

MMAE 536 HW#1 SOLUTIONS

Problem 1 Given the identity

$$\epsilon_{ijk} \epsilon_{pqr} = \begin{bmatrix} \delta_{ip} & \delta_{iq} & \delta_{in} \\ \delta_{jp} & \delta_{jq} & \delta_{jn} \\ \delta_{kp} & \delta_{kq} & \delta_{kn} \end{bmatrix} \dots \dots \dots (1)$$

Show that

(i) $\epsilon_{ijn} \epsilon_{ken} = \delta_{ik} \delta_{je} - \delta_{ie} \delta_{jk}$

PROOF:

$$\begin{aligned} \epsilon_{ijn} \epsilon_{ken} &= \begin{vmatrix} \delta_{ik} & \delta_{ie} & \delta_{in} \\ \delta_{jk} & \delta_{je} & \delta_{jn} \\ \delta_{nk} & \delta_{ne} & \delta_{nn} \end{vmatrix} = \delta_{ik} \delta_{je} \cdot 3 + \delta_{ie} \delta_{jn} \delta_{nk} + \delta_{in} \delta_{jk} \delta_{ne} \\ &\quad - \delta_{in} \delta_{je} \delta_{nk} - \delta_{ie} \delta_{jk} \cdot 3 - \delta_{ik} \delta_{jn} \delta_{ne} \\ &= \underline{3 \delta_{ik} \delta_{je}} + \delta_{ie} \delta_{jk} + \delta_{ie} \delta_{jk} - \delta_{ik} \delta_{je} - \underline{3 \delta_{ie} \delta_{jk}} - \delta_{ik} \delta_{je} \\ &= \underline{\delta_{ik} \delta_{je} - \delta_{ie} \delta_{jk}} \dots \dots \dots (2) \end{aligned}$$

(ii) $\epsilon_{ijn} \epsilon_{kjn} = 2 \delta_{ik}$

PROOF:

Starting from (2) $\Rightarrow \epsilon_{ijn} \epsilon_{ken} = \delta_{ik} \delta_{je} - \delta_{ie} \delta_{jk}$
 let $l=j$ here, then

$$\begin{aligned} \epsilon_{ijn} \epsilon_{kjn} &= \delta_{ik} \delta_{jj} - \delta_{ij} \delta_{jk} \\ &= 3 \delta_{ik} - \delta_{ik} \\ &= \underline{2 \delta_{ik}} \dots \dots \dots (3) \end{aligned}$$

(iii) $\epsilon_{ijk} \epsilon_{ijk} = 6$

PROOF

let $k=i$ in (3) $\Rightarrow \epsilon_{ijn} \epsilon_{ijn} = 2 \delta_{ik} = 2 \times 3$

$$= \underline{\underline{6}}$$