabla^2\varphi$

Problem 3. Determine which one of the following expressions makes no sense. Simplify the rest.

1) $M_{ij}\,\delta_{ij}$
2) δ_{ii}
3) $x_i\,y_j\,\delta_{ij}-x_q\,y_p\,\delta_{qp}$
4) $C_{ijk}\,\delta_{ip}=x_p\,u_i\,v_i$
5) $\delta_{ii}\,\delta_{jj}-\delta_{ij}\,\delta_{ij}$

Problem 4. Given the components of tensor $\underline{\underline{\mathbf{A}}}$ in an orthogonal Cartesian frame **[o; $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$]** as

$$\left[A_{ij}\right]=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 6\\ 3 & 6 & 1\end{bmatrix}$$

1) Find the eigenvalues λ_i (i =1,2,3) (suppose $\lambda_1>\lambda_2>\lambda_3$)
2) Find the corresponding principal directions (eigenvectors) $\mathbf{v}^1$, $\mathbf{v}^2$, $\mathbf{v}^3$
3) Demonstrate that $\mathbf{v}^1$, $\mathbf{v}^2$, $\mathbf{v}^3$ are mutually orthogonal
4) Find the invariants in terms of λ_1, λ_2, λ_3 (Principle Frame).