MAKEUP EXAM

MMAE 530 Advanced Mechanics of Solids

November 10, 2011

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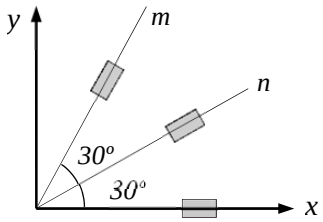
Problem 1. (35 pts) In order to monitor stresses on a critical beam member in a building, a strain gage rosette as shown below is glued onto the surface of the beam. The three strain gages in the rosette, shown as small rectangular boxes, measure extensional strains along their long axes (*x*, *n*, *m*). Designating these strains as $\varepsilon_o = 10^{-3}$, $\varepsilon_{30} = 2\text{x}10^{-3}$ and $\varepsilon_{60} = 3\text{x}10^{-3}$, respectively, find the state of strain in *xy* frame.



(Hint: use $c = cos30 = \sqrt{3}/2;\ s = sin30 = 1/2$; and don't use calculator until the end)

Problem 2. (25 pts) Let $\underline{\nu}$ and $\underline{\mu}$ be the unit vectors and $\underline{t}^{(\underline{\nu})}$ and $\underline{t}^{(\underline{\mu})}$ the surface tractions transmitted at the typical point *P* across the surface elements that are normal to these vectors.

(a) Prove that $\underline{\mu} \cdot \underline{t}^{(\underline{\nu})} = \underline{\nu} \cdot \underline{t}^{(\underline{\mu})}$, and [15 pts]

(b) Comment on the result using a simple sketch. [10 pts]

Problem 3. (40 pts) In a rectangular Cartesian coordinate system **[o; $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$]**, consider the rigid body rotation of a unit cube given by

$$x_1' = x_1 \cos\theta + x_2 \sin\theta$$

$$x_2' = -x_1 \sin\theta + x_2 \cos\theta$$

$$x_3' = x_3$$

where θ is a given angle, and $\underline{x}'$, $\underline{x}$ are the position vectors in the current and reference configurations, respectively.

(a) Find the matrices of deformation gradient tensor $\underline{\underline{F}}$ and the Lagrangian strain tensor $\underline{\underline{E}}$. [10 pts]

(b) Find the displacements associated with this deformation and the matrix of infinitesimal strain tensor $\underline{\underline{\varepsilon}}$ without any additional assumptions about the angle θ. [10 pts]

(c) Use $\underline{\underline{\varepsilon}}$ to find the extensional (or normal) strain in the direction $\mathbf{e}_1$. Find the angle θ for which this strain has a magnitude smaller than 0.01 (i.e., 1% strain). [10 pts]

(d) Comment on the results. [10 pts]

Good Luck,