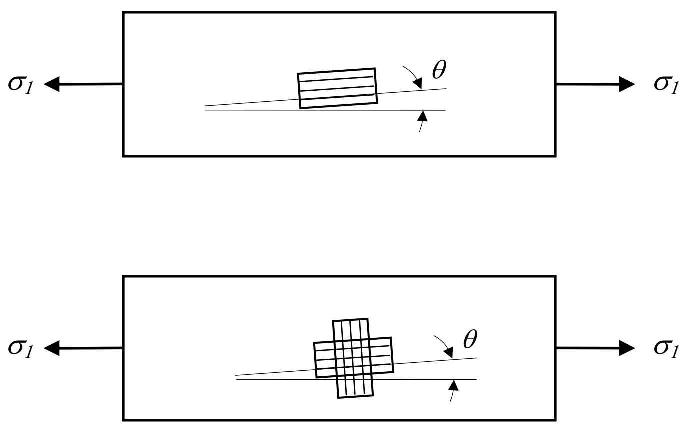
HOMEWORK #5

MMAE 530 Advanced Mechanics of Solids
Fall 2011
Prof. M. Vural
(Due ???)

Problem 1. A student is trying to make a uniaxial compression test using a cylindrical specimen. She glues a strain gage on the surface of specimen along the axial direction to measure axial STRESS. However, she realizes that it is very difficult to glue the strain gage exactly in axial direction and that the strain gage is always misoriented by an angle θ with respect to the axial direction. She asks for help from the lab technician, and he recommends her to use "Tee" gage (Tee gage is composed of two perfectly orthogonal strain gages, and can measure strains in two mutually perpendicular directions as shown below). How can she calculate the axial stress (σ_1) by using the strain measurements from the Tee gage? Please help her.



Initial configuration

θ is unknown misorientation angle.

Recommendation of lab technician

Problem 2. For an isotropic, linearly elastic solid, derive the relation

$$\mu = \frac{E}{2(1+\nu)}$$

using only the stress-strain relation in terms of E, μ, ν (without recourse to the Lame constant λ) and the transformation of coordinate axes.

Problem 3. Consider the following stress-strain relations for an isotropic linear elastic solid:

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij}, \quad \text{where } \lambda \text{ and } \mu \text{ are } \textit{Lame's moduli} \quad \ldots\ldots\ldots (*)$$

(a) Using (*), write the expressions for $\sigma_{11}, \sigma_{22}, \sigma_{12}, \sigma_{13}$ without using the summation convention. What are the units of λ and μ?

(b) Show that for the state of uniaxial stress, we have $\sigma_{11} = E\varepsilon_{11}$ and $\varepsilon_{22} = \varepsilon_{33} = -\nu\, \varepsilon_{11}$ where $E = \dfrac{\mu(3\lambda + 2\mu)}{\lambda + \mu}$ and $\nu = \dfrac{\lambda}{2(\lambda + \mu)}$. E and μ are called the *Young's modulus* and the *Poisson's ratio*, respectively. What are the units of E and μ? Sketch a circular cylindrical body that is homogeneously deformed in the uniaxial stress state with a tensile stress in the direction of the axis of the cylinder. Interpret $\varepsilon_{11}, \varepsilon_{22}$ and the Poisson's ratio in terms of the undeformed and deformed length and radius of the cylinder.

Problem 4. For a linear elastic isotropic solid the constitutive behavior can be expressed with

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij}$$

or alternatively with

$$\sigma_{ij} = C_{ijkl} \varepsilon_{kl}$$

Show that

$$C_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$

Problem 5. Deviatoric stress (S_{ij}) and deviatoric strain (e_{ij}) are defined as in the following

$$S_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij} = \sigma_{ij} + p\delta_{ij}$$

$$e_{ij} = \varepsilon_{ij} - \frac{1}{3}\varepsilon_{kk}\delta_{ij} = \varepsilon_{ij} - \frac{d}{3}\delta_{ij}$$

where $p = -\frac{1}{3}\sigma_{kk}$ is hydrostatic pressure and $d = \varepsilon_{kk}$ is volumetric strain (dilation). Show that, for a linear elastic and isotropic solid, the constitutive behavior can alternatively be expressed with

$$S_{ij} = 2\mu e_{ij} \quad \text{and} \quad p = -K\, d$$

Comment on this alternative expression of constitutive response.